Hydrodynamics and heat and mass transfer during formation of thin metal glass tape

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Abstract. The mathematical model of hydrodynamic and heat transfer processes in a system, which rotates a disk, a metal tape, an environment, is developed. The adequacy of both the mathematical model and the computing algorithm is confirmed by comparison the computing results with the laboratory experiment. It has been shown that increase of the disc's rotational speed reduces the thickness of the amorphous metal tape.

1. Introduction

In production of the amorphous metals, while providing a high melt cooling rate, which results in formation of the glassy state, we cannot completely avoid the influence of the crystallization process on an amorphous material. The glassy state is metastable with respect to the crystalline one and converts into it at various thermal processes. In this regard, one of the central places in the physics of amorphous materials is taken by the study of the melt crystallization processes at high cooling rate. Hydrodynamic processes play a rather significant role in this case. Despite of the short duration of tape production and its small geometric dimensions, hydrodynamic and heat transfer processes significantly affect on the uniformity of the amorphous tape thickness.

2. Problem setting and methods of its implementation

An alloy $Fe_{80}B_{20}$ ($m = 25\,g$) is supplied to a rotating copper disk ($R = 0.3\,m$, $w_0 = 3000\,\text{rpm}$). The disk surface temperature is maintained constant [1]. The disk-melt system is in the helium atmosphere, and therefore there is no heat removal from the top and side borders.

The mathematical model of the process is described by the equations of heat transfer, motion and continuity and by the boundary conditions.

It is deemed that at these cooling rates a high degree of melt supercooling, about 150°C below the solidus temperature, is observed. Due to this, the position of the solid crust is determined by isotherm [2].

The model has the following assumptions:

• the system is two-dimensional;

• metal properties do not depend on temperature (except for the viscosity);

• melt flow is laminar.

The latter statement is justified by the fact that the Reynolds criterion $\text{Re} = 2100$, which is far below the turbulence threshold [3]

Thermophysical properties of the melt are as follows: $T_l = 1173\,K$ –melting point; $\lambda = 71\ W/(m \cdot K)$ – thermal conductivity coefficient; $T_l + 50 \div 100\,K$ – casting temperature; $a = 0.5 \cdot 10^{-5}\ m^2/s$ – thermal diffusivity coefficient; $\nu = 0.2 \cdot 10^{-5}\ m^2/s$ - kinematic viscosity coefficient.

Thermophysical parameters of the substrate are as follows: $c = 544\,J/(kg \cdot K)$ - specific thermal capacity; $\alpha = 2 \cdot 10^5\ W/(m^2 \cdot K)$ - heat transfer coefficient; $\lambda = 20\,W/(m \cdot K)$ - thermal conductivity coefficient; $\rho = 7500\,kg/m^3$ - density.

The mathematical model for solving this problem can be constructed from the equations where velocity and pressure are the unknowns. However, it is more convenient to introduce the other variables: stream function ψ and vorticity ω (Helmholtz variables) that are associated with the speed by vector relations:

$$V_1 = \frac{\partial \psi}{\partial y}; \quad V_2 = -\frac{\partial \psi}{\partial x}; \quad \vec{\omega} = \nabla \vec{V}. \tag{1}$$

In this case continuity equation is satisfied automatically and certain arbitrary rule is allowed in selecting stream function.

Navier–Stokes equation in Boussinesq approximation, continuity and heat transfer equations will be written as follows [4]:

$$\frac{\partial V_1}{\partial t} + \frac{\partial V_1^2}{\partial x} + \frac{\partial V_1 V_2}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial \nu}{\partial x}\frac{\partial V_1}{\partial x} + \frac{\partial \nu}{\partial y}\frac{\partial V_1}{\partial y}; \tag{2}$$

$$\frac{\partial V_2}{\partial t} + \frac{\partial V_1 V_2}{\partial x} + \frac{\partial V_2^2}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\partial \nu}{\partial x}\frac{\partial V_2}{\partial x} + \frac{\partial \nu}{\partial y}\frac{\partial V_2}{\partial y} + g\beta(T - T_0); \tag{3}$$

$$\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} = 0; \tag{4}$$

$$c\rho\left[\frac{\partial T}{\partial t} + \frac{\partial V_1 T}{\partial x} + \frac{\partial V_2 T}{\partial y}\right] = \frac{\partial \lambda}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial \lambda}{\partial y}\frac{\partial T}{\partial y} + \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right); \tag{5}$$

where p – pressure, c – melt heat capacity, ρ – melt density, g – gravity acceleration, β – volumetric expansion coefficient, λ – thermal conductivity of the medium, T_0 – initial temperature. Let us evaluate contribution of thermal convection into the process of amorphous tape formation using Richardson criterion [5] ($Ri = Gr/\mathrm{Re}^2$): $Ri = 0.000722$, which indicates a substantial predominance of dynamic forces over the buoyancy forces. Hence, volumetric forces can be neglected in the equation (3).

