

Modeling and Analysis of Non-Uniform Honeycomb Structures Based on Topology Optimization

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Abstract. Honeycomb structure infilling is an important way to achieve lightweight. Focusing on the deficiency of the non-optimized macro-material distribution in uniform honeycomb structures, a modeling method of non-uniform honeycomb structures based on topology optimization was proposed. The loaded component was topology-optimized and the density results were mapped to the relative density matrix of cells. The rapid and automatic modeling of non-uniform honeycomb structures was realized with the using of User-Defined Features and the cyclic definitions of the reference datums. The simulation results show that the mechanic performance of non-uniform honeycomb structures is better than uniform honeycomb structures and the efficiency of the proposed method is validated.

1. Introduction

In the structural design of aerospace, rail transportation and other industry products, lightweight can not only reduce energy consumption in the production and operation, but also bring higher manoeuvrability and better dynamic and static performance, which are key indicators related to the product competitiveness. In recent years, with the rapid development of material preparation and forming technology, honeycomb structure emerges as a kind of novel multifunctional lightweight structure [1]. It exhibits properties of high strength and rigidity except for weight reduction, realize the shock resistance [2], heat transfer and insulation [3], sound absorption [4], and meet the requirements of multifunction [5]. Traditional honeycomb structure is a periodic structure, with the relative density distribution uniform, and the preparation technologies mature [6]. Although the microstructure is optimized on the cell scale, the uniform structure of the macroscale is not optimized. The uniform distribution of the structure doesn't considers the optimal load transfer path; the lightweight efficiency is not sufficient.

Topology optimization is an important way to achieve lightweight when satisfying performance. The essence of this method is to seek the structural layout with the optimal load transfer path under the action of external force and constraint [7]. Topology optimization plays a significant role in the structural design of automobile, aerospace, industrial equipment and other fields [8, 9]. At present, the most mature commercial application is solid isotropic material penalty model (SIMP) proposed by Rozvany [10]. The density of each element in the design space is used as the design variable and the optimal design result is the density distribution between 1 and 0. We can map the optimal density distribution of topology optimization results to the relative density of honeycomb cells, resulting in



non-uniform honeycomb structure, and realize the optimization design in the macro-scale and meso-scale, to improve the efficiency of lightweight.

There are two ways to model the honeycomb structure. First method is to use CAD platform for interactive modeling, and the problem is that: due to the non-uniform characteristics of the honeycomb cells, it is not easy to directly model with periodic operations of array or mirror. The manual modeling has the difficulties of high cost. Second method is to use rapid prototyping processing software, such as Magics. This kind of software is to fill the model with fixed units based on STL format. The main problem of the scheme is that: it only provides function of uniform-cell filling. Modeling object is STL format, which is not conducive to the modification of structures. If the structure is generated with defects, it must return to the CAD software. Therefore, it is important to develop the auxiliary application for CAD software by programming to realize rapid and automatic modeling.

2. Structural topology optimization

For the topology optimization of nonuniform honeycomb structure, the reference structure is the honeycomb structure with uniform density. The maximum stiffness at the same weight reduction is achieved by the topology optimization. It adapts to the general definition of the minimum compliance problem [11]: under the given material volume constraint, the optimal distribution of the material in the design domain is sought, so that achieve the structural topology optimization of the maximum stiffness. Therefore, the minimum compliance optimization model is established, the relative density of elements is used as the design variable, and the maximum stiffness is obtained under the constraint of volume.

The expression for the compliance is given as:

$$C = F^T U = U^T K U \quad (1)$$

where K is the overall stiffness matrix, which can be expressed as a function of element density. U is the structural displacement matrix. F is the external force. Constraints include volume constraint, element density range constraints, and other constraints. The volume constraint represents that material distribution is arranged in a fixed volume fraction the same as the uniform honeycomb structure. Element density range is $x_{e\min} \leq x_e \leq x_{e\max}$. Taking into account that extremely low density means the cell wall is too thin and extremely high density means the cell center hole is too small, which is both not conducive to manufacturing, so the minimum density and maximum density is given as 0.2 and 0.9. Other constraints include other requirements to be considered in the design and manufacturing, and they are expressed in $g(x_e) \leq 0$. The mathematical model of topology optimization is obtained:

$$\begin{aligned} \min C(x_e) &= F^T U = U^T K U = \sum_{i=1}^m u_i^T k_i^{(e)} u_i \\ \text{s.t. } F &= K U; \\ K &= K(D(x_e)) = K(x_e); \\ \sum_{i=1}^m x_{ei} \cdot v_i &= V; \\ x_{e\min} &\leq x_e \leq x_{e\max}; \\ g(x_e) &\leq 0; \end{aligned} \quad (2)$$

