

# Numerical model of round rod in flat triangle pass in three-roll tandem rolling process based on mathematical analysis

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**Abstract.** In pass design of three-roll rolling process, accurate numerical model in deformation region is very important. Thus, the numerical model of round rod in flat triangle pass in three-roll tandem rolling process was proposed by mathematics analytic method in this paper. The parameters of numerical model, including arc length of the contact field, width of the contact field, rolling reduction(maximum, point) and rolling piece dimension at the exit, were established based on mathematical analysis.

## 1. Introduction

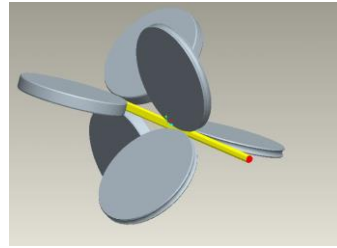
Three-roll mill is also called as Y-type mill, which is consisted of three dish-shape rollers, one of which is inclined to the other at 120 degree. Three-roll mill stands compactly, and temperature decrease is lower, which is benefit to control the rolling temperature [1]. The deformation in three-roll pass is equal, and spread is lower, which is less than 5%. In addition, the restriction of rolling piece is imposed from three directions by three rollers, which is propitious to low plastic, hard-deformed materials, such as titanium alloy, tungsten.

Conventional, deformation region in three-roll tandem rolling process was transferred to that of two-roll system, and mathematics model for pass design was the formulas written for two-roll system, which led to low-precision. Few studies for mathematics models have been carried out in three-roll sequence pass. Cao xifa proposed contact width in flat-triangle pass [2], J.H.Min, H.C.Kwon, Y.Lee, etc. proposed to use the mathematics model of two-roll in three-roll pass system [3-4].

In this study, considering that the spread of three-roll system is 1/3 of two-roll system, which was less than 5%, spread was ignored. The mathematics model was proposed by mathematics analytic method, including include contact arc length, contact width, contact area, rolling reduction(point, maximum), and cross-section area.

In this paper, formula derivation of mathematics model made the following assumptions: (i) ignore the influence of roller elastic flattening and rolled piece elastic deformation, (ii) ignore spread. And round rod in flat-round triangle sequence pass system was shown in Fig. 1.

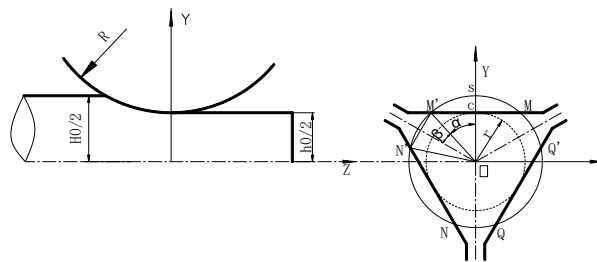




**Figure 1.** Round rod in flat-round triangle sequence pass system

## 2. Mathematics model

The parameters of round rod in flat triangle pass were shown in Fig. 2, The key parameters of round rod in flat triangle pass were shown in table 1.



**Figure 2.** Geometric model of round rod in flat triangle pass

**Table 1.** Rolling parameters of round rod in flat triangle pass

Parameter	Value
Round rod diameter	$H_0$
Roller radius	$R$
Flat triangle Pass inscribed circle radius	$r$

Ignoring spread, the contact points is M and M', included angle between OM' and coordinate axis y is  $\alpha$ .

Flat triangle pass curved surface equation in coordinate system XOY is calculated as Eq. 1:

$$\sqrt{[y - (R + r)]^2 + z^2} = R \quad (1)$$

Round rod curved surface equation in coordinate system XOY is shown as Eq. 2:

$$x^2 + y^2 = \left(\frac{H_0}{2}\right)^2 \quad (2)$$

### 2.1. Contact arc length

Ignoring spread in three-roll pass, the intersection curve between round rod curved surface and flat triangle pass curved surface can be calculated by solving the Eq.1 and Eq.2 simultaneously, as Eq. 3.

$$\begin{cases} \sqrt{[y - (R + r)]^2 + z^2} = R \\ x^2 + y^2 = \left(\frac{H_0}{2}\right)^2 \end{cases} \quad (3)$$

Eq. 3 represents the curved boundary of deformation region of rolling piece. Overhead view curve from Z-axis of deformation region can be determined by setting Eq. 3 plane projection to x-z plane of

eliminating variable  $y$ , as shown in Eq. 4. And then set Eq. 4 line projection to  $z$ -axis of eliminating variable  $x$ . Eq. 5 is the contact arc length in deformation region by definition of the contact arc length [4-5].

$$[\sqrt{(\frac{H_0}{2})^2 - x^2} - (R+r)]^2 + z^2 = R^2 \quad (4)$$

Contact arc length:

$$l = z = \sqrt{R^2 - [\frac{H_0}{2} - (R+r)]^2} \quad (5)$$

### 2.2. Contact arc width

As shown in Fig. 2, line  $MM'$  represents the contact width between rolling piece and roller.  $MM'$  is the intersection line between round rod curved surface and flat triangle pass curved surface of front view in  $x$ - $y$  plane of CS  $XOY$ .

