

# Integrated GPS/DR Vehicle Navigation System Based on Sequential and Square Root Kalman Filters

MOSTAFA Elzoghby<sup>1,\*</sup>, USMAN Arif<sup>1</sup>, FU Li<sup>1</sup> and XI Zhi Yu<sup>1</sup>

<sup>1</sup>School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

\*E-mail: mostafa@buaa.edu.cn

**Abstract.** Global Positioning System (GPS) has become part of many applications in life. In mountainous terrains and around buildings, GPS reception is compromised. In dense urban canyons, signals bounce off the buildings creating multipath reception and provide erroneous measurements. To overcome GPS bandwidth and signal fading problems, Navigation solutions are built on GPS measurements fused with inertial sensors to provide dead reckoning (DR) based position solution. Solution for land vehicle Navigation System using GPS, inertial sensor and odometer is presented. The sensors fusion is performed based on conventional, sequential (SKF) and square root Kalman (SRKF) filters. SRKF based on Cheolesky factorization for covariance matrix P. Simulations are performed on real data, with precisely known covariance's to simulate mathematical stability, performance and processing time required by each method on a high end microprocessor. The results demonstrate integrated system using SRKF has better performance in stability and estimation accuracy than conventional and sequential filter.

## 1. Introduction

Land vehicle navigation systems are one the fastest growing area in-vehicle electronics market [1]. Generally, the land Vehicle navigation system gets the positioning information via GPS. Hence the GPS can provide sufficient positioning information but suffers seriously especially in tunnels or urban canyons where GPS signal gets faded or corrupted; therefore degrading the availability and accuracy for the GPS based positioning information. Many high end GPS antennas and amplifiers have been designed to solve this problem, but these systems increase the cost of the Navigation Solution. There are four main parameters that describe the performance of a navigation system: availability, continuity, integrity and accuracy [2]. In order to fulfill these requirements, the most widely accepted approach is to integration of DR with GPS which increased the integrity and availability of Navigation Solution [3].

Other The principle of DR is to provide the current position of the vehicle based on previous position and velocity, and provides continuous real time navigation information. Using only DR has a drawback: the navigation error accumulates over time and cannot keep up the accuracy requirements. Therefore for a complete navigation solution, DR can provide the vehicle positioning information when GPS is not available, while GPS can correct the accumulated error of DR [4]. Most commonly used DR sensors for land vehicle navigation include odometer, gyro and magnetic compass. In our work, we have utilized a single axis gyroscope which provides the angular rate of change in the heading direction while Odometer which provides absolute velocity information as Dead Reckoning



sensors. Another approach like map matching algorithms with DGPS used for Land Vehicle position determination in [5]. The low cost IMU Integrated with GPS System for Land Vehicle Navigation by using kalman filter is proposed in [6,7]. Cascade filters implemented for DR navigation solution in [8] via using a wheel encoders to provide distance and a magnetic compass to provide direction. Also, differential wheel speed sensor integrated with an inertial measurement and GPS are used in [9] to reduce the errors from sideslip and eliminate the large errors in vehicle motions by using two nonlinear kalman filter forms. And some of another Kalman filter forms used to estimate vehicle position by using heading sensors combined with speed sensors during GPS unavailability in [10-13].

The key problem of GPS/DR integrated system is how to achieve the optimal estimation for the position, moving and states of interest parameters. Our work involves designing land based navigational solution with our prescribed model and investigating still provides any superiority over conventional or batch Kalman filtering as well as to compare the computational time required by each method. We have chosen SKF and SRKF and compared the results with conventional KF in our problem of designing navigation solution for a land vehicle on a high end processor. In the following sections, we begin with describe the mathematical model for GPS/DR integration then a brief summary about the algorithms of the sequential and square root Kalman Filter. The part of the simulation and discussion to show the effect of (KF), (SKF) and (SRKF) on the integration, Simulation results and a comparison of these methods are presented in section 2. Finally, in section 3, these methods are evaluated in theory and simulation.

