

A Modeling Method of Multiple Targets Assignment under Multiple UAVs' Cooperation

Q H Wang ¹, G Wan ¹, X F Cao ¹ and L X Xie¹

¹Zhengzhou Institute of Surveying and Mapping, China

e-mail: dcn_ing@163.com

Abstract. Aiming at the multiple UAVs' cooperation in the complex environment, detailed analysis about targets assignment model is made in the paper. Firstly, three basic situations are discussed according to the quantitative relationship between the UAVs and the targets. Then in order to make the targets model more practical, the probability that the UAVs' damage is also taken into consideration. Following, basic particle swarm optimization algorithm is adopted to solve the model which has great performance in efficiency and convergence. Finally, three-dimensional environment is simulated to verify the model. Simulation results show that the model is practical and close to the actual environment.

1. Introduction

More and more UAVs are put into practical use with their performance improves. They complete a variety of tasks instead of people in dangerous and complex environment, such as targets search, targets attack and etc. In the face of complex tasks, it is necessary of UAVs' cooperation to improve the implementation efficiency. Then targets assignment is value of discussion. Targets assignment is reasonable planning between multiple targets and multiple UAVs based on UAVs' performance constraints, environment constraints, etc. Targets assignment helps to improve the efficiency of UAVs and the probability of tasks' completion. Multiple Targets assignment under multiple UAVs' cooperation is a kind of combinatorial optimization problem.

Targets assignment model is modelling in the specific of some environment. The common targets assignment models include mixed integer linear programming (MILP), network flow model (NFM), vehicle routing problem (VRP) and multiple traveling salesman problem (MTSP) [1-3].

Zengin [4] et al. researched on the problem of the cooperative control and the targets assignment around the target tracking. Bellingham [5] proposed the mixed integer linear programming aiming at the minimum time of tasks completion. Bertuccelli [6] et al. studied the effects of the dynamic uncertain environment on the multiple targets assignment modeling. In the case where UAVs is less than targets. Shaferman [7] et al. paid attention to the multiple tasks assignment which is especially suitable for the modeling under the environment of the city. However, the discussion is made under the situation where targets is more than targets. At present, there is no detailed discussion about the quantitative relationship between the UAVs and the targets in any literature.

For the problem that not all possibilities are discussed when modeling, and the corresponding factors are not comprehensive when constructing cost functions, three basic situations are discussed according to the quantitative relationship between the UAVs and the targets. The probability that UAVs' damage are taken into consideration when constructing cost function. Then Basic particle swarm optimization algorithm is adopted to solve the model. Finally, the three-dimensional environment is simulated to verify the model. The simulation results show that the model is practical

and close to the actual environment. The particle swarm algorithm is easy to implement and has good performance in efficiency.

2. Targets assignment model

2.1. Cost function

It is necessary to solve the assignment problem between targets and UAVs in the multiple UAVs' cooperation. The basic principle is to minimize the overall cost in the premise of completing the tasks.

Numbered the UAVs $1, 2, \dots, U$ and Numbered the targets $1, 2, \dots, T$ where U and T respectively represent the quantities of the UAVs and the targets. The relationship can be described as $U > T$, $U = T$, $U < T$. It must be ensured that each UAV will be assigned to the target and each object must have the corresponding UAV. Assuming that all UAVs' and targets' initial positions are known before assignment, there are three factors we considered in flight cost function, including the total flight voyage of multiple UAVs, the total time of executing tasks and the probability of the UAVs' damage. The three factors are discussed as follows.

The total flight voyage of multiple UAVs

$$d_{\text{sum}} = \sum_{u=1}^U \sum_{i=1}^T d(u \sim v, i \sim j) x(u \sim v, i \sim j) \quad (1)$$

where $u \sim v$ are UAVs; $i \sim j$ are targets; $d(u \sim v, i \sim j)$ are flight voyage.

The total time of executing tasks

$$t_{\text{sum}} = \max \left\{ \sum_{u=1}^U \sum_{i=1}^T t(u \sim v, i \sim j) x(u \sim v, i \sim j) \right\} \quad (2)$$

where $t(u \sim v, i \sim j)$ is the flight time of some UAV which satisfies

$$t(u \sim v, i \sim j) = \frac{d(u \sim v, i \sim j)}{v_{u \sim v}} \quad (3)$$

where $v_{u \sim v}$ is the velocity of UAVs.