Sensitivity analysis is carried out as the key step to solve the optimization problem. The sensitivity of the objective function to the design variables is obtained [11]:

$$\frac{\partial C(x_e)}{\partial x_e} = -U^T \frac{\partial K(x_e)}{\partial x_e} U \quad (3)$$

Based on the sensitivity of the objective function, the OC algorithm is used to update the design variables. The iteration formula for the design variables based on the optimization criterion method is given [12]:

$$x_e^{k+1} = \begin{cases} \max(x_{e\min}, x_e - m) & \text{if } (B_e^k)^\eta x_e^k \leq \max(x_{e\min}, x_e - m) \\ \min(x_{e\max}, x_e + m) & \text{if } \min(x_{e\max}, x_e + m) \leq (B_e^k)^\eta x_e^k \\ (B_e^k)^\eta x_e^k & \text{otherwise} \end{cases} \quad (4)$$

where k is used to represent the number of iterations, $\eta = 0.5$ is the damping coefficient to ensure the stability and convergence of the numerical calculation, and $m = 0.2$ to limit the maximum variation of the density variable for each iteration. B_e^k is the optimal criterion:

$$B_e^k = -\frac{\partial C}{\partial x_e} / \lambda \frac{\partial V}{\partial x_e} \quad (5)$$

where Lagrange multiplier λ must satisfy the volume constraints in the optimization model, which can be searched by bisection method. After the determination of optimal criterion, design variables start iteration until convergence.

3. Cellular density mapping

Aiming to the specific object as shown in figure 1:

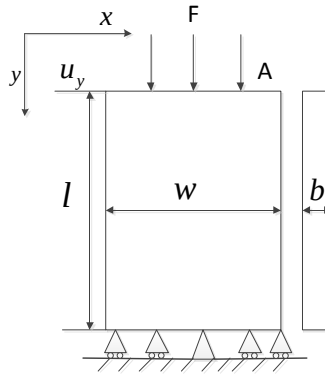


Figure 1. Geometry object ready for design.

It is a solid plate design domain with $l = 120\text{mm}$, $w = 60\text{mm}$ and $b = 10\text{mm}$ under pressure $F = 60\text{N}$ on the top. Because the load is on the direction of the cell wall, refer to the mechanical properties analysis carried out by Wang [13] for the typical honeycomb structures including hexagonal honeycomb, square honeycomb and triangle honeycomb, square honeycomb is choose to construct nonuniform honeycomb structure. Based on the elements density matrix obtained by the optimal solution, the densities of the finite elements are calculated by averaging, and the density distribution matrix of honeycomb cells is mapped out. The specific process, as shown in figure 2, is divided into the following three steps:

(1) Determination of some parameters. The length of the design domain is *length*, the width of the design domain is *width*, the dimension of one single cell is a , the dimension of one single element is r , and the density distribution matrix of elements is x_e .

(2) Determination of the arrangement of cells. With the origin O as the starting point for arrangement, the number of cells in x direction is $m = \text{ceil}(\text{width} / a)$, and y direction is $n = \text{ceil}(\text{length} / a)$.

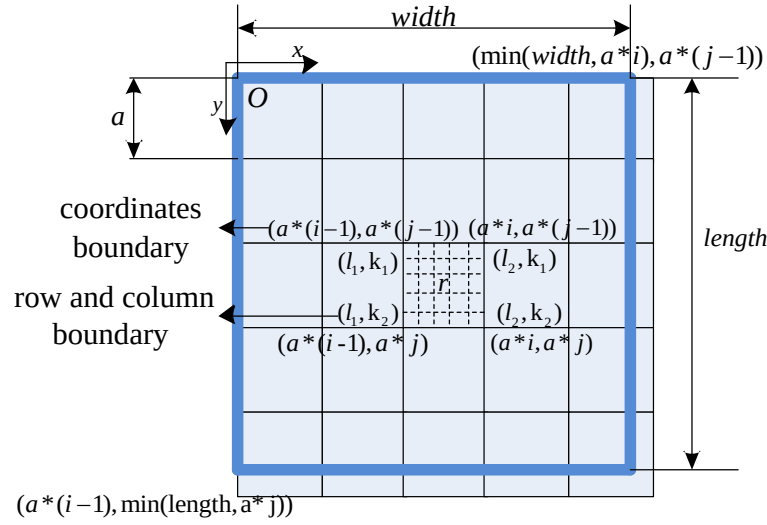


Figure 2. Illustration of map from elements to cells.