## 2. Formulations and equations

### 2.1 GPS/DR Mathematical Model

2.1.1 The state equations. *The east position  $E_x$ , north position  $N_y$ , heading  $\psi$ , speed  $v$ , gyro rate  $\dot{\psi}$ , bias,  $B$  and odometer scale  $S$  can be expressed in state space form as:*

$$E_x(k+1) = E_x(k) + V(k)T\sin(\psi(k)) + w_1 \quad (1)$$

$$N_y(k+1) = N_y(k) + V(k)T\cos(\psi(k)) + w_2 \quad (2)$$

$$\psi(k+1) = \psi(k) + T\dot{\psi}(k) + w_3 \quad (3)$$

$$V(k+1) = V(k) + w_4 \quad (4)$$

$$\dot{\psi}(k+1) = \dot{\psi}(k) + w_5 \quad (5)$$

$$B(k+1) = B(k) + w_6 \quad (6)$$

$$S(k+1) = S(k) + \frac{V(k) - N(k)S(k)}{N(k)} + w_7 \quad (7)$$

A processes noise  $w_k$  is white, zero-mean, uncorrelated, and has known covariance matrix  $Q_k$ .

2.1.2 The observation equations. *The measurements equations are expressed as:*

$$E_{gps} = E_x(k) + v_1 \quad (8)$$

$$N_{gps} = N_y(k) + v_2 \quad (9)$$

$$\psi_{gps} = \psi(k) + v_3 \quad (10)$$

$$V_{speed}(k) = n(k)S(k) + v_4 \quad (11)$$

$$\dot{\psi}_{gyro}(k) = \dot{\psi}(k) + B + v_5 \quad (12)$$

The measurement noise  $v_k$  is a zero mean white noise and the measurement noise covariance matrix  $R_k$ .

### 2.2 Sequential Kalman Filter

This technique is also called “sequential-sensor method” [14]. The approach is to implement the kalman filter while avoiding the matrix inversion during the computation of the kalman gain and

considering every sensor observation as an independent and separate realization. At each time-step the kalman gain and the new prediction are computed at every observation from every sensor.

### 2.2.1 The system and measurement equations.

$$\mathbf{X}(k) = \mathbf{F}(k, k-1)\mathbf{X}(k-1) + \mathbf{w}(k-1) \quad (13)$$

$$\mathbf{Z}(K) = \mathbf{H}(k)\mathbf{X}(k) + \mathbf{v}(k) \quad (14)$$

Where  $\mathbf{w}_k$  and  $\mathbf{v}_k$  uncorrelated and white noise. The measurement covariance matrix  $\mathbf{R}_k$  is a diagonal and given by  $\mathbf{R}_k = \text{diag}(\mathbf{R}_{1k}, \dots, \mathbf{R}_{rk})$ , Uncorrelated measurements in  $\mathbf{Z}(K)$  can be taken as accomplished step by step.

$$\begin{bmatrix} \mathbf{Z}_k^1 \\ \mathbf{Z}_k^2 \\ \vdots \\ \mathbf{Z}_k^r \end{bmatrix} = \begin{bmatrix} \mathbf{H}_k^1 \\ \mathbf{H}_k^2 \\ \vdots \\ \mathbf{H}_k^r \end{bmatrix} \mathbf{X}_k + \begin{bmatrix} \mathbf{v}_k^1 \\ \mathbf{v}_k^2 \\ \vdots \\ \mathbf{v}_k^r \end{bmatrix} \quad (15)$$

