

Determination of dynamic mechanical properties of silica aerogel by resonance and non-resonance method

Z P Qu and M P Sheng*

School of Marine Science and Technology, Northwestern Polytechnical University, Xi'an 710072, China

E-mail: *smp@nwpu.edu.cn

Abstract. Silica aerogel has great potential in aeronautics, building construction, and offshore oil transportation. The dynamic mechanical properties of silica aerogel were measured by the resonance and non-resonance method. The results of the two methods were compared. While the Young's modulus coincided well, the loss factor had big difference. To explain this phenomenon, the sensitivity of the two methods was analyzed. It shows that the loss factor determination of low-loss materials by the non-resonance method is quite sensitive to the experiment uncertainties. This probably results in the loss factor difference between the two methods.

Key words: silica aerogel, dynamic, Young's modulus, loss factor, resonance, non-resonance

1. Introduction

Silica aerogel, due to its excellent thermal insulating capability and low density, has great potential in aeronautics, building construction, and offshore oil transportation. The thermal properties of the material have been studied extensively [1,2]. However, the mechanical properties have only been studied by a few works, which mainly focused on the static modulus [3].

As noise-vibration engineers, we care most how the material properties vary within the frequency range, namely, the dynamic properties. The dynamic modulus and loss factor, representing the energy storage and dissipation capability respectively, are two most important parameters. However, few works about this could be dated.

In this work, we use the resonance and the non-resonance methods to measure the dynamic mechanical properties of silica aerogel. The two methods are usually used to measure the properties of polymers [4,5]. Lately they have also been applied to foam-like materials [6]. First the two methods were briefly described. Then they were used to measure the properties of silica aero, and the results were compared. Both the Young's modulus and the loss factor rise slightly as the frequency increases. The Young's modulus obtained by the two methods coincides well, but the lost factor has big difference. Finally the sensitivity analysis was made, which reveals the difference causes and helps to judge the data reliability.

2. Description of the resonance and non-resonance method



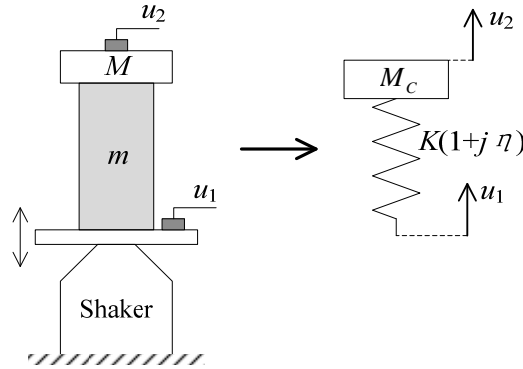


Figure 1. Set-up and equivalent model of the resonance method.

Figure 1 shows the set-up and equivalent model of the resonance method. One end of the specimen is excited by a shaker, while the other end is loaded by a mass. The displacements of the two ends are measured by two accelerometers. Just as the mass is heavy enough, the set-up can be modeled as a mass-spring system near the first resonance frequency. Thus the displacement transfer function from the excited end to the other is

$$H = |H|e^{j\varphi} = \frac{u_2}{u_1} = \frac{K(1+j\eta)}{K(1+j\eta) - M_c} \quad (1)$$

where the spring stiffness is related to the specimen's properties by $K = ES/l$, η is the loss factor of the material, and the equivalent mass is $M_c = M + m/3$. E , S and l represents the Young's modulus, the cross-section area and length of the specimen, respectively. At the resonance frequency, the phase of the transfer function satisfies $\varphi = -\pi/2$. Substituting it into equation (1) gives

$$E = \frac{\omega^2 l M_c}{S} \frac{|H|^2}{|H|^2 + 1} \quad (2)$$

$$\eta = \frac{1}{|H|} \quad (3)$$

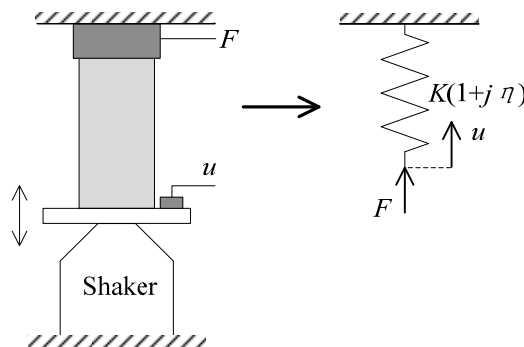


Figure 2. Set-up and equivalent model of the non-resonance method.

Figure 2 shows the set-up and equivalent model of the non-resonance method. One end of the specimen is excited by a shaker, while the other end is fixed to a rigid wall. The displacement of the moving end and the force of the fixed end are measured by an accelerometer and a fore transducer, respectively. The specimen can be modeled as a spring well below the first resonance frequency. Thus the transfer function from the displacement to force is

$$H = |H|e^{j\varphi} = \frac{F}{u} = K(1 + j\eta) \quad (4)$$

So the modulus and the loss factor are

$$E = \frac{|H|l \cos \varphi}{S} \quad (5)$$

$$\eta = \tan \varphi \quad (6)$$

3. Description of the resonance and non-resonance method

The Young's modulus and loss factor of the silica aerogel are investigated. Three cuboid specimens with length 20mm, 40mm and 60mm, as shown in figure 3, are measured by the resonance method. The cross-section of the specimens is a 10mm × 10mm square. The 60mm specimen is also measured by the non-resonance method. The material density is 316 kg/m³ and the loading mass is 11.8g. The ambient temperature is about 20°C.

