Optimal location of piezoelectric patches for active vibration control

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Abstract. This paper focuses on finding the optimal location for a piezoelectric patch for minimizing the settling time of an excited isotropic and orthotropic plate. COMSOL Multiphysics has been used to design and model the plate with PID controller. Classical Optimization tool called Parametric Sweep has been used to achieve the objective of the experiment. Five different stacking sequences were used in the study of orthotropic plate. The results obtained by the FEA software indicated that by placing the piezoelectric patches at the optimal location, the settling time of a plate can decrease by 40% compared to placing it at the centre of the fixed end.

1. Introduction

The use of piezoelectric in the field of active vibration control has gained a lot of attention in the recent years due to their ability to minimize the settling time. Active vibration control is important in space application and in structures where it has low stiffness and high flexibility. In space application, the density of the atmosphere is almost zero which require the structure longer time to settle. In low stiffness structures, the damping effect will be too low to supress the structure in short time. The implementation of active vibration control has shown a good results to restore the structure to its equilibrium state in short time. The piezoelectric patches can be placed in different configurations, either bonded to the upper and lower surface or imbedded between layers of the material. The optimal location depends of the objective of the setup. Kumar et al [1] considered a flexible beam using LQR to control his model. Guadenzi et al [2] studied the effect of open and close loop circuit effect on a cantilever composite Beam. Bruant et al [3] studied a flexible beam and defined different optimal characteristics for the sensor and actuator patches. In his model, the optimal criterion for the sensor was to maximize the energy output from the patch, while, for the actuator the optimal location was based on minimize mechanical energy in the plate. Hac et al [4] considered the optimal location for the piezoelectric patches based on the observability and controllability degree of the structure. Biglar et al [5] tried to find the optimal location placement of a single patch of sensor and actuator over a cantilever steel plate. He implemented the genetic algorithms to increase the controllability and observability of the cantilever plate. Roy et al [6] proposed an improved genetic algorithm to be applied, in order to maximize the controllability of the structure. Kim et al [7] considered the optimal location based on minimizing the sound level generated by a vibrating beam. Nor

et al [8] used ant colony optimization method to determine the optimal placement of the actuator and sensor based on minimizing the energy of the plate.

2. Mathematical Modeling

In this work a cantilever plate made of isotropic/orthotropic material was considered with the dimension a, b and thickness t, as illustrated in Fig. 1
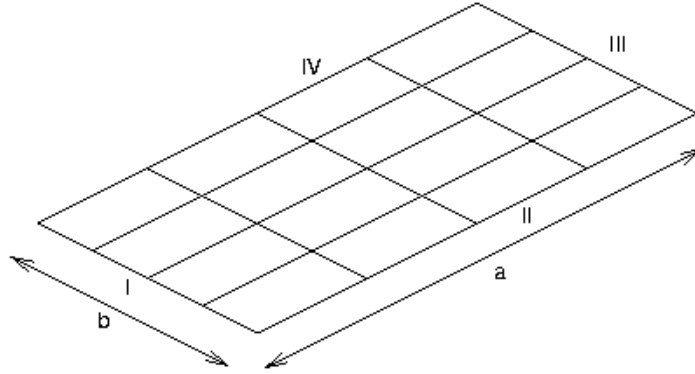

Figure 1. Isotropic plate model

According to Guedenzi [9], the governing equations that describe the piezoelectric effect are

$$\epsilon = S\,\sigma + d^T\,E \quad \text{... (2.1)}$$
$$D = \epsilon\,E + d\,\sigma \quad \text{... (2.2)}$$