2.2.2 Realization of SKF. The state prediction and the corresponding covariance at any one time step for the first observation  $\mathbf{Z}_k^1$  are given by

$$\hat{\mathbf{X}}_k^0 = \hat{\mathbf{X}}(k | k-1) \quad (16)$$

$$\mathbf{P}_k^0 = \mathbf{P}(k | k-1) \quad (17)$$

The next new state estimation made by fusing  $\mathbf{Z}_k^1$  which achieved at time  $k$  by the first sensor is given as

$$\hat{\mathbf{X}}_k^1 = \hat{\mathbf{X}}_k^0 + K_k^1(\mathbf{Z}_k^1 - \mathbf{H}_k^1 \hat{\mathbf{X}}_k^0) \quad (18)$$

$$\mathbf{P}_k^1 = (\mathbf{I} - K_k^1 \mathbf{H}_k^1) \mathbf{P}_k^0 \quad (19)$$

$$K_k^1 = \mathbf{P}_k^0 \mathbf{H}_k^{1T} [\mathbf{H}_k^1 \mathbf{P}_k^0 \mathbf{H}_k^{1T} + \mathbf{R}_k^1]^{-1} \quad (20)$$

To fuse the observation from the second sensor so, it requires  $\hat{\mathbf{X}}_k^1$  to produce a prediction. Generally, the estimation of  $\hat{\mathbf{X}}_k^1$  at time  $k$  on the basis of the observations from all the sensors until time  $(k-1)$  and through first  $i$  sensors until time  $k$  can to calculate from the estimation of  $\hat{\mathbf{X}}_k^{i-1}$ . Update sequentially until the last observation  $\mathbf{Z}_k^r$  then the estimation of  $\hat{\mathbf{X}}_k^r$  at time  $k$  on the basis of the observations from all the sensors until time  $(k-1)$  and through first  $r$  sensors until time  $k$  can to calculate from the estimation  $\hat{\mathbf{X}}_k^{r-1}$  as kalman throughout

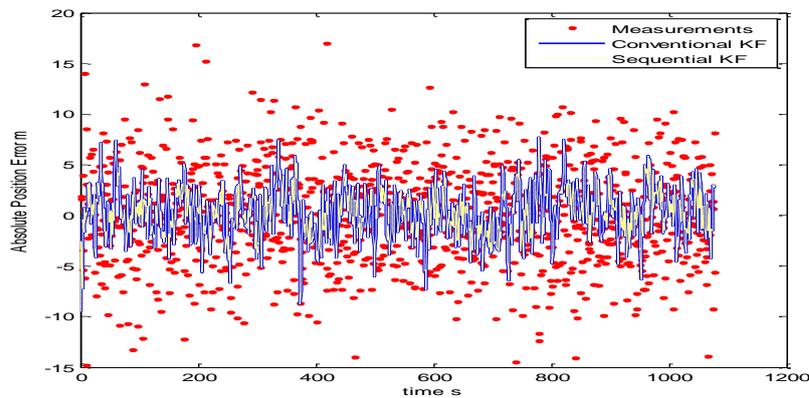
$$\hat{\mathbf{X}}_k^r = \hat{\mathbf{X}}_k^{r-1} + K_k^r(\mathbf{Z}_k^r - \mathbf{H}_k^r \hat{\mathbf{X}}_k^{r-1}) \quad (21)$$

$$\hat{\mathbf{X}}_k^r = \hat{\mathbf{X}}(k | k) \quad (22)$$

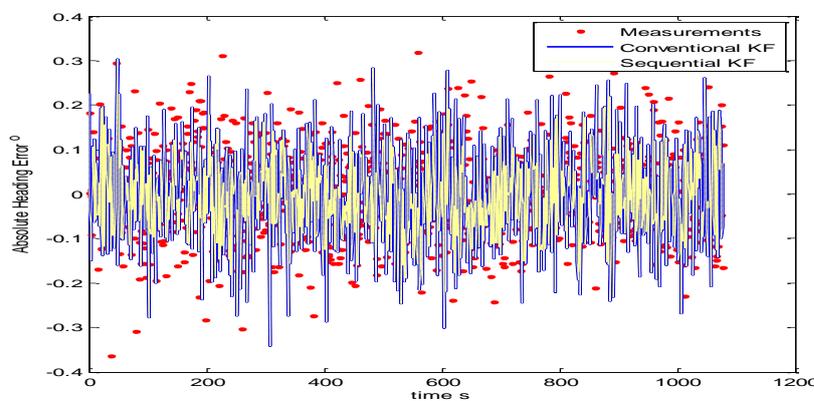
$$\mathbf{P}_k^r = (\mathbf{I} - K_k^r \mathbf{H}_k^r) \mathbf{P}_k^{r-1} \quad (23)$$

