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System Identification and Control Design of an Unmanned Helicopter Using a PI-MPC Controller

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Abstract. This paper presents the study of the system identification and controller design for an unmanned helicopter using the integration of Proportional Integral (PI) and Model Predictive Control (MPC). Since the dynamic model of a helicopter is highly nonlinear and contains many uncertainties, the system identification and control are challenging and complicated. To accelerate the development, the autonomous flight and trajectory tracking of an unmanned helicopter, this study first setup a software simulation environment of the helicopter using the X-Plane flight simulator. The prediction-error minimization (PEM) and subspace methods were applied in this study to identify the dynamic model of the interested flight trim conditions. The lateral, longitudinal, heave, and yaw dynamic models were predicted by using the System Identification Toolbox of MATLAB. To enhance the stability and eliminate the uncertainty of the control system, the Integration of Proportional Integral (PI) and MPC were introduced. The developed control system was then applied to perform the trajectory tracking of a helicopter. The simulation results show that the performance of the proposed approach can track the desired trajectory.

1. Introduction

Comparing with fixed-wing aircrafts, helicopters have many advantages since they are a type of rotorcraft in which lift and thrust are supplied by rotors. Therefore, helicopters have the capability to take off and land vertically, to hover, and to fly forward, backward, and laterally. With the capability of high mobility, unmanned helicopters are applied to many applications such as rescue, transportation, and agriculture. However, it is well-known that the dynamic model of a helicopter is complex and highly nonlinear. The modeling and control of a helicopter are very complicated and need many efforts.

Before designing a model-based controller, it is required to study the system dynamics. The first step is to determine the system model. Many system identification methods have been investigated in [1]. Several identification methods and software packages are used for helicopter model estimation such as CIFER® (Comprehensive Identification from Frequency Responses) [2], System Identification Programs for Aircraft (SIDPAC) [3], extended and unscented Kalman filter, neural networks, and so on. These techniques often are applied to large systems. Moreover, there are also some popular system identification methods used to determine the helicopter model such as PEM method and subspace method [4]. These methods are available in the system identification toolbox of MATLAB. As mentioned above, the aerodynamic characteristics of helicopters are highly nonlinear. There are forty unknown parameters of the helicopter dynamic model used in [5]. It is difficult to determine all the parameters at the same time. The helicopter model can be divided into sub-models, which are lateral, longitudinal, heave, and yaw dynamic models [5]. System identification of these sub-models can be done by using system identification toolbox. After getting the dynamic model of the helicopter, the model-based controller can be designed and implemented. Many control methods have been adopted to perform flight control of helicopters, such as MPC, PID, Linear-Quadratic Regulator (LQR), Linear-Quadratic-Gaussian (LQG), fuzzy logic, and neural networks. In this paper, a PI-MPC controller was proposed. MPC has been applied to many systems with highly complex, multivariable processes and real-time applications [6,7]. In this study, MPC was integrated with PI to enhance the performance of the flight control system for the unmanned helicopter. The used control scheme consists of the inner and outer loops. PI was used in the outer-loop to improve the response time, and MPC was used in the inner loop to increase the robustness of flight control. In this study, four PI-MPC controllers were applied to the sub-models.

The rest of this paper is organized as follows. Dynamic model, system identification, flight control design for the helicopter presented introduced in Section 2, 3, and 4, respectively. Results and discussions are described in section 5. Finally, conclusions are presented in Section 6.

2. Dynamic model of the small-scale helicopter

The linearized model of a helicopter has the following form:

$$\dot{x} = Ax + Bu \quad (1)$$

$$Y = Cx \quad (2)$$

where x is the state vector, which contains $[u\ v\ p\ q\ \phi\ \theta\ a\ b\ w\ r\ r_{fb}]^T$, and u, v, and w are the longitudinal, lateral, and vertical velocities, respectively. p, q, and r are roll, pitch and yaw angular rates, respectively. a and b are longitudinal and lateral flapping angles, respectively. ϕ and θ are roll and pitch angles, respectively. u is the control input, $[\sigma_{lat}\ \sigma_{lon}\ \sigma_{col}\ \sigma_{ped}]^T$, where σ_{lat} , σ_{lon} , σ_{ped} are roll, pitch, and yaw control inputs, respectively. σ_{col} is height control. Y is the output vector, which contains $[u\ v\ w\ p\ q\ r\ \phi\ \theta]^T$. The adopted A and B matrices are presented in Equation (3) and (4). Referring to the study [5], the entire model can be divided into sub-models as lateral, longitudinal, heave, and yaw dynamic models as shown in Equation (6), (7), (8), (9), respectively.