The probability of the UAVs' damage

$$th_{\text{sum}} = \sum_{u=1}^U \sum_{i=1}^T p(u \sim v, i \sim j) x(u \sim v, i \sim j) \quad (4)$$

where $p(u \sim v, i \sim j)$ is the probability to the damage of some UAVs.

According to [8], here is the probability when the u -th UAV flied to the j -th target,

$$p_u = \begin{cases} \exp\left(-\frac{(R - r_{\text{radar}}^{\max} / 2)^2}{2\left(\sqrt{2}r_{\text{radar}}^{\max}\right)^2}\right), & R \leq r_{\text{radar}}^{\max} \\ 0, & R > r_{\text{radar}}^{\max} \end{cases} \quad (5)$$

where $r_{\text{radar}}^{\max}$ is the effective radius of radars, R is the distance between the UAVs and radars.

From (5), without the effective range of the radar, the probability of the UAVs' damage is 0. Within the effective range, the probability obeys Gauss distribution.

$x(u \sim v, i \sim j)$ is the decision variable whose value is 0 or 1. It represents the corresponding relationship between the UAVs and the targets and following is the three situations.

1) When $U = T$, the relationship between the UAVs and the targets is one to one, then

$$x(u \sim v, i \sim j) = x(u, i). \text{ In that case, } \forall u = 1, 2, \dots, U, \sum_{i=1}^m x(u, i) = 1; \forall i = 1, 2, \dots, T, \sum_{u=1}^n x(u, i) = 1.$$

2) When $U > T$, each UAV has the only one corresponding target. It means that some target may be assigned to several UAVs, then $x(u \sim v, i \sim j) = x(u \sim v, i)$. In that case, For any UAV or UAV groups which means the several UAVs assigned to the same target, $\sum_{i=1}^m x(u \sim v, i) = 1$.

3) When $U < T$, each target has the only one corresponding UAV. It means that some UAV may be assigned several targets, then $x(u \sim v, i \sim j) = x(u, i \sim j)$. In that case, for any target or target groups which means the several targets assigned to the same UAV, $\sum_{i=1}^m x(u \sim v, i) = 1$.

Considering the above factors, the flight cost function is given as follows:

$$F(x) = w_1 \alpha_1 d_{\text{sum}} + w_2 \alpha_2 t_{\text{sum}} + w_3 th_{\text{sum}} \quad (6)$$

w_1, w_2, w_3 are the weight factors within (0,1) and $w_1 + w_2 + w_3 = 1$. The weight factors are given by experts according to the specific tasks. α_1, α_2 are the ratio factors which make the total voyage cost, the total time cost and the threat cost remain the same magnitude level.

2.2. Analysis of constraints

There are constraints as follows,

1) Voyage constraints

The single voyage of each UAV is limited. It is influenced by the UAV's performance or the oil supplement. The voyage constraint can be expressed as

$$\sum_{i,j=1}^T d(u, i \sim j) x(u \sim v, i \sim j) \leq D_u \quad (7)$$

where D_u is the maximum voyage of the u-th UAV, $u = 1, 2, \dots, U$.

2) Flight time constraints

Similar to the voyage, the single flight time of each UAV is limited, which can be expressed as follows

$$\sum_{i,j=1}^T t(u, i \sim j) x(u \sim v, i \sim j) \leq T_u \quad (8)$$

where T_u is the maximum flight time of the u-th UAV, $u = 1, 2, \dots, U$.

3) Target execution order constraints

In most cases, the concept of target value is fuzzy. The commander or experts seldom provide the accurate value while they describe the target value with words like 'high' and 'low'. For convenience, the target value is divided into seven grades, 'especially high', 'very high', 'high', 'medium', 'low', 'very low', 'especially low'.

In the paper, the seven grades are transmit into quantitative numbers [7] within the range of [0, 10]. The correspondence is as follows, 'especially high=[9,10]', 'very high=[8,9]', 'high=[7,8]', 'medium=[5,7]', 'low=[3,5]', 'very low=[1,3]', 'especially low=[0,1]'. This will provide a quantitative reference for the experts so that they can determine the value of the targets, which has an effect on the order of the target execution.