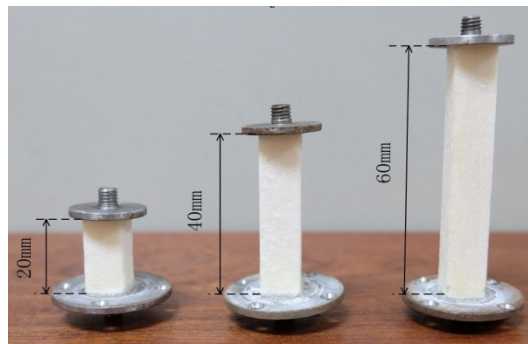


Figure 3. Three experimental specimens.

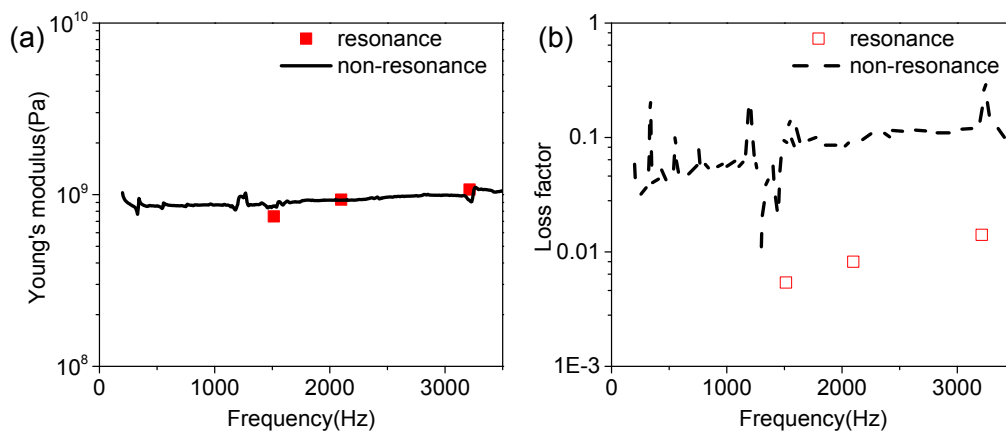


Figure 4. Experimental results, (a) Young's modulus and (b) loss factor of the silica aerogel.

Figure 4 shows the measured Young's modulus and loss factor of the silica aerogel. Three separate resonance frequencies are measured, each for one specimen. The continuous curves are the results of the 60 mm specimen. The Young's modulus coincides well. It increases slightly with the frequency increasing. However, the loss factor departs largely. The non-resonance result is about one order greater than the resonance result. In addition, the loss factor of the non-resonance method fluctuates more severely than the Young's modulus.

4. Discussion based on the sensitivity analysis

For the resonance method, the errors in E and η caused by the uncertainties in $|H|$, M_c , l , S and ω can be represented as

$$\begin{aligned}\Delta E/E &\approx D_{E|H|}(\Delta|H|/|H|) + D_{EM_c}(\Delta M_c/M_c) + D_{El}(\Delta l/l) \\ &\quad + D_{ES}(\Delta S/S) + D_{E\omega}(\Delta\omega/\omega) \\ \Delta\eta &\approx D_{\eta|H|}(\Delta|H|/|H|)\end{aligned}\quad (7)$$

where D_{yx} relates to the partial derivatives of y by x , and are called the error coefficients. They are computed as follows

$$\begin{cases} D_{E|H|} = (|H|/E)(\partial E/\partial|H|) = 2\eta^2/(1+\eta^2), & D_{EM_c} = (M_c/E)(\partial E/\partial M_c) = 1 \\ D_{El} = (l/E)(\partial E/\partial l) = 1, & D_{ES} = (S/E)(\partial E/\partial S) = -1 \\ D_{E\omega} = (\omega/E)(\partial E/\partial\omega) = 2, & D_{\eta|H|} = (|H|/\eta)(\partial\eta/\partial|H|) = -1 \end{cases}\quad (8)$$

For the non-resonance method, the errors in E and η caused by the uncertainties in $|H|$, ϕ , l and S can be represented as

$$\begin{aligned}\Delta E/E &\approx D_{E|H|}(\Delta|H|/|H|) + D_{E\phi}\Delta\phi + D_{El}(\Delta l/l) + D_{ES}(\Delta S/S) \\ \Delta\eta &\approx D_{\eta\phi}\Delta\phi\end{aligned}\quad (9)$$

where D_{yx} are computed as follows

$$\begin{cases} D_{E|H|} = (|H|/E)(\partial E/\partial|H|) = 1, & D_{E\phi} = (\phi/E)(\partial E/\partial\phi) = -\eta \\ D_{El} = (l/E)(\partial E/\partial l) = 1, & D_{ES} = (S/E)(\partial E/\partial S) = -1 \\ D_{\eta\phi} = (1/\eta)(\partial\eta/\partial\phi) = \eta + 1/\eta \end{cases}\quad (10)$$