So, the positions of  $M$ ,  $M'$  in coordinate system  $xoy$  are show as Eq. 6 and Eq. 7.

$$x_m = \sqrt{(\frac{H_0}{2})^2 - r^2}, y_m = r \quad (6)$$

$$x_{m'} = -\sqrt{(\frac{H_0}{2})^2 - r^2}, y_{m'} = r \quad (7)$$

Therefore, contact width  $MM'$  can be determined as Eq. 8.

$$k = 2 \times \sqrt{(\frac{H_0}{2})^2 - r^2} \quad (8)$$

### 2.3. Rolling reduction (point, maximum)

As shown in Fig. 2, line  $SC$  represents the maximum rolling reduction in deformation region, as shown in Eq. 9. And the reduction for every point in arc  $MM'$  can be calculated by Eq. 10.

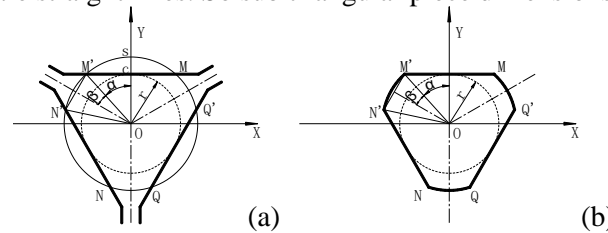
$$\Delta h_{\max} = \frac{H_0}{2} - r \quad (9)$$

$$\Delta h_x = \sqrt{(\frac{H_0}{2})^2 - x^2} - r \quad (10)$$

Where,  $x$  represents the  $x$ -position in arc  $MM'$

### 2.4. Rolling piece dimension at the exit

Round rod before rolled in flat triangle pass was shown in Fig. 3 (a). The shape of round rod at the exit of flat triangle pass has been reduced to sub triangular piece as Fig. 3 (b), ignoring spread.  $MM'N'NQQ'$  represents the profile of rolling piece rolled in flat triangle pass.  $M'N'$ ,  $NQ$ ,  $Q'M$  are arc, and  $M'M$ ,  $N'N$ ,  $Q'Q$  are straight lines. So sub triangular piece dimensions are show in Table 2.



**Figure 3.** a) Round rod before rolled in flat triangle pass; b) Sub-triangular piece exiting flat triangle pass

**Table 2.** Rolling piece dimension for arc M'N', NQ, Q'M

Parameter	Center of arc	Radius of arc	Included angle
M'N', NQ, Q'M	(0,0)	$\frac{H_0}{2}$	$2\beta = 2 \times [60^\circ - \arccos(\frac{2 \times r}{H_0})]$

### 2.5. Cross-section area and contact area

As shown in Fig. 2, are  $M'SMM'$  are presents maximum cross-section area in deformation region,

$S_h = \int_{x_m}^{x_m} \Delta h dx = 2 \int_0^{x_m} \Delta h dx$ . Combining with Eq. 10, the mathematics model of  $S_h$  can be considered as Eq. 11.

$$S_h = 2 \times [\frac{x_m}{2} \times \sqrt{(\frac{H_0}{2})^2 - x_m^2} + \frac{(\frac{H_0}{2})^2}{2} \times \arcsin(\frac{2 \times x_m}{H_0}) - r \times x_m] \quad (11)$$

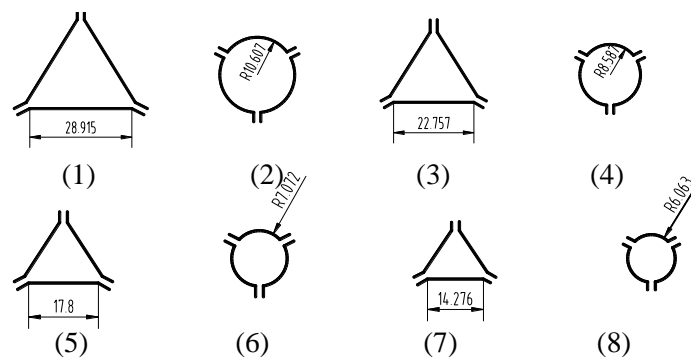
Mean rolling reduction in deformation region can be considered as  $\bar{\Delta h} = \frac{S_h}{k}$ , Where,  $k$  represents contact width and  $S_h$  represents cross-section area

As shown in Fig. 1, the geometric shape of deformation region can be reduced to rectangle. Therefore, contact area  $S_j$  can be calculated by multiplying the contact arc length  $l$  by contact width  $k$ , as shown in Eq.12.

$$S_j = 2 \times \sqrt{R^2 - [\frac{H_0}{2} - (R+r)]^2} \times \sqrt{(\frac{H_0}{2})^2 - r^2} \quad (12)$$

### 3. Application of mathematics model in deformation region

In this paper, mathematics model of flat triangle-round triangle sequence pass system in three-roll tandem rolling process has been proposed, which has solved the pass design of three-roll tandem rolling process. In the case of the pass system for  $\Phi 25 \sim \Phi 12$  titanium alloy rod(TC4)product line, the parameters in deformation region was calculated with mathematics model. Titanium alloy rod (TC4)  $\Phi 25$  was rolled to rod  $\Phi 12$ , and the eight-passes flat triangle-round triangle sequence pass system was shown in Fig.4. The entrance velocity of the flat triangle-round triangle sequence pass system was 1m/s, initial rolling temperature of rolling piece was 950°C.

