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Displacement of a Free Tyre in Natural Frequencies to Determine Noise Structure

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Abstract. Tyre vibration represents the main source to generate the noise for light vehicles. The physical aspects which are involved in studying tyre noise are tyre vibrations and structure borne sound. In this paper, radial tyre displacements have been investigated for tyre vibration characterization in natural frequency. A cylindrical shell model has been used to analyse flexural wave's propagation. Two important features of flexural wave propagation on a cylindrical shell were analysed, lower order modes of the second and third radial mode sets are in the range where the membrane effects in sidewall are relatively large. The tread band and sidewall responses of the tyre have been simulated and compared. As a result, the concentration of displacements has been observed in the vicinity of tyre structure. For the simulation purposes, frequency analysis has been performed, showing the influence of the rotating speed on the vibrations level and natural frequency content. The study gives a physical insight on generation mechanism of tyre radial vibrations.

1. Introduction

It is important to improve the quality of driving by reducing the vibration of vehicles. A geometric non-linear analysis was used for the non-inflation process. From this equilibrium state of reference, the natural frequencies and mode shapes were determined for the linear vibration about a tyre pre stressed configuration. Generally various sources of vibration in an automobile exist when it operates normally. The rate of vibration effect has gradually increased. Especially, the tyre is not only as part of the initial rotating contact that transfers impacts from road surfaces to the main body of the automobile into the automobile's interior but also, the tyre has a great influence on enhancing the quality of riding.

Reviewing the established studies on a tyre vibration, Böhm [2] extracted the equations of tyre motion by reviewing both stationary and moving characteristics while assuming the tyre to be an elastic ring. Böhm validated his equations using experimental methods. Another study by Kropp [6] modelled a tread band as a 2D circular ring. This study investigated several aspects of the vibration of unloaded smooth tyres. The study results showed that the influence of the area on which the force gives respect to the admittance. However, the higher wave modes, associated with displacement variations across the width of the tread band were not considered. After a few years, Perisse [7] investigated tyre road interaction and radial tyre vibration on a rolling smooth tyre with test laboratory facilities. It was shown that the tyre vibration could be composed into the motion of bending wave

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produced by unsteady tyre road interaction forces. Then Pinnington [8] assumed the tyre to be the circular beam model. It was found that bending, tension, shear and stiffness control the characteristics of the tyre vibration. Kindt [5] also studied experimentally the impact response on a rotated tyre. The results showed an excitation amplitude dependency of the structural response for road impact. This effect, which called the Payne effect, should be included in a tyre model that predicts the response to impact excitations. Johnson [4] identifies an indicator of tyre bead area for use in tyre durability testing. Wavelet maps analysis indicated that damage progression and showed potential for locating damage circumferentially on a tyre. Byoung Sam Kim [1] modelled a tyre as a thin ring and studied the natural frequency of the tyre, considering the quality and the aspect of geometry. The results show that experimental conditions can be considered as the parameters that shift the natural frequency and damping ratio. However, the results using the analytical and numerical methods for interpreting the displacement in natural frequency of the tyre have quit a difference to their experimental results. In addition, the existing experimental results differ from the naturally occurring frequency conditions of the automobile tyre. This is because the tyre was equipped with an adapter to make it stationary. In this respect, natural frequency of the radial direction in the tyre-wheel system were simulated to survey not only the displacement of tyre equipped in the automobile, but also the influence in which boundary condition affected radial directional natural frequency in the tyre wheel system.

Tyre noise is generated by several mechanisms. With a modern tyre, wall vibration, air pumping and air resonant radiation are all considered to be important. However tyre noise generating mechanisms are still not clear due to the complication of tyre vibration behaviour. Vibrations of the tyre shell are the combination of several different wave types, which appear at different frequencies. In a low frequency range, where the tyre behaves like an elastically supported beam, the circular ring model is used to analyse the dispersion relations. Above 300 Hz, which is the transition point from one-dimensional to two-dimensional wave-guide properties of the passenger car tyre, a cylindrical shell model is used to analyse flexural wave's propagation [9].

In this paper, two important features on the wave propagation, wave-guide behaviour and the curvature effect of the tyre wall are analysed. In consideration of noise radiation from tyre waves, most of the tyre waves observed in this study is inefficient sound radiators since their wave numbers are larger than the acoustic wave number. As a result, one of the most important features in sound radiation of a tyre shell is acoustically excited wave motion of the tyre wall.

2. Development of the Tyre Vibration Model

This study is focused on the conception analysis of sound radiation of the tyre modes, so that it is only necessary to use simply vibration theory. A pneumatic tyre is a structure of the shape of a toroid filled with air. The arrangement of the principal components of a tyre is shown in Figure 1.