Where $\boldsymbol{\sigma}$ is the stress vector, $\boldsymbol{S}$ is the compliance matrix, $\boldsymbol{\epsilon}$ the dielectric permittivity at zero stress, $\boldsymbol{D}$ is the electrical displacement and $\mathbf{E}$ is electrical field vector. For an isotropic plate the equation that relates the stress to the voltage applied can be expressed as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{E_p}{1-v_p^2} & \dfrac{v_p E_p}{1-v_p^2} & 0 \\ \dfrac{v_p E_p}{1-v_p^2} & \dfrac{E_p}{1-v_p^2} & 0 \\ 0 & 0 & \dfrac{E_p}{2(1-v_p)} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix} - \begin{bmatrix} \dfrac{E_{pz}}{1-v_{pz}^2} & \dfrac{v_{pz} E_{pz}}{1-v_{pz}^2} & 0 \\ \dfrac{v_{pz} E_{pz}}{1-v_{pz}^2} & \dfrac{E_{pz}}{1-v_{pz}^2} & 0 \\ 0 & 0 & \dfrac{E_{pz}}{2(1-v_{pz})} \end{bmatrix} \begin{bmatrix} d_{31} \\ d_{32} \\ d_{33} \end{bmatrix} \dfrac{V(t)}{t_{pz}} \quad \text{... (2.3)}$$

where $\boldsymbol{E_p}, \boldsymbol{v_p}, \boldsymbol{E_{pz}}$ and $\boldsymbol{v_{pz}}$ represent the modulus of elasticity, possion's ratio of the plate, modulus of elasticity of the piezoelectric patch and possion's ratio of the piezoelectric patch respectively.

In case of composite material, a different representation is required due to the fiber orientation. The governing equation in the case of composite plate is given as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix} - \begin{bmatrix} \dfrac{E_{pz}}{1-v_{pz}^2} & \dfrac{v_{pz} E_{pz}}{1-v_{pz}^2} & 0 \\ \dfrac{v_{pz} E_{pz}}{1-v_{pz}^2} & \dfrac{E_{pz}}{1-v_{pz}^2} & 0 \\ 0 & 0 & \dfrac{E_{pz}}{2(1-v_{pz})} \end{bmatrix} \begin{bmatrix} d_{31} \\ d_{32} \\ d_{33} \end{bmatrix} \dfrac{V(t)}{t_{pz}} \quad \text{... (2.4)}$$

The Q matrix is the reduced stiffness matrix that depends of the fibre orientation in the lamina.

In this paper, the optimal location of a single isotropic sensor and actuator patches to minimize the settling time of a cantilever orthotropic and isotropic plate. Proportional integral derivative controller has been adopted for this model. The gain values for the PID controller were calculated based on Ziegler–Nichols method theory of control. Six cases will be presented in this paper, Aluminium plate, Graphite/Epoxy plate with the following stacking sequences, $[0/0]_s$, $[90/90]_s$, $[45/45]_s$, $[0/90]_s$ and $[45/-45]_s$. The plates are modelled under the plane stress assumption i.e. $\sigma_z = \tau_{zx} = \tau_{zy} = 0$

3. Benchmark model

3.1 Controller model
The selected controller was PID. The governing equation of the PID controller is given by:

$$V_{applied} = K_p(V_{Measured} - V_{set}) + K_i \int_0^t (V_{Measured} - V_{set})\, dt + K_d \frac{\partial}{\partial t}(V_{Measured} - V_{set}) \quad \dots (3.1)$$

where K_P, K_I and K_D are the gain values for the PID controller, respectively. V_{set} is set to equal zero because the aim of the controller is to return the structure to its equilibrium condition. The following figure illustrates the relation between the generated voltage by the model and the applied controller.
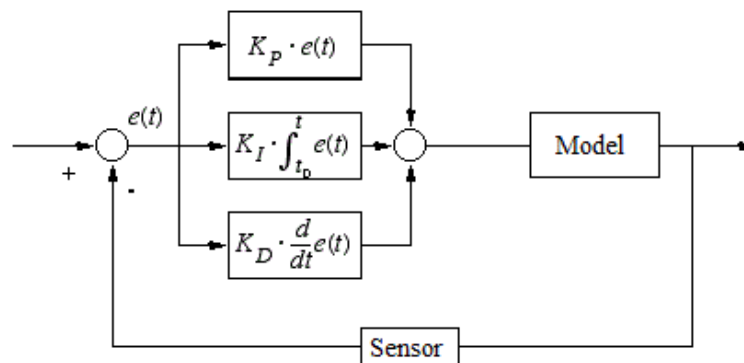

Figure 2. PID controller block diagram.