$$A = \begin{bmatrix} Xu & 0 & 0 & 0 & 0 & -g & -g & 0 & 0 & 0 & 0 \\ 0 & Yv & 0 & 0 & g & 0 & 0 & g & 0 & 0 & 0 \\ Lu & Lv & 0 & 0 & 0 & 0 & La & Lb & 0 & 0 & 0 \\ Mu & Mv & 0 & 0 & 0 & 0 & Ma & Mb & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1/\tau_a & Ab & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & Ba & -1/\tau_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Za & Zb & Zw & Zr & 0 \\ 0 & 0 & Np & 0 & 0 & 0 & 0 & 0 & Nw & Nr & Nrf \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Kr & Krf \end{bmatrix} \quad (3) \qquad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Alat & Alon & 0 & 0 \\ Blat & Blon & 0 & 0 \\ 0 & 0 & Zcol & 0 \\ 0 & 0 & Ncol & Nped \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{\phi} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} Yv & 0 & g & g \\ Lv & 0 & 0 & Lb \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1/\tau_a \end{bmatrix} \begin{bmatrix} v \\ p \\ \phi \\ b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ Blat \end{bmatrix} \delta_{lat} \quad (6)$$

$$\dot{w} = Z_u u + Z_w w + Z_{col} \delta_{col} \quad (7)$$

$$\begin{bmatrix} \dot{u} \\ \dot{q} \\ \dot{\theta} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} Xu & 0 & -g & -g \\ Mu & 0 & 0 & Ma \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1/\tau_a \end{bmatrix} \begin{bmatrix} u \\ q \\ \theta \\ a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ Alon \end{bmatrix} \delta_{lon} \quad (8)$$

$$\begin{bmatrix} \dot{r} \\ \dot{r}_{fb} \end{bmatrix} = \begin{bmatrix} N_r & N_{rf} \\ K_r & K_{rf} \end{bmatrix} \begin{bmatrix} r \\ r_{fb} \end{bmatrix} + \begin{bmatrix} N_{ped} \\ 0 \end{bmatrix} \delta_{ped} \quad (9)$$

3. System identification

3.1. System identification procedure
The adopted system identification procedure is illustrated in Figure 1. Logging data includes two parts, estimation data and validation data. Estimation data are used to identify the dynamic model. Validation data are utilized in the last step to perform model validation. The identified model was validated by using different data. The validation data were used to compare the response of the real system and estimated model. If the outputs of the real system and estimated model are similar, the identified model is precise. In contrast, an alternative procedure needs to be performed. In this paper, subspace and PEM methods were used to determine the helicopter model. Subspace method was employed to determine the initial model, and PEM method was used to improve the accuracy of the identified model. The details of subspace method can refer to the study [8].

