Heuristic for Critical Machine Based a Lot Streaming for Two-Stage Hybrid Production Environment

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Abstract. Lot streaming in Hybrid flowshop [HFS] is encountered in many real world problems. This paper deals with a heuristic approach for Lot streaming based on critical machine consideration for a two stage Hybrid Flowshop. The first stage has two identical parallel machines and the second stage has only one machine. In the second stage machine is considered as a critical by valid reasons these kind of problems is known as NP hard. A mathematical model developed for the selected problem. The simulation modelling and analysis were carried out in Extend V6 software. The heuristic developed for obtaining optimal lot streaming schedule. The eleven cases of lot streaming were considered. The proposed heuristic was verified and validated by real time simulation experiments. All possible lot streaming strategies and possible sequence under each lot streaming strategy were simulated and examined. The heuristic consistently yielded optimal schedule consistently in all eleven cases. The identification procedure for select best lot streaming strategy was suggested.

1. Introduction

Hybrid Flow type production environment or Hybrid Flowshop [HFS] is commonly encountered in many manufacturing environments in which two stage HFS is prominent. Lot streaming is an effective tool for time-based manufacturing strategy. Lot streaming is a practice of subdividing the production lots into smaller sub-lots in a multi-stage production systems. By use of this technique the operations of a given lot can be overlapped and minimized the manufacturing makespan [2]. However, this kind of problems received less attention to the researchers [1]. Tsubone et al. [3] investigated the impact of sequencing rules, scheduling scenarios and lot size, on makespan, work-in-process inventory and resource utilization by using simulation in a two stage HFS with lot streaming. Zhang et al. [4] considered m-1 HFS lot streaming problem with special cases of the equal-sublot version and proposed two heuristics which enumerated the number of sublots and scheduling them. This work proposed only one heuristics for both lot streaming and scheduling for all cases of HFS considered.

Chao-Tang Tseng and Ching-Jong Liao [5] dealt a single-job lot streaming problem in a two-stage hybrid Flowshop in which '*m*' identical machines at the first stage and only one machine in second stage. They used a rotation method for sequencing and scheduling then lot sizes were

optimized by using linear programming.   Ming Cheng et.al., [6] developed a comprehensive mathematical model for scheduling the hybrid flowshop under lot streaming. They also studied a two-stage hybrid flow shop in which only one machine at first stage and the second stage has two identical parallel machines. For solving this problem they developed mathematical programming-based heuristic methods. Mohsen Nejati et al. [7] addressed a two-stage assembly scheduling problem under lot sizing environment in which they considered 'm' machines at the first stage and 'n' assembly machines on the second stage. Their objective was to minimize the sum of weighted completion times with better machine utilization and they proposed simulated annealing and genetic algorithm for computing the optimal sequence for scheduling. Biao Zhang et al [8] proposed a mathematical model and an effective modified migrating bird's optimization (EMBO) to solve the hybrid flowshop; hybridizing with lot streaming problems within an acceptable computational time.

   The shortest waiting time rule was preferred to schedule jobs which arrived concurrently for processing. Quan-Ke Pan et al., [9] discussed a rare case of hybrid flowshops with due windows. The authors aimed to minimize the weighted earliness and tardiness from the due window and presented a comprehensive computational campaign for solving the problem. This work is unique. Here the work order is initially divided into six equal groups. In each group containing similar parts and having same sequence of processing and processing times at the respective stages. Then the sub-lots are prepared by combining the groups or considering single group to process them in the two-stage HFS environment. This paper investigates lot streaming strategies in two-stage HFS to optimize lot streaming by developing a heuristic with local optima and globalize the solution based of set out successful results.

2. Two-Stage hybrid Flowshop scheduling Problem

Ruiz et al., [10] discussed in detailed about the scheduling for the flow shops with multiple parallel machines per stage usually referred to as the Hybrid Flow Shop. It is a complex combinatorial problem. Djellab and Djellab [11] hybrid flowshop encountered in many real world scenarios like the industries of paper, Electronics, photographic film, Concrete production, Container handling, textile, and internet service architectures etc.. These kinds of flowshops are common manufacturing environments in which 'J' number of jobs to be processed in serial order at 2 stages. Ether stages may have more than one parallel processor, but at least one stage must have only one processor. Job may skip the stages, but it must be processed at least only one stage. The configuration of two-stage hybrid flowshop is furnished in figure1. According to this problem the jobs provided for processing of sub-lot as per group schedule. The problem is to find a group schedule which optimizes a given objective function. The hybrid flowshop problem is, in most cases, NP-hard [12]. For instance, the two stage hybrid flow shop containing two machines at one stage and the other one a single machine, However, the problem might be polynomially solvable with some special properties and precedence relationships [13].
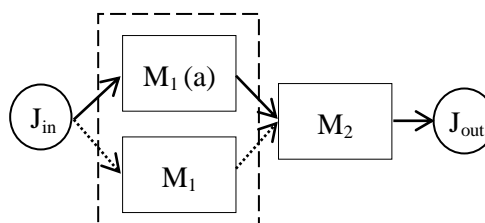