The gain values used in this simulation were:

Table 1: PID Gain Values used

	K_p	K_i	K_d
Gains	1.023	0.85	0.1

The gains are calculated based on the initial position which was centre position near the fixed end.

3.2 Validation

Before optimization of location of patches on isotropic and orthotropic plates using the COMSOL software, the software's capability in handling piezoelectric patches and composite laminates was first tested and validated against the following 2 standard benchmark case studies results
Case 1: The model of the Nechibvute et al [10] in which it evaluated the analytical voltage produced due to a force applied at the tip of a cantilever beam as shown in Figure 3. An Aluminium cantilever beam was considered with PZT-5H attached to the upper face of the cantilever beam. Using COMSOL Multiphysics, this problem was modelled and validated against Nechibvute et al results as shown in Table 2. It can be seen the results are very closer.
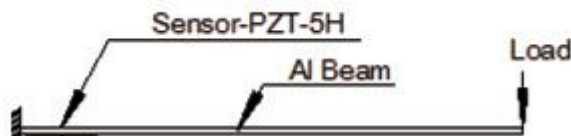

Figure 3. Composite Plate Geometry and Boundary Condition [10].

Table 2: Comparison between Nechibvute et al result and simulation results.

Amplitude of AL Beam tip (mm)	Open Circuit Voltage (V)	
	Nechibvute et al Experimental	Simulated
0.33	3.19	3.24
0.44	4.10	4.23
0.58	5.80	5.94
0.86	8.11	8.23
1.04	9.87	10.04
1.27	11.99	12.17
1.50	14.22	14.33

Case 2 : Reddy [11] model for finding the centre deflection for an orthotropic laminated plate under uniform pressure as shown in Fig. 4.
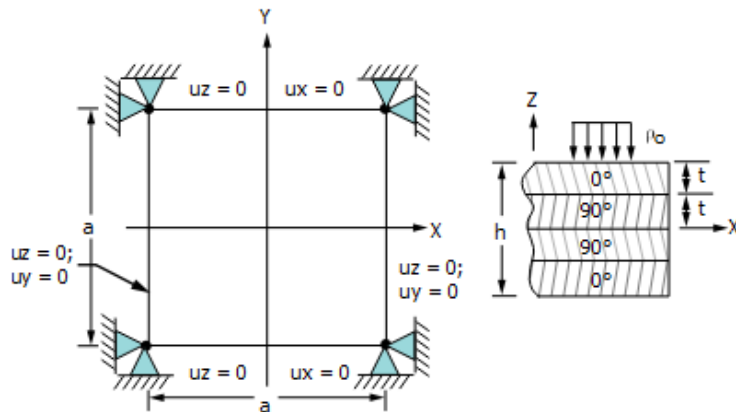
Figure 4. Composite Plate Geometry and Boundary Condition.

Composite material are defined by the orientation of the fibre with respect to the initial frame. In order to simulate the composite material in COMSOL we need to define a base vector system that is rotated with the desired orientation. The results that were obtained by COMSOL Multiphysics for the above problem is validated against the Reddy [11] results. The displacement at the centre of $[0/90]_s$ plate was evaluated by COMSOL and it was very close to the theoretical results of Reddy [11] with the error of 0.12966 %.

3.3 Optimization technique
In this model, classical optimization method called parametric sweep is adopted. Parametric sweep method is where the FEA software will work out on the optimal patch location for minimizing the settling time. For all plates, a force is applied at the free end for 5 seconds, which will ensure the structure is fully deflected, and then released to measure the time needed to reach settling condition. The isotropic plate, was divided into 16 location, for sensor and actuator to be tested, where x_1, y_1 and x_2, y_2 are the coordinates of the centre of the piezoelectric sensor and actuator, respectively.
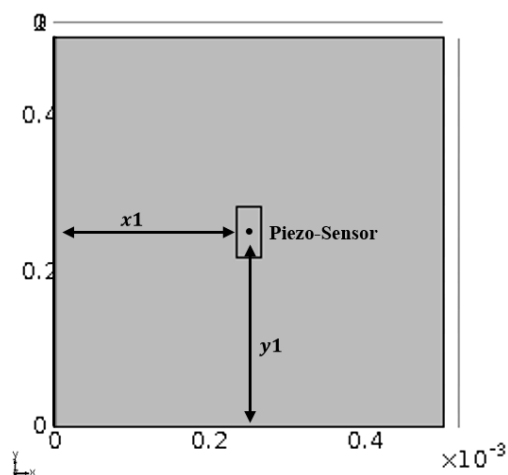