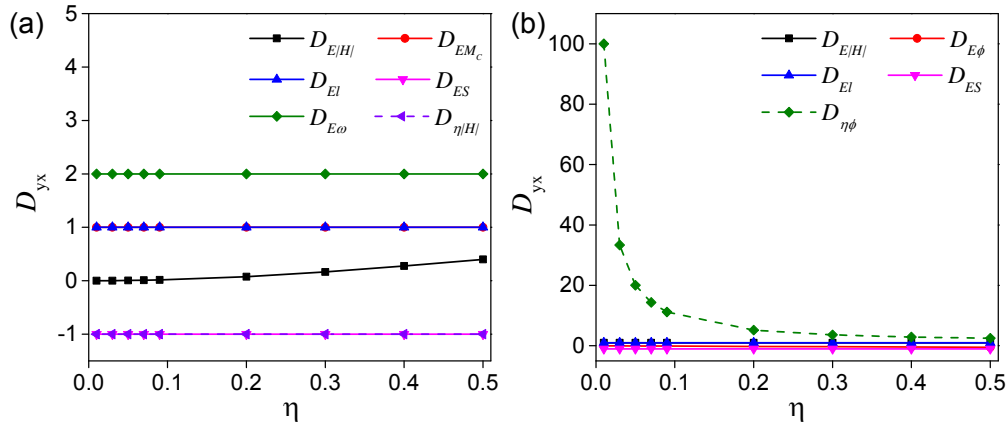


Figure 5. Error coefficients, (a) Resonance method and (b) non-resonance method.

As is evident from equations (8) and (10), the error coefficients D_{yx} are determined by the loss factor η individually. Figure 5 shows how D_{yx} vary with η for the two methods. From figure 5 (a) we see that the absolutes of D_{yx} are small for the resonance method. Apart from $D_{E|H|}$, all the coefficients are constant. $D_{E\omega}$ is the biggest with a value of 2, which means that the resonance frequency ω uncertainties will be magnified by 2 times. D_{EM_c} , D_{El} , D_{ES} and $D_{\eta|H|}$ are smaller with a value of 1 or -1. $D_{E|H|}$ rises slightly as η increases, but is always smaller than 1. So the experimental

uncertainties are magnified by no more than 2 times. As the experimental uncertainties are generally little, we can say that it is quite reliable to investigate both the Young's modulus and the loss factor by the resonance method.

Figure 5(b) shows the error coefficients of the non-resonance method. We see that D_{yx} relating to E are quite small, with absolutes of no more than 1. So the Young's modulus can be determined quite accurately. However, $D_{\eta\phi}$ rises sharply as η decreases. If $\eta > 0.1$, $D_{\eta\phi}$ is only moderately big (less than 10). This will not result in too big errors, only if the phase ϕ uncertainties can be controlled at a low level. But if $\eta < 0.01$, $D_{\eta\phi}$ can reach up to 100. This means tiny errors of ϕ will be magnified by 100 times or higher. And ϕ should be the main error origin in the experiment. So though it is suitable to investigate the loss factor of high-loss materials by the non-resonance method, the determination of low-loss materials may lead to big errors.

The above analysis helps to explain why the loss factor diverges for the two methods, and why the loss factor obtained by the non-resonance method fluctuates so much. The phase uncertainties, resulting probably from the rigid wall vibration, are very likely the cause of the errors. The fluctuations can be simply amended by removing the peaks and by curve fitting techniques. However, the large deviation cannot be corrected without full knowledge of the experimental set-up's energy dissipation mechanism. This may be a worthwhile subject for future work.

5. Discussion based on the sensitivity analysis

The Young's modulus and the loss factor of the silica aerogel are measured at room temperature by resonance and non-resonance method. The Young's modulus is about 10^9 Pa, and the loss factor is about 0.01, both rising slightly as the frequency increases. The Young's modulus obtained by the two methods coincides well, but the loss factor is quite different. The sensitivity analysis shows that the loss factor of the resonance method is reliable, but that of the non-resonance method is not trustworthy as the error coefficient is big for low-loss materials.

References

- [1] Padmanabhan S K, Haq E U and Licciulli A 2016 *Ceram. Int.* **42** 7216
- [2] Wei G, Wang L, Xu C, Du X and Yang Y 2016 *Energ. Build* **118** 226
- [3] Wong J C H, Kaymak H, Tingaut P, Brunner S and Koebel M M 2015 *Micropor. Mesopor. Mat.* **217** 150
- [4] Pritz T 1980 *J. Sound. Vib.* **81** 317
- [5] Oyadiji S O and Tomlinson G R 1995 *J. Sound. Vib.* **186** 623
- [6] Jaouen L, Renault A and Deverge M 2008 *Appl. Acoust.* **69** 1129