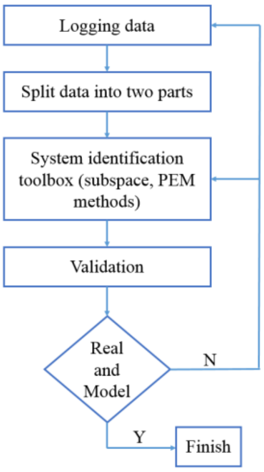

Figure 1. The used system identification procedure

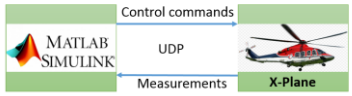

Figure 2. Communication between Simulink and X-Plane

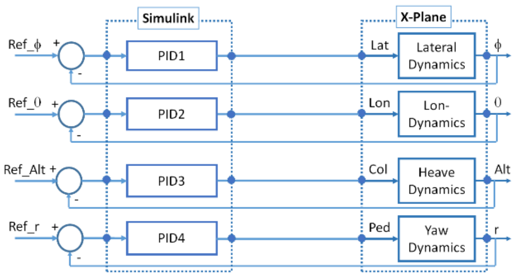

Figure 3. Structure of the used software-in-the-loop

3.2. Experimental setup and logging data
The simulation was performed by using a virtual helicopter on X-Plane. The adopted control system was developed by Simulink. Figure 2 presents the communication between Simulink and X-Plane via UDP protocol. Before collecting data, the helicopter must operate in trim condition. Referring to [9], four PID controllers were designed. The structure of the adopted controllers is shown in Figure 3. PID1, PID2, PID3, and PID4 controllers correspond to the lateral, longitudinal, heave, and yaw dynamic model, respectively. The control responses of the helicopter in hovering are showed in Figure 4. The designed inputs of the system identification procedure are shown in Figure 5. The collected flight data will be used to identify the used four sub-models.

3.3. PEM method
PEM is based on numerical optimization to minimize the *cost function*, a weighted norm of the prediction errors, defined as follows for scalar outputs [10]:

$$V_N(G,H)=\sum_{t=1}^{N}e^2(t) \quad (10)$$

where *e(t)* is the difference between the measured output and the predicted output of the model. For a linear model, the error is defined as:

$$e(t)=H^{-1}(q)[y(t)-G(q)u(t)] \quad (11)$$

where the cost function $V_N(G,H)$ is a scalar value. The subscript N denotes that the cost function is a function of the number of data samples and becomes more accurate for larger values of N.

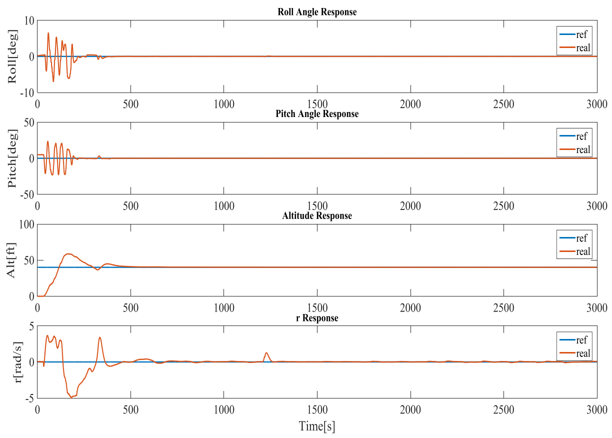

Figure 4. The responses of the helicopter in simulation

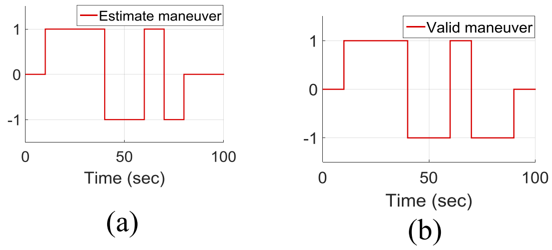

Figure 5. Designed Inputs for (a) estimation and (b) validation

4. Flight control design

The trajectory tracking control was applied to verify the developed system identification procedure and controller design. The structure of the developed control system is illustrated in Figure 6. The system includes six blocks. Reference block creates a reference trajectory. Inertial to Body block transfer from inertial coordinates to body coordinates. PI- MPC controller block processes signals and creates control commands. Plant block includes four sub-models, which was determined in section 3. Body to Inertial block transfers vectors from body coordinates to inertial coordinates. In this paper, the control commands of the developed PI-MPC are the velocities of the helicopter in body coordinate frame. Figure 7 presents the structure of the developed PI-MPC controller. V_{ref} is the target velocity; e is the difference between actual and target velocity; u_{ref} is the control input of MPC controller; u is the control inputs of lateral, longitudinal, heave, and yaw dynamic model; V is the output velocity of the Plant block. There are two feedback signals. One is the measured velocity for PI controller, and the other is the measured state vectors (attitude, velocities, and angular rates) for MPC controller. The designed control parameters are presented in Table 1.