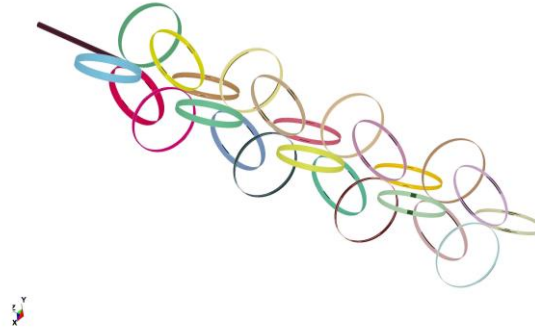


**Figure 4.**  $\Phi 25 \sim \Phi 12$  titanium alloy rod (TC4) eight-passes sequence pass system

#### 3.1. Finite element analysis

Elastic-plastic finite element method was used in metal pressure process to predict shape and parameters during deforming process. And the result of FEM simulation can get high precision, and fits the practice results. Therefore, to prove the accuracy of mathematics model derivated in this paper,

the eight-pass flat triangle-round triangle sequence rolling process of  $\phi 25 \sim 12$  titanium alloy rod(TC4)product line was simulated with elastic-plastic finite element software, as shown in Fig. 5 [6-7].



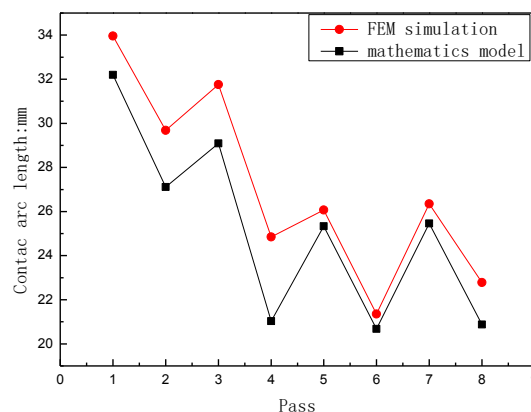
**Figure 5.** The finite element model of  $\Phi 25 \sim \Phi 12$  titanium alloy rod tandem rolling

### 3.2. Comparison of contact arc length between FEM simulation and mathematics model

As is shown in Table 3, error of contact arc length between FEM simulation and mathematics model was below 10%, and the contact arc length of mathematics model was smaller, as the result of ignoring the spread.

**Table 3.** Comparison between FEM simulation and mathematics model

Pass	contact arc length with simulation(mm)	contact arc length with Mathematics model(mm)	Error (%)
1	33.962	32.193	5.20
2	29.684	27.10919	8.67
3	31.755	29.08512	8.40
4	22.846	21.03437	7.92
5	26.073	25.33204	2.84
6	21.355	20.677	3.17
7	26.345	25.45635	3.373
8	22.786	20.87822	8.37



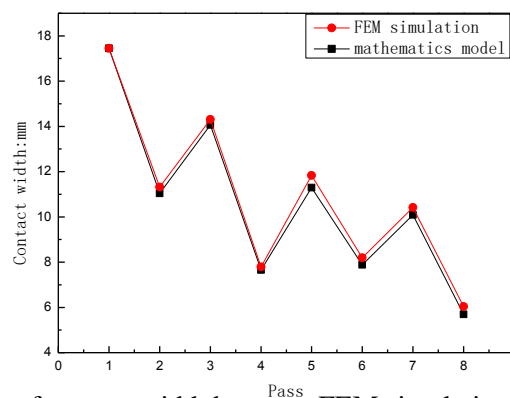
**Figure 6.** Comparison of contact arc length between FEM simulation and mathematics model

### 3.3. Comparison of contact width between FEM simulation and mathematics model

As is shown in Table 4, error of contact width between FEM simulation and mathematics model was mainly below 5%, and the contact width of mathematics model was smaller, as the result of ignoring the spread.

**Table 4.** Comparison between FEM simulation and mathematics model

Pass	contact width with simulation (mm)	contact width Mathematics model(mm)	Error(%)
1	17.95	17.4525	2.77
2	11.318	11.041	2.44
3	14.31	14.065	1.71
4	7.791	7.6535	1.76
5	11.836	11.29422	4.57
6	8.192	7.886542	3.72
7	10.417	10.08894	3.14
8	6.035	5.691089	5.69



**Figure 7.** Comparison of contact width between FEM simulation and mathematics model

## 4. Summary

Mathematics model of round rod in flat triangle pass in three-roll rolling process was proposed in this paper, including arc length of the contact field, width of the contact field, rolling reduction(maximum, point) and rolling piece dimension at the exit. Therefore, Mathematics model of other pass system, such as flat triangle-spherical triangle pass system, spherical triangle-round triangle, can also be derivated by mathematics analytic method.

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