Except the target value which influences the execution order, some target may have the specific execution order. For example, if the i-th target must be executed after the j-th target, then the constraint can be expressed as

$$t_j \geq t_i + \Delta t$$

Where t_i represents the time when the i-th target is executed, the same as the t_j . Δt is the minimum interval between the t_i and t_j .

3. Solutions to the model

Particle swarm algorithm(PSO) is an intelligent optimization algorithm proposed by Dr. R.Eberhart and Dr. J.Kenedy in 1995, which stems from the simulation of birds flock's looking for food [9, 10]. It is a kind of calculation methods of artificial life. However, it is different from other evolution algorithms, which generates the optimal solution by the group cooperation mechanism instead of the group competition mechanism. PSO algorithm possesses the advantages of simplified, rather quick convergence speed, global optimization performance, and less controlling parameters, et al. PSO has attracted more and more attention recently.

In the paper, PSO is adopted to solve the model. Each particle represents a kind of solution to the targets assignment, which is denoted as $\bar{x} = \langle \bar{p}, \bar{v} \rangle = \langle \text{target assignment solution, velocity vector} \rangle$, $\bar{p}, \bar{v} \in S \subset R^D$ (D-dimensional space), that is to say that the position and velocity of the i-th particle is respectively $\bar{p}_i = (p_{i1}, p_{i2}, \dots, p_{id}, \dots, p_{iD})$ and $\bar{v}_i = (v_{i1}, v_{i2}, \dots, v_{id}, \dots, v_{iD})$.

Step 1 (Initialization): $t=1$, generate K particles $\bar{x}_i(t) = \langle \bar{p}_i(t), \bar{v}_i(t) \rangle$ randomly to form the initial particle group.

Step 2 (Fitness value calculation): calculate the individual extremum which corresponding to the best fitness value of each particle, denoted as $pbest_i(t)$ and the optimal solution found by the whole particle swarm, called the global extremum $gbest_i(t)$.

Step 3 (Evolutionary learning): $t = t + 1$, update the positions and velocities according to (9)(10)

$$v_i(t+1) = v_i(t) + c_1 rand_1 [pbest_i(t) - p_i(t)] + c_2 rand_2 [gbest_i(t) - p_i(t)] \quad (9)$$

$$p_i(t+1) = p_i(t) + v_i(t+1) \quad (10)$$

where $v_i(t)$ is the velocity of the current particle, $rand_1$ and $rand_2$ are random numbers within [0,1].

c_1 and c_2 are learning factors (also called acceleration constant), which represent the memory learning and the motivation of learning from others. Recalculate $pbest_i(t)$ and $gbest_i(t)$.

Step 4 (Algorithm end) If $t > \max_gen$ or the end conditions are satisfied, output $gbest(t)$, otherwise go to Step 3.

4. Simulations and results analysis

Target assignment model and solving algorithm are verified by MATLAB simulation. Firstly, the environment area where the UAVs and the targets exist is constructed.

Though the two-dimensional simulation is relatively easy [11], the description is not actual enough and will influence the effectiveness of the targets assignment. Therefore, the three-dimensional simulation is made. Three-dimensional environment is built based on the standard digital elevation model data, and the radar threat is taken into consideration at the same time. The related parameters are shown in appendix.

Table 1 and table 2 are the initial parameters of the UAVs.

Table 1. Parameters of UAVs when $U < T$

Number of the UAV	Three-dimensional coordinates (km)	The maximum voyage (km)	The longest flight time (min)
1	(75,20,13)	500	50
2	(110,30,8)	700	70
3	(100,25,9)	600	60
4	(45,50,12)	600	60

Table 2. Parameters of UAVs when $U = T$ and $U > T$

Number of the UAV	Three-dimensional coordinates (km)	The maximum voyage (km)	The longest flight time (min)
1	(75,20,13)	500	50
2	(110,30,8)	700	70
3	(100,25,9)	600	60
4	(45, 0,12)	600	60
5	(35,65,10)	650	65
6	(65,35,8)	610	61

Table 3 and table 4 are the initial parameters of the targets.