$$\mathbf{P}_k^r = \mathbf{P}(k | k) \quad (24)$$

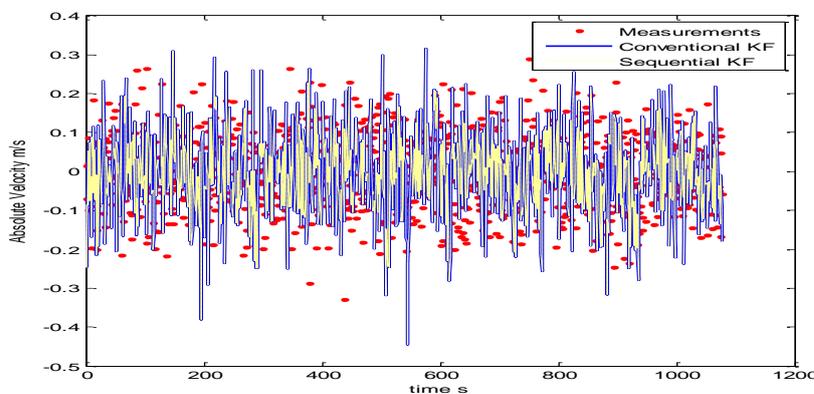
$$K_k^r = \mathbf{P}_k^{r-1} \mathbf{H}_k^{rT} [\mathbf{H}_k^r \mathbf{P}_k^{r-1} \mathbf{H}_k^{rT} + \mathbf{R}_k^r]^{-1} \quad (25)$$



**Figure 1.**  
Conventional and Sequential KF Results for absolute position error



**Figure 2.**  
Conventional and Sequential KF Results for absolute heading error



**Figure 3.**  
Conventional and Sequential KF Results for absolute velocity error

### 2.3 Square Root Kalman Filter

SRKF effectively increases the accuracy of the Kalman filter by reducing the divergence and instability, but on the other side increase the computational cost [15]. Generally, square root filter is stable numerically more than the conventional filter; also, it is less affected by numerical problems [16]. This technique is also known as the covariance square root filter. It works by calculating square root of covariance matrix  $P$  using Cholesky decomposition, given by:

$$P = SS^T = \begin{bmatrix} S_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ S_{n1} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} S_{11} & \cdots & S_{n1} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_{nn} \end{bmatrix} \quad (26)$$

Where

$$S_{ii} = \sqrt{P_{ii} - \sum_{j=1}^{i-1} S_{ij}^2} \quad (i < j) \quad (27)$$

$$S_{ij} = \sqrt{\frac{1}{S_{jj}} (P_{ij} - \sum_{k=1}^{j-1} S_{ik} S_{jk})} \quad (i > j) \quad (28)$$

$$S_{ij} = 0 \quad (i < j) \quad (29)$$

Mathematically, there is no difference between the square root and conventional filter time update equation for P but the difference is to update and propagate S instead of P [15]. In ill-conditioned cases this approach can yield double effective precision as compared to the standard filter [17]. And successfully keeps the error covariance as a positive semi definite which yields the numerical stability as well. The difficulties in the numerical calculations maybe cause  $P_k$  to be indefinite or non symmetric but cannot cause the same for  $SS^T$  [15]. Numerical problems may arise and The Error covariance matrix can become ill conditioned if one state variable has a much higher uncertainty than another. The square root Kalman filter works effectively especially if the estimation problem is ill conditioned or computer word length is limited.