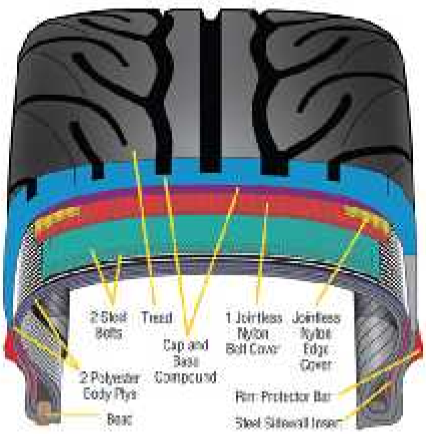

Figure 1. Tyre structural composition. [https://gr8autotech.files.wordpress.com]

A typical tyre carcass consists of rubber, plies and reinforcing belts. The ply and belt are made up of flexible filaments of a high modulus cord and a matrix of rubber. In addition to the carcass rubber, a tyre has tread and sidewall rubber. The material is both non-isotropic and non-homogeneous due to the composite structure of the rubber and laminated plies. In the absence of a built-up tread region such as in aircraft tyres, the tyre can be considered as a toroidal shape with a doubly curved surface. In steel belted radial tyres such as a passenger tyre, however, the tread band forms a relatively rigid flat band whose largest principal curvature lies in the plane of the wheel. The tyre is usually categorized into the bias and the radial types in terms of the angle of the carcass; the carcass cord in the bias tyre makes a rather large angle with the tread centreline while the carcass cord in the radial tyre lies perpendicular to the tread centreline.

In certain frequency range, the tyre behaves like a spring-mass system (the sidewall acts as a spring and the tread band as a mass). For the radial first mode, the equation of motion in the in-plane vertical direction is

$$M\frac{d^2v}{dt^2} = -K_r\ v\int_0^{2\pi}\sin^2\theta\ R_0\ d\theta \quad = \quad -K_t\ v\int_0^{2\pi}\cos^2\theta\ R_0\ d\theta \tag{1}$$

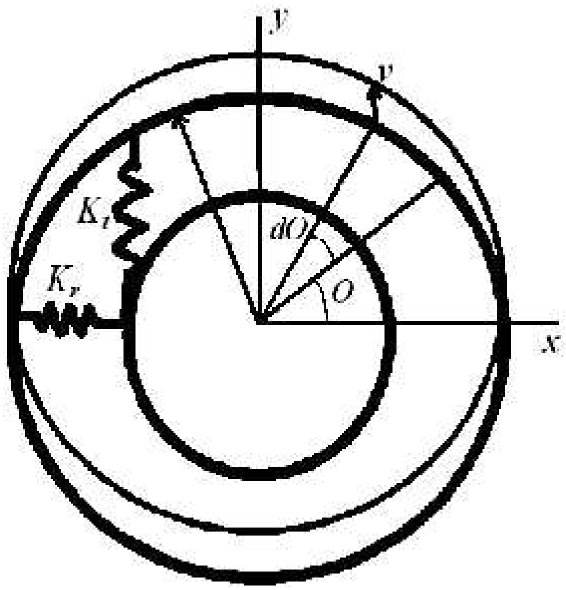

Figure 2. Radial vibration mode

Where v is displacement of the rigid tread plate in the vertical direction, K_r and K_t are sidewall spring constants in the direction of radial and traction direction, respectively, as shown in Fig. 2. Equation (1) becomes

$$M\frac{d^2v}{dt^2} + \pi R_0\left(K_r + K_t\right)v = 0 \tag{2}$$

For the first lateral mode and the first traction (Fore/aft) mode, equations of motion in the axial and traction direction are given by

$$M \frac{d^2 x}{dt^2} = -K_x 2®R_0 x \quad (3)$$

$$I \frac{d^{2—}}{dt^2} = -K_t \ 2®R_0^{—} \quad (4)$$

Where K_x and K_t are sidewall spring constants in the direction of axial and traction respectively. From equation of motion, equations (2, 3, 4) the eigen-frequencies of the first radial, lateral and traction modes obtained are listed below

$$µ_r = \sqrt{\frac{®R_0 \left(K_r + K_t\right)}{M}} \quad (5)$$

$$µ_x = \sqrt{\frac{2® \ R_0 \ K_x}{M}} \quad (6)$$

$$µ_t = \sqrt{\frac{2® \ R_0 \ K_t}{I}} \quad (7)$$

From the observation of modal analysis results, the vibrations on the shell of passenger car tyre are the combination of several different wave types, which appear at different frequencies [3]. There are some modes, which are mostly related to sidewall stiffness. Vibrations in the composite structure of tires as sources of noise are not so easy to identify experimentally. Through the finite element analysis, it can be traced the formation of various types of waves, which will propagate and generate sound.

3. Finite Element Model

The tyre consists of more than 10 different material sets; belts, plies, bead wire, tread rubber, sidewall rubber, inner liner rubber, apex, etc. The belt layers are made up of steel cords and rubber, and the ply layers made up of polyester cords and rubber [10]. In the finite element analysis, linear material properties were used throughout the analysis. The equivalent composite isotropic material prosperities were used to model the tread and sidewall, while isotropic material properties were used to model the wheel rim. The material properties of the tread band and sidewall are shown in Table 1. These properties were estimated based on physical reasoning or direct measurement and adapted from the work of Kropp [6].