Table 3. Target parameters when $U > T$

Number of the UAV	Three-dimensional coordinates (km)	Targets value (0~10)	Order of targets execution
1	(70,190,8)	5	[2 1],[4 1],[1 3]
2	(120,170,3)	8	
3	(100,175,3)	2	
4	(160,180,3)	7	

Table 4. Target parameters when $U = T$ and $U < T$

Number of the UAV	Three-dimensional coordinates (km)	Targets value (0~10)	Order of targets execution
1	(70,190,8)	5	[3 4],[5 2],[6 4]
2	(65,110,8)	3	
3	(120,170,3)	8	
4	(100,175,3)	2	
5	(160,180,3)	7	
6	(80,195,5)	7	

200 Monte Carlo simulations have been carried out. Figure 1 is the three-dimensional simulation results when $U = T$. In the figure 1, sphere represents the radar threat in the flight environment, diamond sign indicates the starting position of the UAV and the asterisk sign is the positions of the targets. The lines between the UAVs and the targets represent the flight tracks. Figure 2 is the contour map of the simulations. The solutions of the targets assignment under multiple UAVs' cooperation are also shown in table 5.

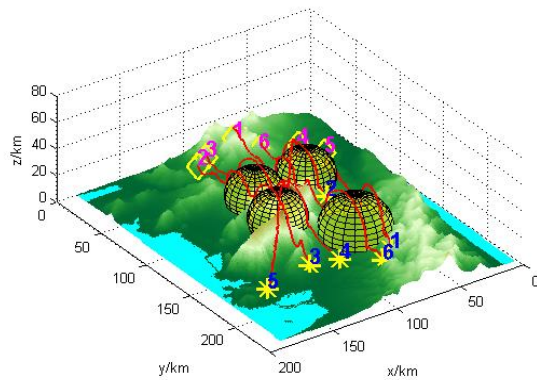


Figure 1. Three dimensional simulation results when $U = T$

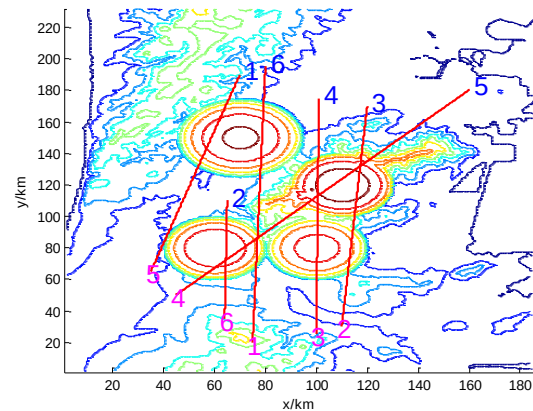


Figure 2. Contour map when $U = T$

Table 5. Targets assignment solutions when $U = T$

Number of the UAV	1	2	3	4	5	6
Number of the target	6	3	4	5	1	2

When $U > T$, the three-dimensional simulation results are shown in figure 3. The corresponding contour map is shown as figure 4. The targets assignment solutions are shown in table 6.

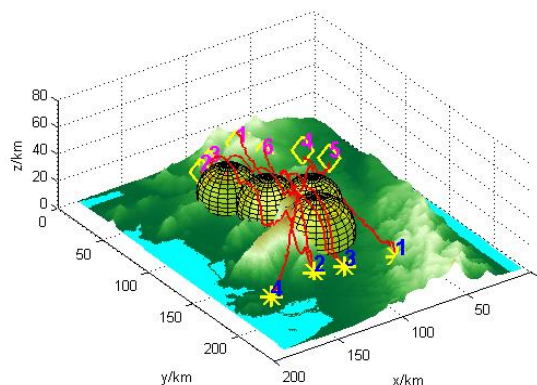


Figure 3. Three dimensional simulation results when $U > T$

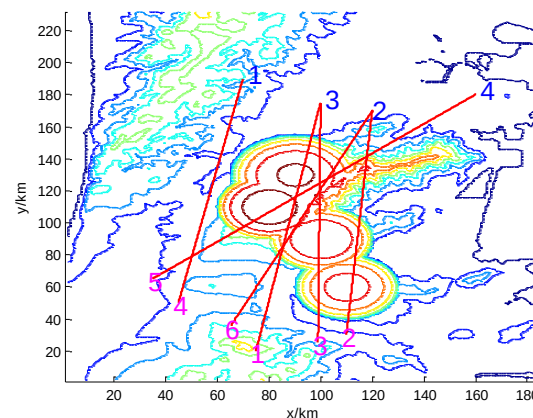


Figure 4. Contour map when $U > T$

Table 6. Targets assignment solutions when $U > T$

Number of the UAV	1	2	3	4	5	6
Number of the target	3	2	3	1	4	2

When $U < T$, the three-dimensional simulation results are shown in figure 5. The corresponding contour map is shown as figure 6. The targets assignment solutions are shown in table 7.