(3) Solution for the relative density matrix of cells. The boundary coordinates of the cell in row j and column i are calculated as:

$$\begin{aligned} x_1 &= a*(i-1), x_2 = \min(\text{width}, a*i), i=1, 2, \dots, m \\ y_1 &= a*(j-1), y_2 = \min(\text{length}, a*j), j=1, 2, \dots, n \end{aligned} \quad (6)$$

The number of rows and columns the cell locates in the density matrix of elements are given as:

$$\begin{aligned} \text{row boundary: } k_1 &= y_1 / r + 1, k_2 = y_2 / r \\ \text{column boundary: } l_1 &= x_1 / r + 1, l_2 = x_2 / r \end{aligned} \quad (7)$$

Then the relative density of the cell can be calculated with average method in the boundary of rows and columns:

$$x(j, i) = \frac{\sum_{l=l_1}^{l_2} \sum_{k=k_1}^{k_2} x_e(k, l)}{(l_2 - l_1 + 1) * (k_2 - k_1 + 1)} \quad (8)$$

4. Nonuniform honeycomb modeling

With the arrangement and density matrix of cells as input, modeling of nonuniform honeycomb structure is carried out. The PTC Creo is choose as structural modeling CAD software. User-Defined Feature (UDF) is used to simplify the modeling process. With the support of Pro/Toolkit library of Creo, we can develop the automatic modeling auxiliary application for nonuniform honeycomb structure.

The general flow of nonuniform honeycomb structure modeling is shown in figure 3. Firstly, we need to establish parametric model and references of the honeycomb cell features, and so on to complete the creation of cell UDF. The schematic diagram of square cell UDF is shown in figure 4. The key parameters and references in the diagram are shown in table 1, which constitute a parameterized UDF

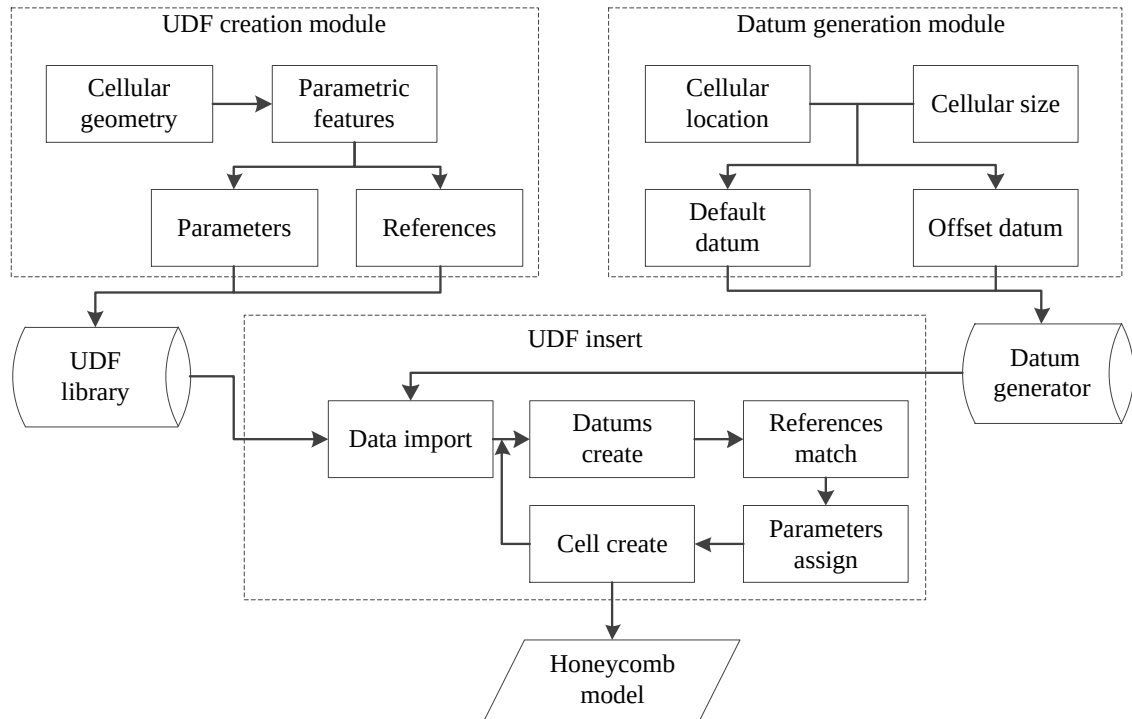


Figure 3. Flow chart of non-uniform honeycomb structures modeling.

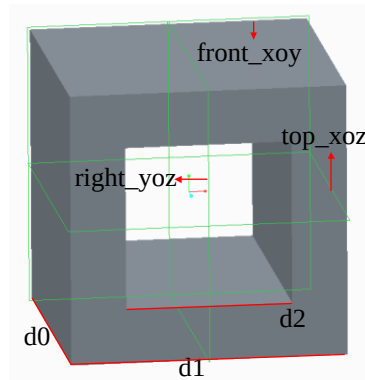


Figure 4. UDF of square cells.

Table 1. Parameters and references of square cells.