Figure 1. Two stage hybrid Flowshop

3. Mathematical Modelling

3.1. Parameters

J_g = The job J_g (J_g=1, 2,…, 200) of part family type, i.e., group type **g (g = 1, 2,…, 6),** where **200** total number of jobs to be processed in a group i.e., group size; Where **6** is total number group types are to be scheduled. $J_g \in J$; Where J=1200 is total Number of jobs to be

scheduled.

$(P_i)_{Jg} =$ Processing times of job J_g of group type **g** at i^{th} stage, **i(i=1,2,…s)** where **s =2**; that is the total number of stages.

m_i = Number of parallel identical processor available in stage **i**.

T = Total number of time units in scheduling jobs

$(C_i)_{Jg}=$ Completion time of job **J** of type **g** at the stage **i.**

It is a time point $(C_i)_{Jg} = t$ means that the operation completes at the end of the time unit **t**.

3.2. Decision variables

$(\delta_i)_{Jg} t = $ {1   if **Jg** is processed at the stage **i** in the time unit **t;**
            0   other wise   }

With the above notation the hybrid flow shop problem (HFSP) under the consideration can be formulated as follows.

Note that $(C_2)_{Jg}$ in the model is the final completion time of the male job **J** of type **g**, C_k after processed at require **2** stages, in other words it is the completion time of a job of group type **g.**

3.3. Mathematical Model

Usually in the batch processing the makespan is the completion time of the last job of the group. If there are '**g**' numbers of group means, then the makespan will be the completion time of the last job of the last group. Hence it is clear that the objective is to minimize the completion time of all jobs of all group types. Mathematically, the objective function can be defined as

$$\textbf{Minimize} \sum_{g=1}^{6} \sum_{J_g=1}^{200} (C_2)_{Jg} \tag{1}$$

Subject to

$$(C_i)_{Jg} \leq (C_i)_{Jg} + 1 - (P_i)_{Jg}$$

$i\ (i = 1,2);\ g\ (g = 1,2,…,6);\quad J_g\ (J_g=1,2,…,200);\qquad J_g \in J;\ J=1200; \tag{2}$

$$\sum_{t=1}^{T_m} (\delta_i)_{Jg} t = (P_i)_{Jg}$$

$,\ i\ (i = 1,2);\ g\ (g = 1,2,…,6);\ J_g\ (J_g=1,2,…,200);\ J_g \in J;\ J=1200;\ t = 1,2,…,T; \tag{3}$

$$t * (\delta_i)_{Jg} t \leq (C_i)_{Jg},$$

$i(i = 1,2);\ g(g = 1,2,…,6);\ J_g(J_g=1,2,…,200);\ J_g \in J;\ t=1,2,…,T;\ J=1200; \tag{4}$

$$(C_i)_{Jg} - (P_i)_{Jg} + 1 \leq t + T(1 - (\delta_i)_{Jg} t),$$

$i\ (i = 1,2,…,s);\ g\ (g = 1,2,…,x);\ J_g\ (J_g=1,2,…,200);\ J_g \in J;\ t = 1,2,…,T;\ J=1200 \tag{5}$

$$\sum_{Jg=1}^{e} (\delta_i)_{J_g} t \leq m_i,$$

$i\ (i = 1,2);\ g\ (g = 1,2,…,6);\ J_g\ (J_g=1,2,…,200);\ J_g \in J;\ t = 1,2,…,T;\ J=1200 \tag{6}$

$$(\delta_i)_{Jg} t \in \{0,1\},$$

$i\ (i = 1,2);\ g\ (g = 1,2,…,6);\ J_g\ (J_g=1,2,…,200);\ J_g \in J;\ t = 1,2,…,T;\quad J=1200 \tag{7}$

$(C_i)_{Jg} \in \{1,2,…,T\},\qquad i\ (i = 1,2);\ J_g(J_g=1,2,…,200);\qquad J_g \in J;\ J=1200 \tag{8}$

In the above formulation, the objective is to determine a schedule that minimizes the total completion time, while satisfying all the constraints as listed above. The objective function also helps to achieve reduced work-in- process inventory, avoid delays group submission [tardy job reduction] and improved customer services, etc. The constraint [2] represents an operation precedence among the stages of a job and ensures that an operation cannot be started until the operation of the same job at its preceding stage is finished. Constrains [3] – [5] define the time intervals for which a job is processed on a machine at a stage. Constraint [6] indicates that all these machine requirements was satisfied with the number of available machines at that time. Constraints [7] & [8] define value ranges of the variables.