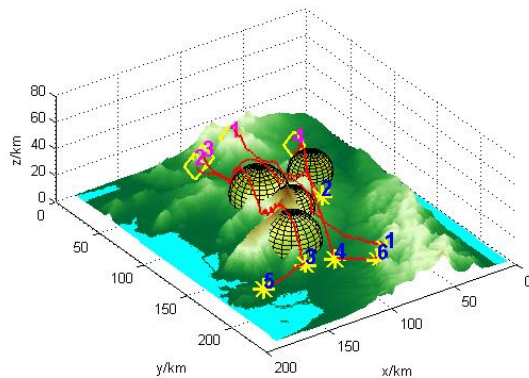


Figure 5. Three dimensional simulation results when $U < T$

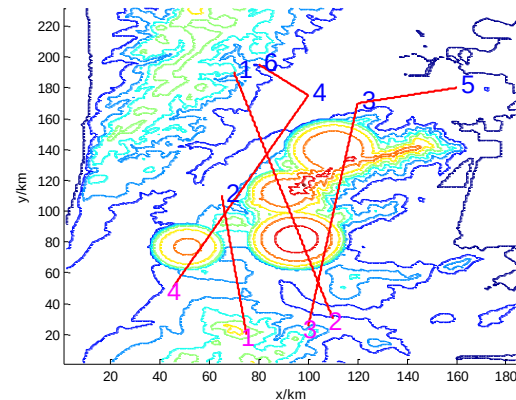


Figure 6. Contour map when $U < T$

Table 7. Targets assignment solutions when $U < T$

Number of the UAV	2	1	3	4	3	4
Number of the target	1	2	3	4	5	6

5. Conclusion

Multiple targets assignment of the multiple UAVs' cooperation is researched on in the paper. Firstly, the quantitative relationship between UAVs and targets is discussed in detail and the probability that the UAVs' damage is taken into consideration. Thus the model is more practical. Basic particle swarm optimization algorithm is adopted to solve the model which has great performance in efficiency. Finally, three-dimensional environment is simulated to verify the model. In the future, it is necessary to consider the effect of dynamic threat on the modeling. In addition, more intelligent optimization algorithms are in multi UAV cooperative application. The algorithm can also be combined with other intelligent optimization algorithms to further improve the performance.

6. References

- [1] Griggs B J, Parnell G S and Lehmkuhl L J 1997 *Oper. Res.*, **45** 662-76.
- [2] O'Rourke K P, Carlton W B, Bailey T G and Hill R R 2001 *Military Operations Research* **6** 5-30.
- [3] Beard R W, McLain T W and Goodrich M 2003 *IEEE Trans. Rob. Autom.* **18** 911-22.
- [4] Zengin U and Dogan 2013 *In Proc. of AIAA Guidance, Navigation and Control Conference and Exhibit* (USA: Chicago) **12** p2006.
- [5] Beilingham J, Tilterson M and Richards 2003 *Cooperative Control: Models, Applications and Algorithms* (US: Springer) p23.
- [6] Bertuccelli L, Choi H L and Cho P 2009 *AIAA Guidance, Navigation, and Control Conference* (USA: American Institute of Aeronautics and Astronautics) **10** p2009.
- [7] Shaferman, V and Shima T 2013 *In Proc. of AIAA Guidance, Navigation and Control Conference and Exhibit* (USA: Chicago), **10** p2009.
- [8] Washburn A and Kress M *Combat Modelling* (US: Springer) p 39.
- [9] Kennedy J and Eberhart R 1995 *In Proc. of International Conference on Neural Networks* **4** p1942.
- [10] Eberhart R and Kennedy J 1995 *International Symposium on MICRO Machine and Human Science* p39.
- [11] Bahar M R B, Bahar H B and Hashemzadeh F 2011 *International Conference on Automation and Computing IEEE* p75.