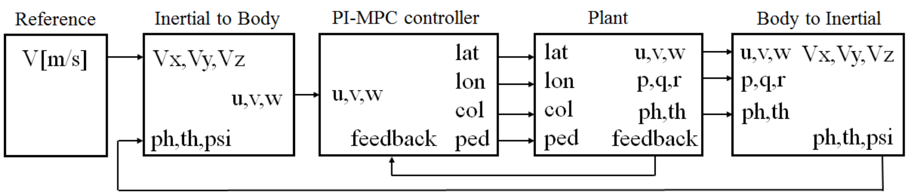

Figure 6. The structure of the system

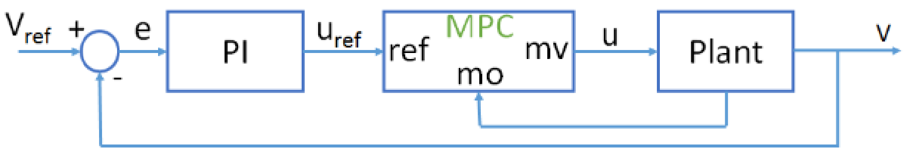

Figure 7. The structure of the controller

Table 1. Values of Kp, Ki

	Lateral	Longitudinal	Heave	Yaw
Kp	0.2	0.4	0.3	0.3
Ki	4.5	3.5	0.96	2.0

5. Results and discussions

The identified system matrices of lateral, longitudinal, and heave dynamic models are shown as follows:

$$Alat = \begin{bmatrix} -0.4214 & 0.7099 & 9.441 & -4.812 \\ 0.7506 & 0.0843 & -4.11 & 4.128 \\ 1.594 & -6.022 & -62.16 & 27.8 \\ -1.615 & 2.353 & 31.24 & -16.15 \end{bmatrix} \quad (12) \qquad Blat = \begin{bmatrix} 0.6425 \\ -0.3183 \\ -6.052 \\ 3.007 \end{bmatrix} \quad (13)$$

$$Alon = \begin{bmatrix} -0.02961 & 0.1246 & 0.6048 & -0.2666 \\ -0.02099 & 0.008143 & -0.02384 & -0.01824 \\ -2.783 & 1.179 & -5.878 & -5.204 \\ -1.007 & 0.458 & -0.6565 & -3.06 \end{bmatrix} \quad (14) \qquad Blon = \begin{bmatrix} -0.1317 \\ 0.03987 \\ 8.562 \\ 2.559 \end{bmatrix} \quad (15)$$

$$Aheave = \begin{bmatrix} 9.632e-05 \end{bmatrix} \quad (16) \qquad Bheave = \begin{bmatrix} -1.583e-05 \end{bmatrix} \quad (17)$$

$$Ayaw = \begin{bmatrix} 0.1321 & -1.089 \\ 0.4828 & -0.5013 \end{bmatrix} \quad (18) \qquad Byaw = \begin{bmatrix} 0.1109 \\ 0.04702 \end{bmatrix} \quad (19)$$

Comparing these system matrices with those presented in (6, 7, 8, 9), it reveals that they are different since the discrete models were used. Although the discrete models are usable for controller design, it needs to improve the precision of the models for dynamics analysis. It needs to find out the good initial values of A and B matrices and then fix values of some elements to estimate others. Identification results of the lateral and longitudinal dynamic models are shown in Figure 8, and 9, respectively. The performance of the developed PI-MPC controllers is shown in Figure 10, 11, 12, and 13.