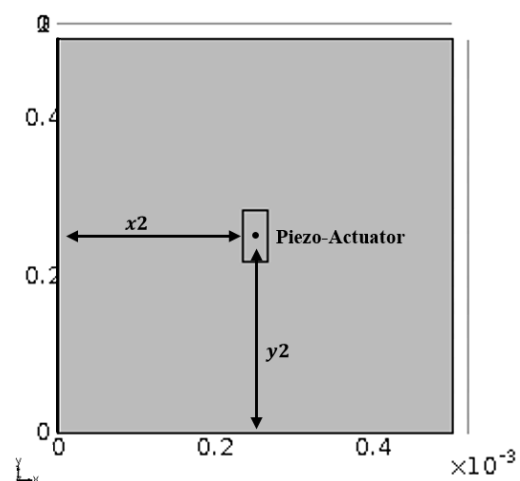**Figure 5.** Top view. Measurement of x_1, y_1 to the centre of the piezoelectric sensor patch.

Figure 6. Bottom view. Measurement of x_2, y_2 to the centre of the piezoelectric actuator patch.
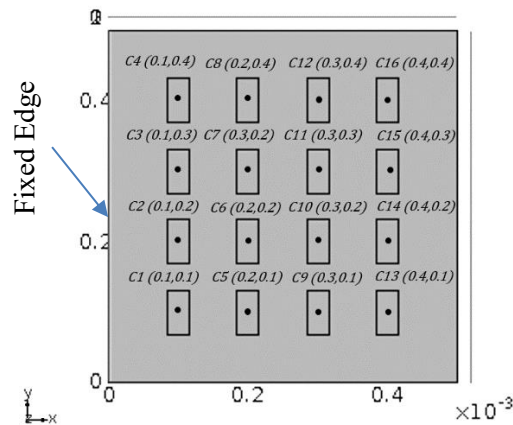
Figure 7. Top View. Location of PZT Sensor to be tested on the isotropic plate.
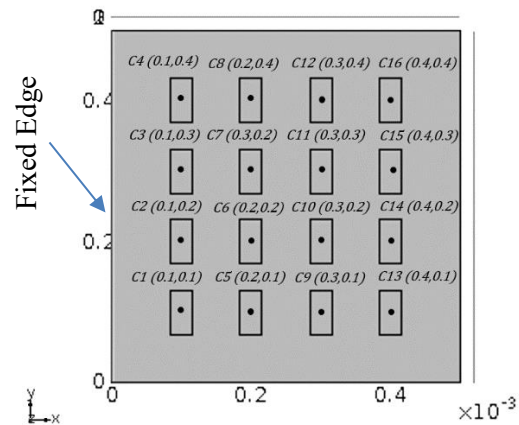
Figure 8. Bottom View. Location of PZT Actuator to be tested on the isotropic plate.

The orthotropic plate is defined in the same manner, but, the dimensions of the plate was changed so that the number of possible location combination of sensor and actuator patches is reduced. In the isotropic 265 location combination was tested to determine the best location to minimize the settling time. In the orthotropic, the number of location combination reduced to 81 per case, as this model will test five different cases for the orthotropic plate.
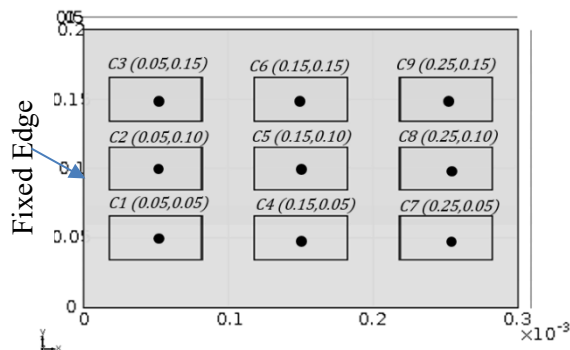