Parameters	Description	References	Description
d0	Out-of-plane thickness	front_xoy	Front view datum plane
d1	Cell length	right_yoz	Right view datum plane
d2	Cell hole dimension	top_xoz	Top view datum plane

The relationship between cell size parameters and cell density is satisfied as below, which can determine the basic parameters of each cell:

$$\frac{d1^2 - d2^2}{d1^2} = x \Rightarrow d2 = d1 * \sqrt{1 - x} \quad (9)$$

Secondly, the function of generating datums with location and size information is encapsulated as

datums generation module. The type of datums includes default datum plane and offset datum plane. Default datum plane is the standard orthogonal plane in the default coordinate system, which is the reference of the initial cell. Offset datum planes are referred by other cells. Based on the default datum, offset value and arrangement number are determined, where offset value is the cell length d_1 , and arrangement of the number in x and y direction is specified in Sec 3.

Finally, combine cell UDF with datums generation module, and import data of design domain size, cellular dimensions and cellular density matrix. With these data, datums can be generated, cellular references can be matched and values of cell parameters are assigned, so as to circular definite location and geometry of each cell. And nonuniform honeycomb CAD model is obtained.

5. Results and discussion

The object is shown in figure 1 and the Young's modulus and Poisson's ratio of cell wall material are $E = 7.2 \times 10^4 \text{ Pa}$ and $\nu = 0.3$. Optimization results and nonuniform honeycomb structure model are shown in figure 5 and figure 6. The model via density mapping and CAD modeling is complete without defects, and it is consistent with the density distribution of topology optimization.



Figure 5. Optimization results.

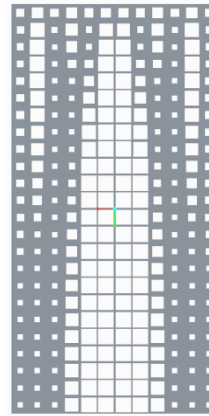


Figure 6. Honeycomb structure.

With ANSYS as the simulation tools, the compare of compressive stiffness of uniform and nonuniform honeycomb structures is carried out by FEA simulation. Compressive stiffness can be expressed as $k = F / u_y$, where u_y is the average displacement in y direction on the top, and F is the pressure. Deformations of uniform and nonuniform honeycomb structures are shown in figure 7 and figure 8.

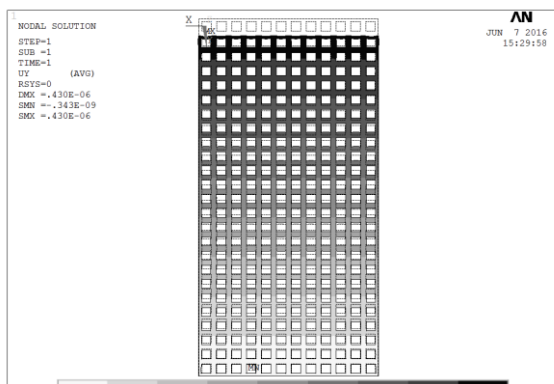


Figure 7. Deformation of uniform honeycomb.

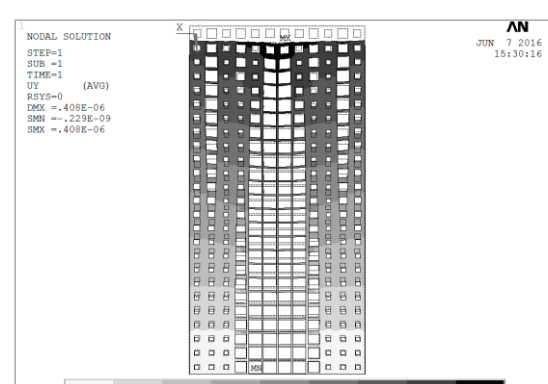


Figure 8. Deformation of nonuniform honeycomb.

Compressive stiffness of uniform honeycomb structure is calculated as 145197 N/mm , and nonuniform honeycomb structure is 164383 N/mm . At the same weight reduction, stiffness of

optimized nonuniform honeycomb structure is higher than uniform structure and the extent is 13.2%. The efficiency of the proposed modeling method is validated by FEA simulation.

6. Conclusion

(1) In this paper, a modeling method of non-uniform honeycomb structures based on topology optimization is proposed. Loaded structure is topology optimized, and the optimized density results are mapped to honeycomb cellular relative density distribution matrix. The rapid and automatic modeling of non-uniform honeycomb structures is realized with the using of User-Defined Features and the cyclic definitions of the reference datums.

(2) After density mapping and CAD modeling, the non-uniform honeycomb structure model is complete with no defects, which can reflect the density distribution of topology optimization well.

(3) Paper analyzes and compares the compressive stiffness of uniform honeycomb structure and non-uniform honeycomb structure. The results show that performance of optimized non-uniform honeycomb structure is better, and the effectiveness of uniform honeycomb structure modeling method based on topology optimization is verified.

7. References

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