Equations (2), (3) and (5) may be written in the criterial form in the variables of vortex (ω) – stream function (ψ) [4]:

$$\frac{\partial \omega}{\partial Fo} + \frac{\partial V_1 \omega}{\partial x} + \frac{\partial V_2 \omega}{\partial y} = \frac{\partial \Pr(\nu)}{\partial x}\frac{\partial \omega}{\partial x} + \frac{\partial \Pr(\nu)}{\partial y}\frac{\partial \omega}{\partial y}; \tag{6}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega; \tag{7}$$

$$\frac{\partial \theta}{\partial Fo} + \frac{\partial V_1 \theta}{\partial x} + \frac{\partial V_2 \theta}{\partial y} = \frac{\partial a}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial a}{\partial y}\frac{\partial \theta}{\partial y} + a\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right); \tag{8}$$

where Fo, $\Pr$, Gr – Fourier, Prandtl and Grashof numbers [3], $\theta = T/T_0$, T – current temperature, a – thermal diffusivity coefficient.

Initial conditions:

$$Fo = 0: \quad \psi = 0; \quad \omega = 0; \quad \theta = 1.$$

Boundary conditions:

$$y = 0: \quad \frac{\partial \psi}{\partial y} = 0; \quad \psi = 0; \quad \frac{\partial \theta}{\partial y} = 0;$$

$$y = L_y: \quad \frac{\partial \psi}{\partial y} = V_m; \quad \psi = 0; \quad \theta = \theta_{disk};$$

$$x = 0: \quad \psi = 0; \quad \frac{\partial \psi}{\partial y} = 0; \quad \frac{\partial \theta}{\partial x} = 0;$$

$$x = L_x: \quad \frac{\partial^2 \psi}{\partial y^2} = 0; \quad \frac{\partial \psi}{\partial y} = 0; \quad \frac{\partial \theta}{\partial x} = 0.$$

The short duration of the process ($0.001\,s$), high temperature (up to $1600^0 C$) and metal aggressiveness do not allow to solve the problem by means of the experiment. The problems of this class can only be solved by numerical methods, in particular using finite differences [4]. The essence of this method lies in the fact that the region of the continuous variation of the argument is replaced by a finite set of points (nodes) forming the spatial-temporal finite-difference grid. In order to implement numerically the mathematical model of the processes, it is necessary to formulate a computational algorithm which is a logical sequence of operations in which the solution of the problem can be found. Proceeding from the requirements of solution accuracy and stability, as well as machine time efficiency, the method of alternating directions was chosen.

The algorithm for the software product is as follows: using the input data (the boundary conditions), vorticity and stream function are defined on the base of the solution of Navier-Stokes and continuity equations. When stream function field is known, it is possible to calculate the distribution of the velocity components.

Temperature distribution on the tape surface is determined using the velocity field. The process is repeated at a new time step. Graphical output of fields of stream functions, velocity and temperature at different times is used for easing the analysis of the obtained data.

3. Research results

In [6] the authors consider the classical representations of the stresses that arisen in the process of tape formation. They suggested that the non- solidified section of the volume will separate from the solidified one at stresses making approximately 10^8 Pa. In this case the horizontal component of velocity and viscosity are determining parameters. Since stress is defined as

$$\tau = \eta \left| \frac{\partial V_1}{\partial y} \right|, \tag{9}$$

an assumption can be made that separation will be observed at the point where product of viscosity by the derivative of horizontal velocity is maximized.

As Figure 1 shows, the tape layers, adjacent to the disc, have the values of the horizontal component of velocity close to the linear velocity of disc rotation. At the point of separation the velocity value sharply decreases, and in some cases it has nay an opposite sign. It happens at the point of a sharp velocity variation that there exists the boundary of tape solidification, as we have assumed.

Varying the disc rotation velocity leads to changes in the tape thickness. To confirm this, several options of linear rotational velocity were calculated on the disk surface:

1) $U = 70.7\ m/s$; 2) $U = 47.1\ m/s$; 3) $U = 23.5\ m/s$; 4) $U = 11.8\ m/s$.