Figure 9. Top View. Location of PZT Sensor to be tested on the orthotropic plate.
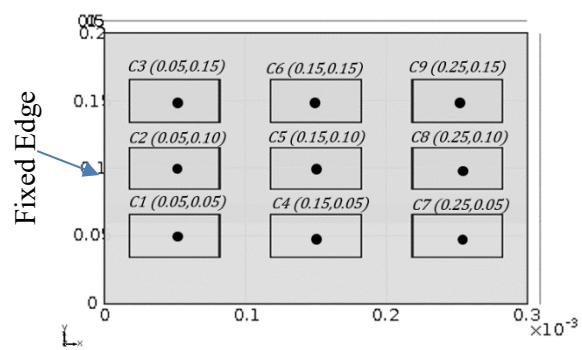
Figure 10. Bottom View. Location of PZT Actuator to be tested on the isotropic plate.

4. Results

4.1 Plate model

The following Tables 3 & 4 show the material properties of aluminium plate, piezoelectric (PZT-5H) patch and graphite/epoxy composite used in this analysis.

Table 3: Material Property of Al Plate and PZT-5H Patch

Parameter	Plate (Aluminium)	Piezoelectric (PZT-5H)

Length	0.5 [m]	31.8 [mm]
Width	0.5 [m]	63.5 [mm]
Thickness	2 [mm]	0.86 [mm]
Density	$2700\ Kg/m^3$	$7500\ Kg/m^3$
Young's Modulus	69 GPa	70 GPa
Passion's ratio	0.33	0.31
Piezoelectric Constant, d_{31}	-	-2.74e-010 [C/N]
Piezoelectric Constant, d_{32}	-	-2.74e-010 [C/N]
Loss factor	0.06	0.06

Table 4: Material Property of Graphite/Epoxy Plate

	Length	Width	Thick-ness	E_l	E_t	v_{12}	G_{lt}	ρ	α	β
Gr/Ep	0.3 [m]	0.2 [m]	0.25 [mm]	181 [GPa]	10.3 [GPa]	0.28	7.17 [GPa]	1600 Kg/m^3	1.0	5×10^{-5}

4.2 Isotropic Aluminium plate

A force of 10 N in applied at the free end for 5 seconds to allow full deflection of the plate and then released to measure the settling time. The results for the aluminium plate indicated that the best location is near the fixed end that would enable the actuator to produce a counter moment to supress the oscillation. The optimal location was to place the sensor is at C_1 in Fig. 5 while the actuator at C_4 in Fig. 6 . The following Fig. 11 shows the difference in settling time between optimal location compared to if it was placed mid plane near to the fixed end.
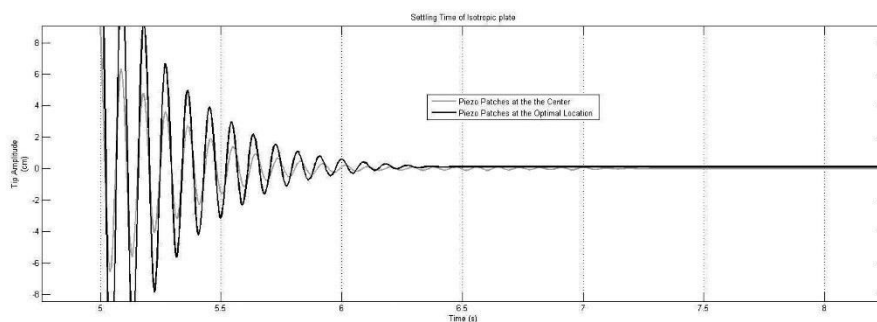

Figure 11. Comparing initial guess and optimal location for isotropic plate.

4.3 Orthotropic plate

In this section the results obtained for the orthotropic plate will be presented. All plates are made of 4 lamina with different stacking sequence. The initial guess for all of the cases was by placing the patches at C_2 as illustrated in Fig. 7 for both sensors and actuators. A force of 10 N in applied at the free end for 5 seconds to allow full deflection of the plate and then released to measure the settling time.