4. Solution Procedure

4.1. Assumption

1. Total job can be classified as equal sized g number of groups based on similar processing time as sequence.
2. First sub-lot in the group schedule available at zero time. Subsequent sub-lots will be dispatched to the shop floor as soon as the completion of the existing sub-lot.
3. The sub-lots may or may not be equal in size [containing number of groups].
4. The pre-emption not permitted.

4.2. Heuristic Approach

Step 1: Calculate the cumulative processing time of each job Ji.

$$J_i = \sum_{j=1}^{M} P_{i,j} \forall N; i \in N$$

Step 2: Calculate the cumulative processing time of all the jobs Tp.

$$Tp = \sum_{t=1}^{g} \sum_{i=1}^{N} J_{ti}$$

Where *t* is no of job types, *i* is no of jobs in each type.

Step 3: Compute the group mean m where Tp is total processing time

$$m = \frac{Tp}{k}$$

Where '*k*' is desirable number of sub-lots

Step 4: Sub-lot formation

To find the optimal group schedule to form the sub-lots as such their total processing time deviate from the mean 'm' on both directions [increase as well as decrease] must be equal as well as minimal and hence algebraic sum of the deviations of the sub-lot processing times zero. The sum of deviations of sub-lots from it their mean value nearly equal to zero will offer near optimal solution.

The heuristic needs to be validated and verified with well known problem. The two-stage hybrid flow shop problem is discussed in Numerical example.

4.3. Simulation Model

The simulation modeling and analysis give real time value and economical to verify and validate the strategies, schedules etc. [Saravanan and Raju 2010]. The authors used such modeling analysis to verify and validate their heuristic solution for dynamic parallel hybrid flowshop scheduling problems. However, in the majority of the situations, simulation is used as a performance evaluation tool. Simulation offers the flexibility to model the complexities adequately, while the gradient computation helps in identifying a good solution quickly. Besides using simulation as a tool for optimization, this work also makes use of it to compare the performance of different capacity management schemes,

providing clear conclusions about their relative performances. Simulation is a very powerful tool for the analysis and evaluation of complex systems. In many circumstances, it is used when analytical models are neither available nor easy to obtain. Traditionally, simulation is used to compare configurations, different policies, validate models, and many other qualitative features with the purpose of answering what if questions. Hence, for verifying and validating the proposed heuristic for a two stage hybrid Flowshop, the simulations modeling and analysis technique was employed. The modeling was done in extend v6. The model was verified and validated properly.

4.4. Makespan Computation

Makespan computation procedure for the example group schedule Gs = *{(l₁& l₂) - (l₃& l₄) - (l₅& l₆)}* which contain three sub-lots with equal group size of two groups per sub-lot.

$$C_j = \max\{\sum_{j1=1}^{200}(C_2)_{j1}, \sum_{j2=1}^{200}(C_2)_{j2}\} +, \max\{\sum_{j3=1}^{200}(C_2)_{j3}, \sum_{j4=1}^{200}(C_2)_{j4}\} + \max\{\sum_{j5=1}^{200}(C_5)_{j5}, \sum_{j6=1}^{200}(C_5)_{j6}\}$$

Makespan computation for the example group schedule *{(l₁, l₂ & l₃) - (l₄, l₅ & l₆)}*, which contain two sub-lot with equal sub-lot size of three groups per sub-lot, is:

$$C_j = \max\{\sum_{j1=1}^{200}(C_2)_{j1}, \sum_{j2=1}^{200}(C_5)_{j2}, \sum_{j3=1}^{200}(C_2)_{j3}\} + \max\{\sum_{j4=1}^{200}(C_2)_{j4}, \sum_{j5=1}^{200}(C_2)_{j5}, \sum_{j6=1}^{200}(C_2)_{j6}\}$$

.