As a consequence, the solidification velocity decreases. This result agrees qualitatively with the experimental studies conducted in the Donetsk Physico-Technical Institute of the National Academy of Sciences of Ukraine. The error amounted to 16%. Figure 1 shows that the change of the tape thickness is linear to the velocity value of 47.1 *m/s;* at higher velocities, the curve monotonically gets broken.

Figure 2 shows the effect of the disk rotation velocity on the distribution of the horizontal component of velocity along the thickness tape.

As can be seen, under the influence of the viscous forces, the velocity decreases along the tape thickness, while at the outermost sections of the substrate the reverse motion is observed. Moreover, the larger the disk rotation velocity becomes, the higher reverse motion speed is. In turn, such a velocity distribution affects on the stresses in the tape (Fig. 3), and therefore, on the possibility for the non-solidified portion of tape volume to be separated. The higher the disk rotation velocity becomes, the greater the likelihood of separation is.

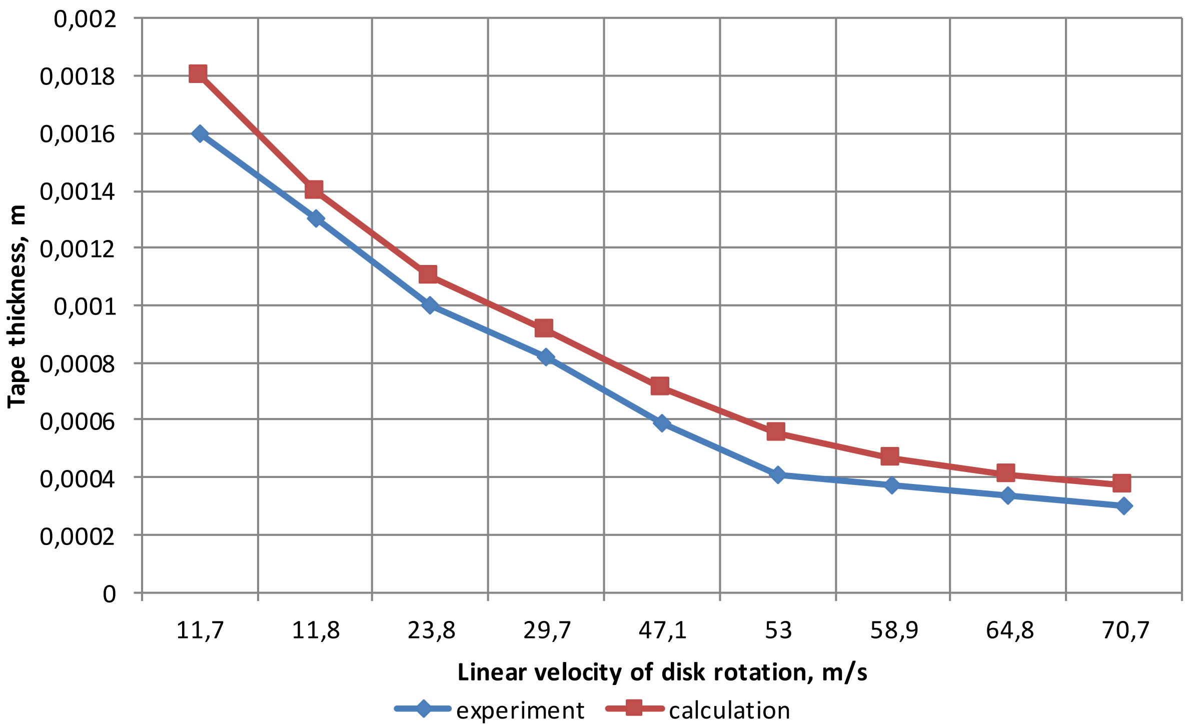
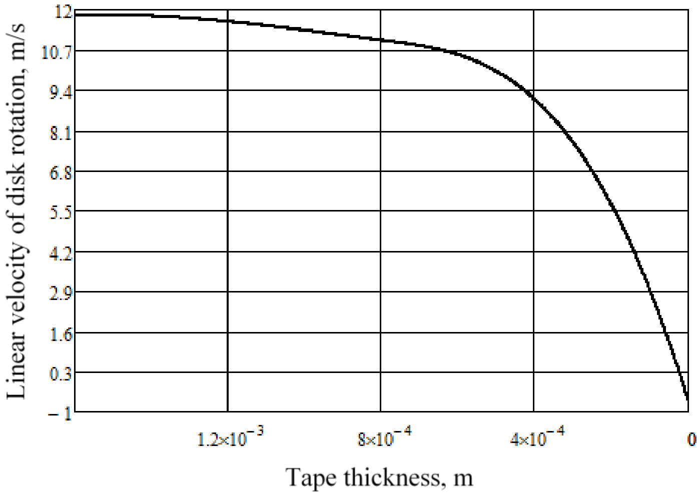
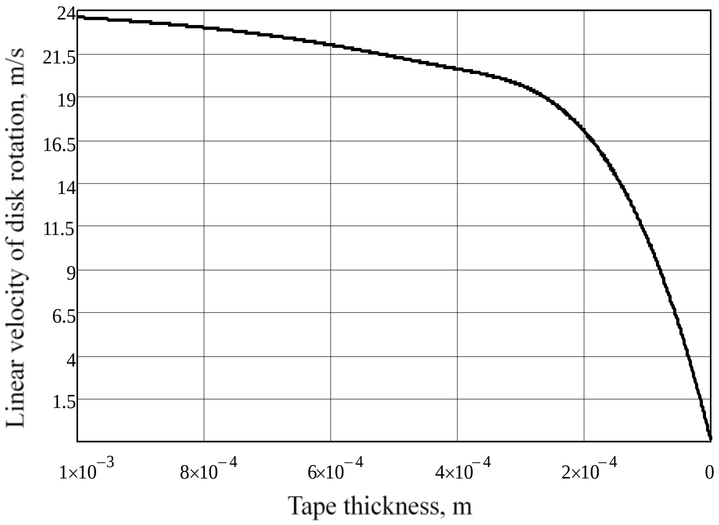