*2.3.1 The square root time update equations. The filter of Square root covariance propagate Cholesky factor of a priori error covariance matrix  $P(k | k - 1)$ .*

$$\mathbf{P}(k | k - 1) = \mathbf{S}(k | k - 1) \mathbf{S}^T(k | k - 1) \quad (30)$$

Where  $\mathbf{S}(k | k - 1)$  is square root of a priori error covariance matrix  $\mathbf{P}(k | k - 1)$  and chosen to be lower triangular. As well,  $\mathbf{S}(k | k)$  is square root of a posteriori error covariance matrix  $\mathbf{P}(k | k)$  and initially  $\mathbf{S}(0 | 0) \mathbf{S}^T(0 | 0) = \mathbf{P}(0 | 0)$ . However, there are 2 cases related with the system model if it has a dynamic noise for all time ( $Q_k \neq 0$ ) or zero process noise ( $Q_k = 0$ ). The time propagation with neglecting control inputs in case of ( $Q_k = 0$ ) will be as follows:

$$\hat{\mathbf{X}}(k | k - 1) = \mathbf{F}(k, k - 1) \hat{\mathbf{X}}(k - 1 | k - 1) \quad (31)$$

$$\mathbf{P}(k | k - 1) = \mathbf{F}(k) \mathbf{P}(k - 1 | k - 1) \mathbf{F}^T(k) \quad (32)$$

By letting  $\mathbf{P}(k | k - 1) = \mathbf{S}(k | k - 1) \mathbf{S}^T(k | k - 1)$  and  $\mathbf{P}(k - 1 | k - 1) = \mathbf{S}(k - 1 | k - 1) \mathbf{S}^T(k - 1 | k - 1)$ , equation (32) can be rewritten as

$$[\mathbf{S}(k | k - 1)] [\mathbf{S}^T(k | k - 1)] = [\mathbf{F}(k) \mathbf{S}(k - 1 | k - 1)] [\mathbf{S}^T(k - 1 | k - 1) \mathbf{F}^T(k)] \quad (33)$$

From this equation it is clear that the appropriate time propagation for the square root filter would be

$$\hat{\mathbf{X}}(k | k - 1) = \mathbf{F}(k, k - 1) \hat{\mathbf{X}}(k - 1 | k - 1) \quad (34)$$

$$\mathbf{S}(k | k - 1) = \mathbf{F}(k) \mathbf{S}(k - 1 | k - 1) \quad (35)$$

The time propagation with neglecting control inputs in case of ( $Q_k \neq 0$ ) will be as follows:

$$\hat{\mathbf{X}}(k | k - 1) = \mathbf{F}(k, k - 1) \hat{\mathbf{X}}(k - 1 | k - 1) \quad (36)$$

$$\mathbf{P}(k | k - 1) = [\mathbf{F}(k) \mathbf{S}(k - 1 | k - 1)] [\mathbf{S}^T(k - 1 | k - 1) \mathbf{F}^T(k)] + \mathbf{G}(k) \mathbf{Q}(k) \mathbf{G}^T(k) \quad (37)$$

In this case will compute  $P(k | k - 1)$  firstly and then generate  $S(k | k - 1)$  as its Cholesky lower triangular square root and this method called root sum square (RSS) and this case is simulated in the paper.

2.3.2 The square root measurements update equations.

$$\hat{X}(k | k) = \hat{X}(k | k - 1) + K(k)(Z(k) - H(k)\hat{X}(k | k - 1)) \tag{38}$$

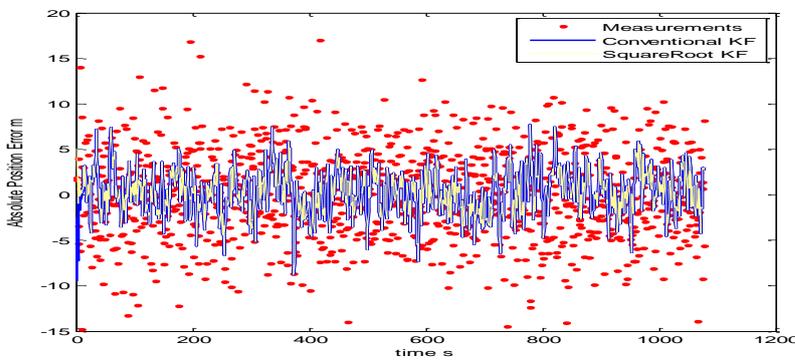
$$K(k) = b_k S(k | k - 1) a_k \tag{39}$$