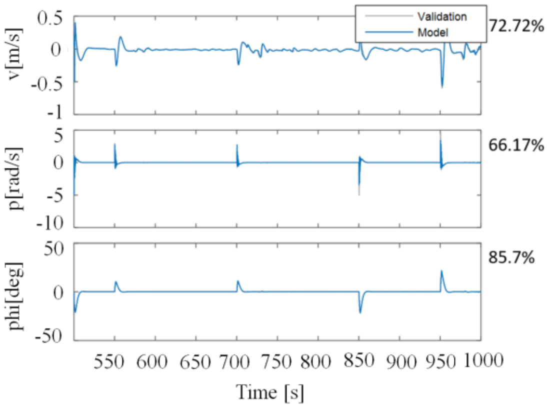

Figure 8. Lateral dynamics

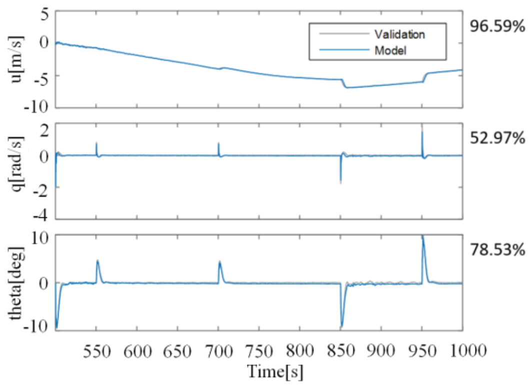

Figure 9. Longitudinal dynamics

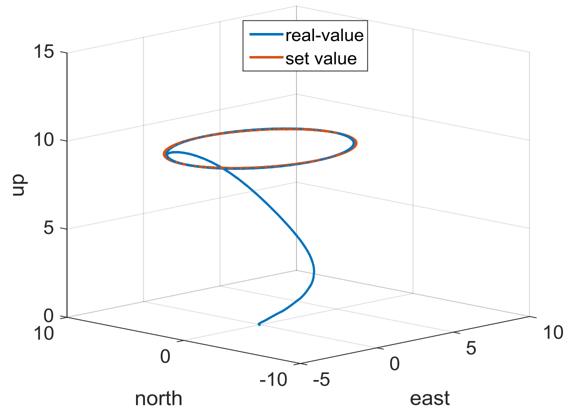

Figure 10. Trajectory tracking performance

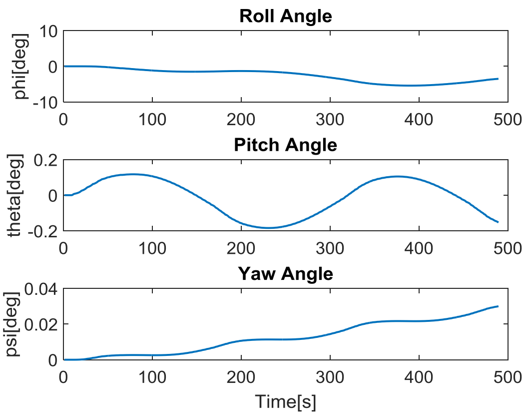

Figure 11. Euler angles

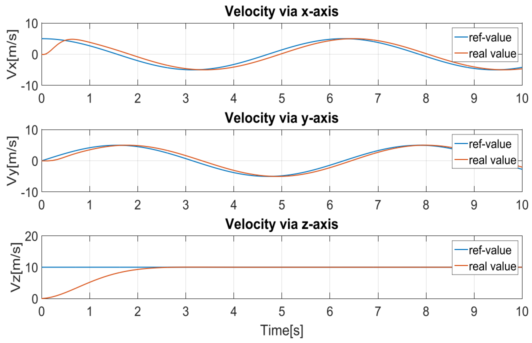

Figure 12. Velocity tracking performance

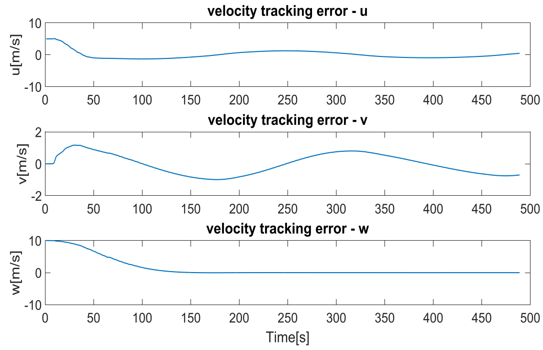

Figure 13. Velocity tracking error

6. Conclusions

A system identification and control design approach for unmanned helicopters has been developed in this paper. A proposed PI-MPC control scheme was proposed for the control design of the used sub-models, which are lateral, longitudinal, heave, and yaw dynamic models. The performance of the designed PI-MPC flight controller is viable to perform trajectory tracking control of the helicopter. The results of software-in-the-loop simulation show that the develop system identification procedure and developed control scheme can achieve the goal of flight control design of the helicopter. Although, the identification results are acceptable, the precision of the identified model need to be improved in further study for dynamics analysis.

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Acknowledgments

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