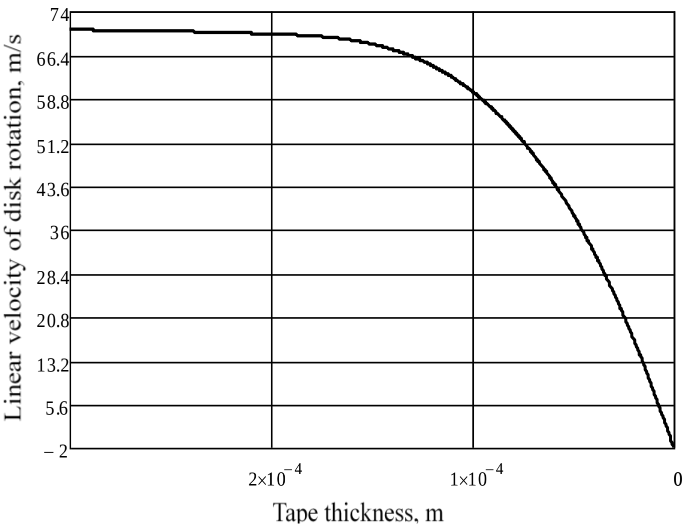
Figure 1. Amorphous tape thickness vs. rotational velocity.



a)



b)



c)

Figure 2. Horizontal component of the velocity distributed along the tape thickness with linear rotational velocity: a) $11.8\ m/s$; б) $23.5\ m/s$; в) $70.7\ m/s$.