4.3.1 $[0/0]_s$

The unidirectional 0 degree plate, indicated that placing the patches at the optimal location would decrease the settling time required for the plate by 0.83 seconds. The 0 degree oriented fiber plate, needed more time to settle due to the lower density and internal damping as compared to the isotropic

plate. The optimal location of the patches for this plate turned out to be C_1 for the sensor patch and C_3 for the actuator patch.
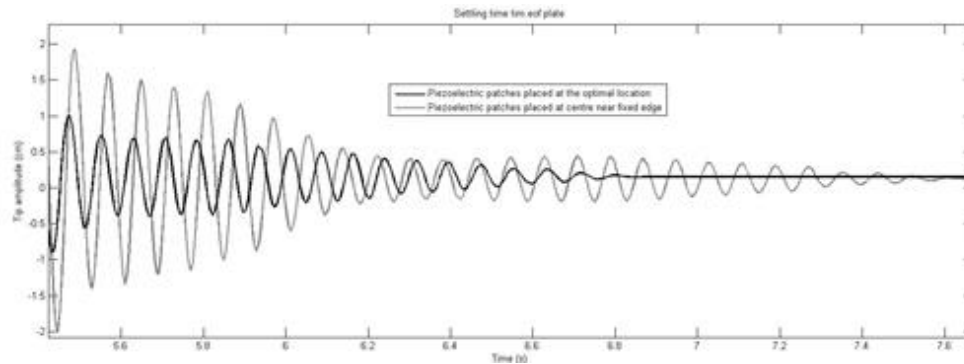

Figure 12. Comparing initial guess and optimal location for 0 degree plate.

4.3.2 $[90/90]_s$

The 90 degree laminate, required the longest time to settle. The reason that the moment being applied from the actuator isn't in the fiber direction. Placing the patches at the optimal location reduced the settling time to 2.76 second compared to placing the patches at the initial guess. The optimal location for this plate was also C_1 for sensor and C_3 for the actuator. Switching the location of the sensor and actuator gave similar results.
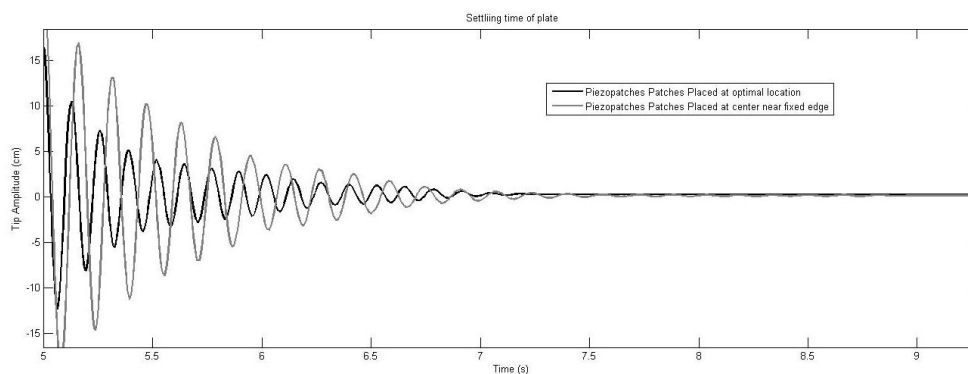