Makespan computation for the example group schedule {*(l₁, l₂, l₃ & l₄) - (l₅ & l₆)*} which contain two sub-lot with equal sub-lot sizes of four and two groups per sub-lot respectively, is:

$$C_j = \max\{\sum_{j1=1}^{200}(C_2)_{j1}, \sum_{j2=1}^{200}(C_5)_{j2}, \sum_{j3=1}^{200}(C_2)_{j3}, \sum_{j4=1}^{200}(C_2)_{j4}\} + \max\{\sum_{j5=1}^{200}(C_2)_{j5}, \sum_{j6=1}^{200}(C_2)_{j6}\}$$

Makespan computation for the example group schedule {*(l₁, l₂, l₃, l₄ & l₅) - (l₆)*}which contain two sub-lot with sub-lot sizes of the five groups and one group per sub-lot respectively, is:

$$C_j = \max\{\sum_{j1=1}^{200}(C_2)_{j1}, \sum_{j2=1}^{200}(C_2)_{j2}, \sum_{j3=1}^{200}(C_2)_{j3}, \sum_{j4=1}^{200}(C_2)_{j4}, \sum_{j5=1}^{200}(C_2)_{j5}\} + \{\sum_{j6=1}^{200}(C_2)_{j6}\}$$

Makespan computation for the example group schedule {*(l₁, l₂, l₃, l₄, l₅ & l₆)*} which contain only one lot of sub-lot size of six groups is:

$$C_j = \max\{\sum_{j1=1}^{200}(C_5)_{j1}, \sum_{j2=1}^{200}(C_5)_{j2}, \sum_{j3=1}^{200}(C_5)_{j3}, \sum_{j4=1}^{200}(C_5)_{j4}, \sum_{j5=1}^{200}(C_5)_{j5}, \sum_{j6=1}^{200}(C_5)_{j6}\}$$

5. Numerical Example

The two stage hybrid flowshop which contains two machines in the first stage and only one machine in second stage. There are 1200 jobs to be processed. The jobs can be classified as uniform groups as 6 types with each group has equal amount of jobs 200. The processing time of each job type are presented in the Table 1. The maximum possible eleven types of lot streaming strategy were furnished in Table 2 and the number of lot streaming schedules available are also shown against the each case.

Let's discuss some example cases. If the group schedule is {$(g_1, g_2, g_3, g_4, g_5 \& g_6)$} means the sub-lot sizes are {1,1,1,1,1&1} and number of sub lots 6 similarly for group schedule {$(g_1\&g_2) - (g_3\&g_4)$ -

(g_5&g_6) is {2, 2, & 2} and number of sub-lots is 3, for group schedule (g_1, g_2 & g_3) - (g_4, g_5 & g_6) is {3 & 3} and number of sub-lots is 2, for group schedule (g_1, g_2, g_3 & g_4) - (g_5 & g_6) is {4 & 3} and number of sub-lots is 2, and for group schedule {(g_1, g_2, g_3, g_4& g_5) - (g_6)} is {5 & 1} and number of sub-lots is 2. The makespan is the completion time of the last job in the group schedule. The makespan depends on factors such as processing time, processing sequence, job availability, machine availability and intermediate waiting time. The scheduling is the task of allocating resources by the way optimizing above parameter to achieve the minimum makespan. Hence the problem is to find optimal the group schedule. Group scheduling involves two levels. The scheduling the sub-lot is Level-II, which includes sub-lot formation and order of processing. The scheduling jobs within the sub-lot is called as Level-I.

5.1. Heuristic Approach for Lot Streaming
The Lot streaming is processes of forming sub-lots and group scheduling. The heuristic is applied here to form the sub-lots. There are 11 possible strategies of lot streaming cases were derived. In those strategies all possible lot streaming schedules (***Gs***) were simulated and makespan computed as discussed below. The optimal schedules were found and listed in the Table 2 for strategy wise. The mean ***m*** deviation of each sub lot processing times '***d***' and algebraic sum of those deviation ∑***d*** were tabulated in Table 2. The Sub-lot sizes were possibly varied from 1 group [200 jobs] to 6 groups [1200 jobs]. The optimal schedules posses sub-lots which deviate uniformly up and down and hence their cumulative deviations are zero and nearly zero. Hence it is proved that the heuristic solution provides optimal schedule in all 11 possible strategies by yielding minimal makespan. Such schedules are verified as validated by simulating all possible cases in under each strategy.