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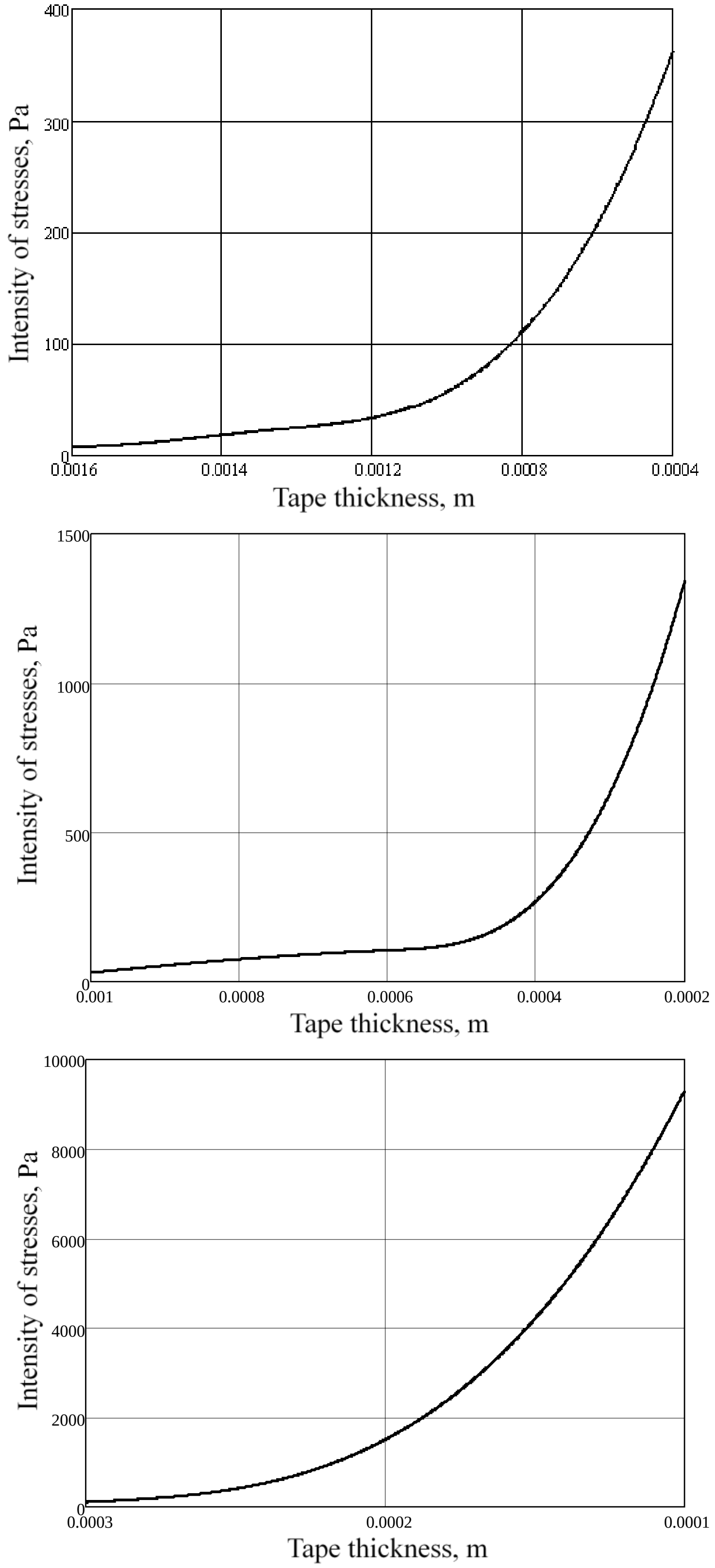

a)

b)

c)

Figure 3. Stresses distributed along the tape thickness with linear rotational velocity: а) $11.8\,m/s$; б) $23.5\,m/s$; в) $70.7\,m/s$.

4. Conclusions

1. The mathematical model and computational algorithm for calculating the hydrodynamic and heat transfer processes have been developed for the amorphous tape production.
2. The software has been developed which is capable to calculate the fields of temperature, velocity and stresses, as well as to visualize the distribution of velocity, stream function and voltage components on the display at any given time.
3. Increase in the disc rotation velocity results in decreased thickness of the solidified tape.
4. Comparison of the results of numerical experiments with the laboratory research described in [6] confirms the adequacy both of the developed mathematical model and of the computational algorithm.

5. References

[1] Naberezhnye V P, Tkach V I et al. 1987 *The analysis of metallic glasses of the crystallization process and* $Fe_{80}B_{20}$ $Fe_{40}Ni_{40}F_{14}B_6$ *in non-isothermal conditions* (Donetsk: Preprint)
[2] Nemoshkalenko V V, Romanov A V, et al. 1987 *Amorphous metal alloys* (Kiev: Naukova Dumka)
[3] Bilousov V V and Bologna N I 2003 *Fundamentals of heat and mass transfer and thermal physics of closed volumes* (Donetsk: Southeast)
[4] Bilousov V V, Nedopekin F V, Hrychikov V E, Leibenzon V A, et al. 2009 *Solidification of metals and metal compositions* (Kiev: Naukova Dumka)
[5] Bilousov V V, Babanin A Y, Belov B F and Pavlov Y V 2015 *Processing of iron-carbon melts highly active agents* (Donetsk: East Publishing House)
[6] Kester W and Gerold W 1983 *Crystallization of metallic glasses Met. Art.* (Moscow: Mir) 325-367
[7] Yavari A R, Hicter P and Desre P 1981 The effect of quenching stresses on the kinetics of crystallization and structure of metallic glasses *Proc. 4th Conf. on Rapidly Quenched Metals* 713-716