$$S(k | k) = S(k | k - 1)[I - b_k r_k a_k a_k^T] \tag{40}$$

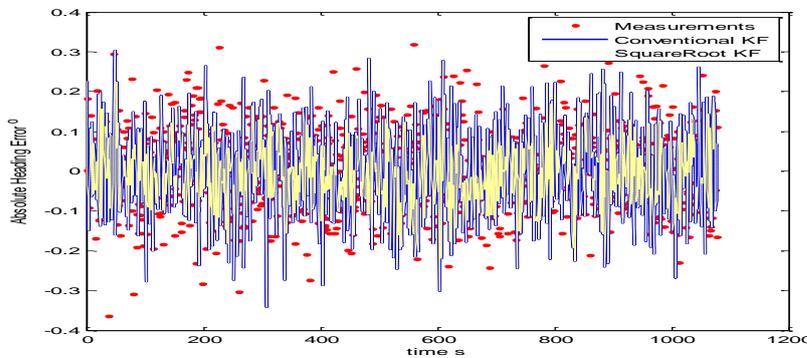
$$a_k = S^T(k | k - 1) H^T(k) \tag{41}$$

$$b_k = [a_k^T a_k + R_k]^{-1} \tag{42}$$

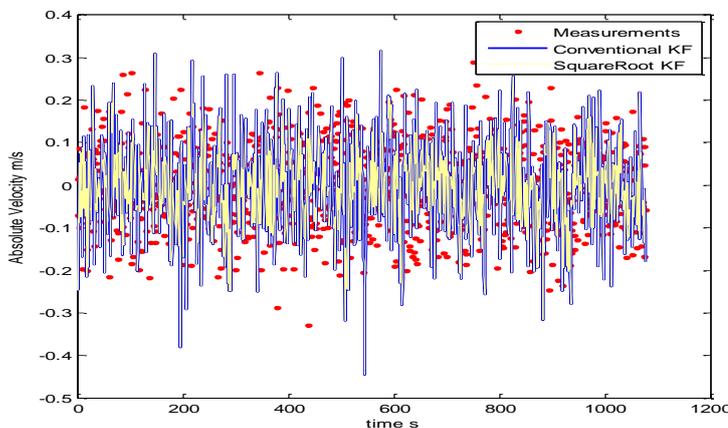
$$r_k = [1 + [b_k R_k] \frac{1}{2}]^{-1} \tag{43}$$



**Figure 4.** Conventional and Square Root KF Results for absolute position error



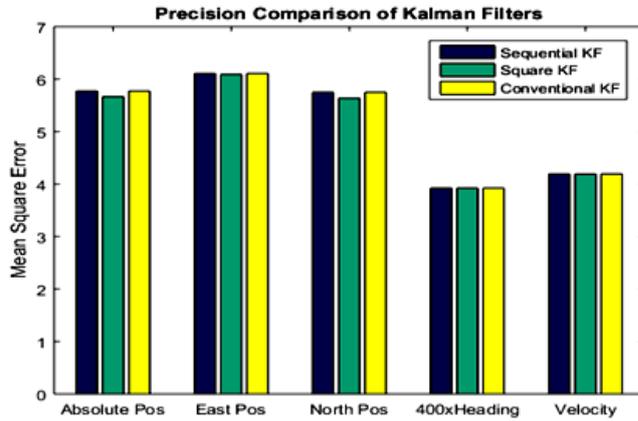
**Figure 5.** Conventional and Square Root KF Results for absolute heading error