Table 1 Processing Time of Job types

Jobs	Stage 1	Stage 2	Total
J_1	5.42	1.50	6.92
J_2	2.70	4.94	7.64
J_3	2.86	2.90	5.76
J_4	4.22	3.94	8.16
J_5	5.86	1.94	7.80
J_6	4.50	3.66	8.16

6. Results & Discussion
From the illustrative case it is clear that the optimal or near optimal makespan can be achieved by using heuristic for any strategy of lot streaming. The makespan variation influence of the stage wise processing time, the number of sub-lots, groups in the sub-lot etc. The increase of the number of sub lot may increases the makespan. The optimal makespan performance based on strategies was shown in figure 2. The queue status at the critical machine with respect to strategy was illustrated in the figure 3 and figure 4. It is noticed that the lot streams case 5 and case 8 experienced lesser average queue lengths and average job waiting time than other strategies at the critical machine.
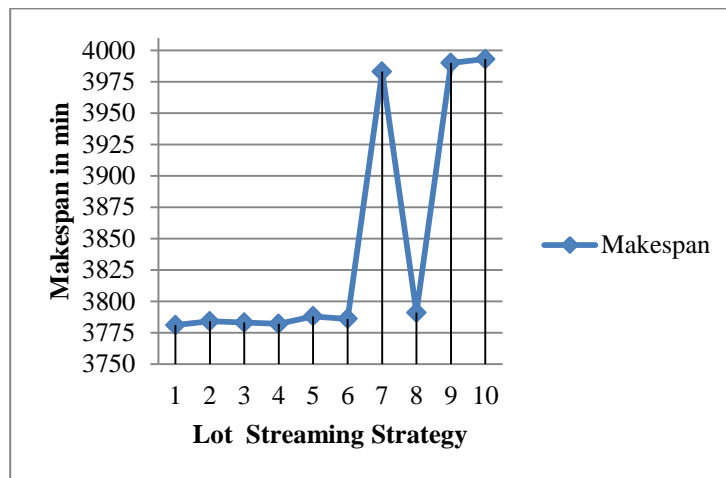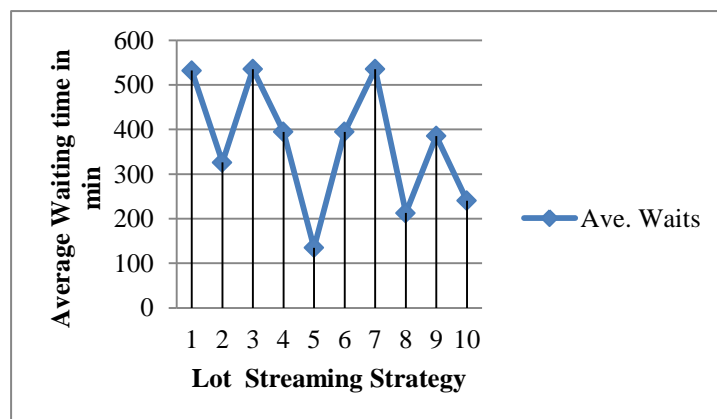
Figure 2. Lot streaming Vs optimal Makespans

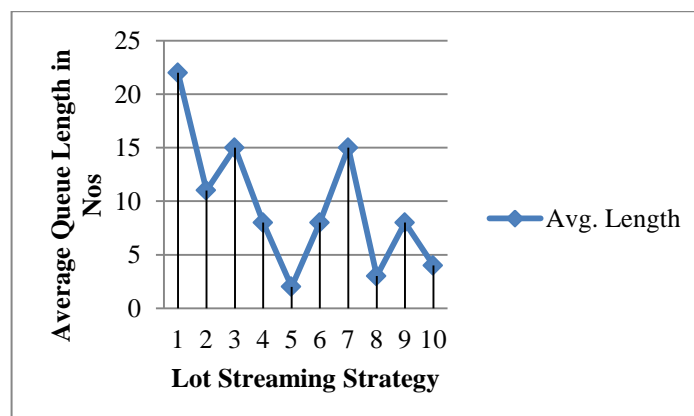**Figure 3.** Lot streaming Vs Average Waiting time at critical
Machine

Figure 4. Lot streaming Vs Average Queue length at critical
Machine

But case 5 experiences minimal queue status because the sub-lots formed in case 5 has 2 groups per sub lot equally and their processing times deviations are minimized and a number of sub-lots minimum [Three sub lots] compared case 11. Hence the best lot streaming strategy has evenly distributed groups

in the sub-lots and the sub-lots deviation from its mean is minimized and possibly a less number of sub-lots.

7. Conclusion

The proposed heuristic for lot streaming type group scheduling for two stage hybrid flow shop problem was discussed. The heuristic yielded minimal makespan among all possible group schedules under that strategy. It works well, irrespective of strategy of lot streaming. The best of best strategy selection also recommended. This heuristic solution may suit for similar problems irrespective of number of groups

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