Figure 13. Comparing initial guess and optimal location for 90 degree plate

4.3.3 $[45/45]_s$

In the 45 degree laminate, the moment being applied from the actuator isn't in the fiber direction. Placing the patches at the optimal location reduced the settling time by 0.96 seconds compared to placing the patches at the initial guess. The optimal location for this plate was also C_1 for sensor and C_3 for the actuator. Switching the location of the sensor and actuator gave the same results for settling time.(Fig. 14)
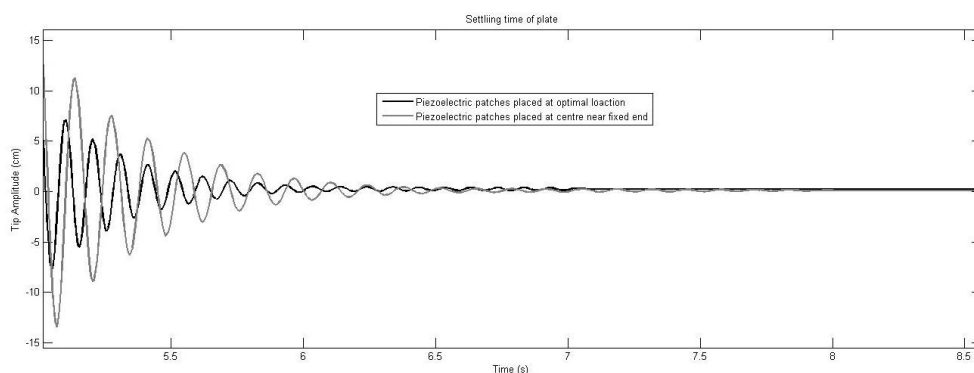
Figure 14. Comparing initial guess and optimal location for 45 degree plate

4.3.4 $[0/90]_s$

Vibration of the cross ply plate settled in shorter time by placing it at the optimal location. The optimal location for this plate turned out to be C_1 for sensor and C_3 for actuator. The settling time reduction between the optimal location and initial guess turned out to be around 45%. The optimal location allowed the structure to settle within 1.23 second from being released.
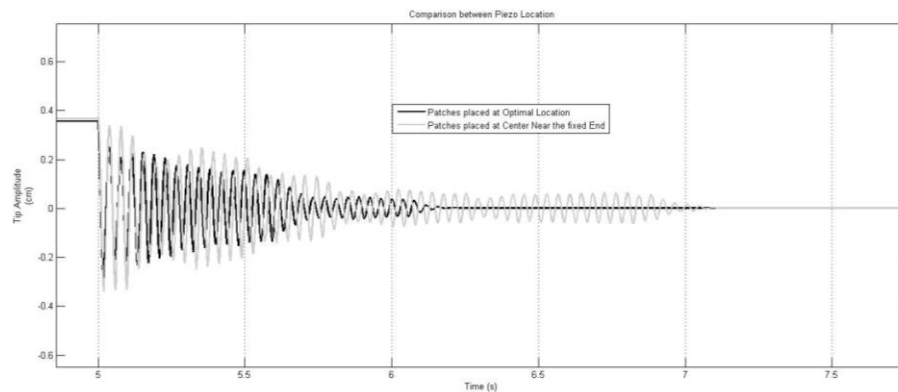

Figure 15. Comparing initial guess and optimal location for cross-ply plate

4.3.5 $[45/-45]_s$

The angle ply laminate, required more time to settle compared to the cross ply laminate. The plate required 2.54 seconds to settle even after placing the patches in the optimal location. The optimal location turned out to be, C_1 for sensor and C_3 for actuator. While switching the location of the patches gave the same output results.
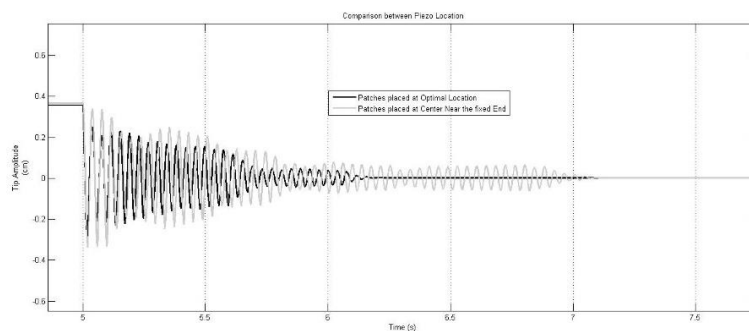

Figure 16. Comparing initial guess and optimal location for $[45/-45]_s$ plate

5. Conclusion

This study focused on finding the optimal location of patches to reduce the settling time of isotropic plate and orthotropic plate with different stacking. This study indicated that the settling time of the structure can be minimized by placing the patches at the optimal location. By reducing the settling time, the structure faces less internal force and shear stress which will increase the reliability of the structure during its life cycle. The optimal location for both actuator and sensor patch was near the fixed edge as the actuator will produce moment to counter the tip oscillating.

6. References

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