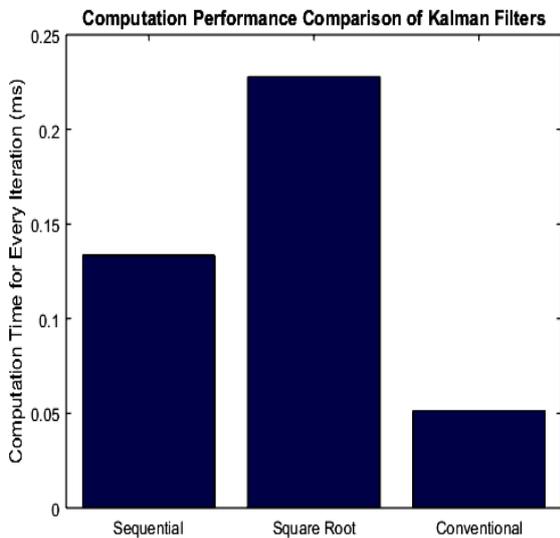
**Figure 6.** Conventional and Square Root KF Results for absolute velocity error

### 3. Experimentation and Results

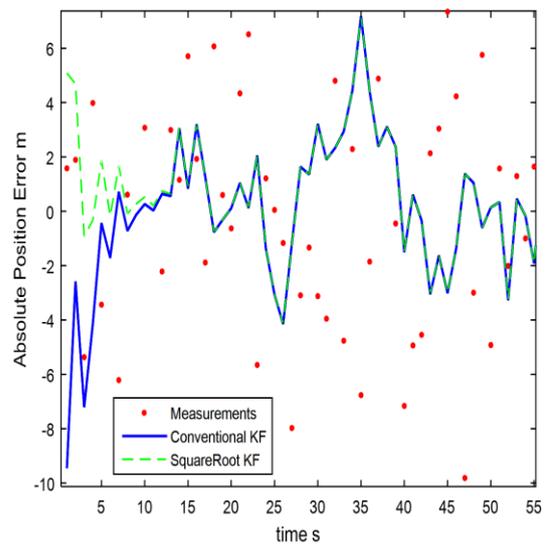
A GPS receiver is mounted on a land vehicle which provides east and north position, velocity, and heading angle in the navigation frame. MEMS gyroscope sensor is used for yaw rate measurement along with odometer for speed measurement. The measurement noise covariance is already known precisely. The data is recorded and is processed offline on a mini processing unit. Three separate Kalman filters namely conventional, SKF and SRKF are implemented separately with same process and measurement covariance matrices.



**Figure 7.** Mean Square error Comparison between Three Filters



**Figure 8.** Computational Time Comparison of Three Filters



**Figure 9.** Convergence Vs Time for Conventional and Square Root KF

The absolute position, velocity and heading are shown in the figures (1, 2 and 3) for conventional and SKF. In figures (4, 5 and 6), a comparison of conventional with SRKF is shown. A complete comparison for all the states for the three sorts of Kalman filter methods is summarized in the bar graph shown in figure 7. It is clear that the SRKF has slightly better performance than the other two Kalman Filters. Figure8 summarizes the computation time utilized by each method for single iteration.

Computation time of conventional is smallest while SRKF is largest. In figure 9 the convergence speed comparison of SRKF and conventional is shown. The superiority of SRKF over conventional is due to its lower convergence time. The convergence time of SKF and conventional is same.

#### 4. Conclusion

The results clearly show that the SRKF method based on the Cholesky factorization performs slightly better than the other two methods which perform equally. This is because the convergence rate of SRKF is better than conventional and SKF. If we increase the number of states, the SRKF method may provide an increased efficiency. The computation time for SRKF is four times that required for conventional Kalman filter. Therefore, if computational time requirements are not stringent and numbers of states are more, then we can use SRKF while for less number of states, ordinary Kalman filter performs quite well. These results are only valid for high end processor based applications. For low computational power embedded systems and SRKF will obviously work far better, even for reduced number of states.

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