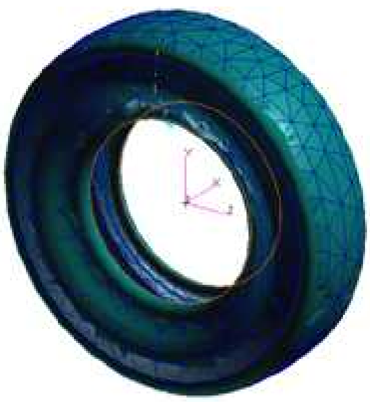

Figure 3. Tyre FE model

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The finite element analysis of a tyre was carried out using general-purpose program. A passenger car steel belted radial tyre of 195/70R14 size has been modelled together with shell elements for the tread and sidewall areas. For the tyre, layers of ply are used through the tyre carcass. The two reinforced belt layers are located in the tread area. Figure 3 shows the finite element model of the tyre, which consists of nodes and elements. For the simulation purposes, the wheel rim has been connected as a contact relation to the tyre.

Table 1. List of material properties used for tyre model

	Side wall	Tread band
Young's Moduli	E=4.8x108 Pa	E=4.8x108 Pa
Poisson's Ratio	v_{12} = 0.45	v_{12} = 0.45
Density	800 kg/m3	1200 kg/m3

The cross section finite element model of the tyre also shown in Figure 4. Rotating the cross section geometry through 360o and defining appropriate angular increments generate a three-dimensional model of tyre. For the purpose of analysis simplification and computer time reduction, the tyre bead has been rigidly connected.

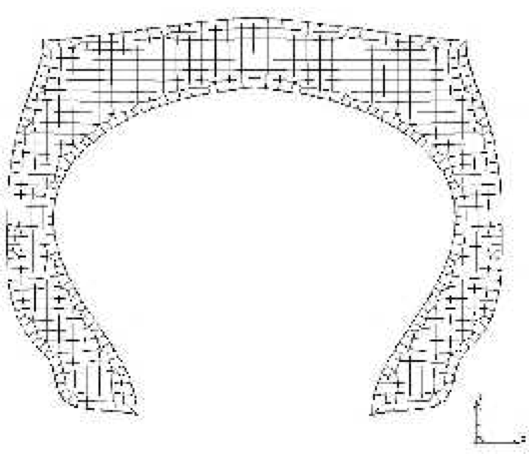

Figure 4. Cross-Section geometry of Tyre

The variety shapes and sizes of mesh describe a typist structure of tyre composite. The mesh sizes of cross-section are smaller than shell models. Its mean the composite constructed to support more axial than lateral and radial loads.

4. Results and Discussion

Simulation conditions were classified into two cases: (1) in which a tyre is not rotating and (2) while a load is not applied. The first case is the boundary condition under which tyres maintain a free unfixed condition. In the second case, tyres were fixed into an adapter, simulating tyre fixed into a vehicle. Displacement in natural frequency of tyre has been obtained from the amplitude of frequency response functions reviewing quality aspects of tyre.

Figure 5 shows the simulation of distinguish displacement for the tread band of smooth tyre in natural frequency. The variety of wave propagated by structure vibration and oscillated around tyre circumstance. Through this type of wave propagation, the natural frequency in radial direction is changed by the difference wave number and displacements structure in the tyre. The wave propagation in the tire structure vibration is one of the sources that generate the noise.

Figure 5. The tread band displacement of smooth tyre in natural frequency

Table 2. Tread band behaviour

Frequency (Hz)	Wave Number	Max. Displacements (mm)	Displacement Direction	Position
48,2	0,5	0,856	y, r	
213,63	0,5	1,59	± y	
308,44	1	1,8	± y, z	
313,57	1	1,25	± z	
332	1	4,42	± x	+ z
335,63	1	4,4	± x	- z
479,52	3	1,34	yz	
485,47	4	1,45	yz	
658,11	4	1,42	yz	

Table 2 presents the tread band wave numbers and displacement behaviour of natural frequency. The natural frequency of tyre tread structure generated in the range number between 48 – 658 Hz. The circumferential wave number is the number of waves around the tyre tread circumference. In the meridian direction, it was observed that the symmetric and asymmetric modes appeared alternatively. This meridian wave numbers and the circumferential has been classified the eigenvectors of the displacement in natural frequency. This waves also propagate in the space direction (x, y, z) alternatively. When the vibration of this tread structure propagated in between tyre pneumatic and atmosphere, it will be one of the tyre noise sources.

The simulation of free sidewall in figure 6 shows some distinguish displacement work on natural frequency and produce several type wave numbers. Tyre side-wall define as thin membrane and generate base on limited flexible polymer materials, its will deflect softer and larger displacements compare to tyre tread. There are two wave types generated by side wall displacement. The first pair vibration (a, b) propagate diagonally waves in y and x axis. The second (c, d) and third (e, f) pairs indicated oscillation waves that generated by displacement radially and laterally around the side-wall. Base of that variety propagation waves, a different point of extreme displacement can be produced for all wave numbers.

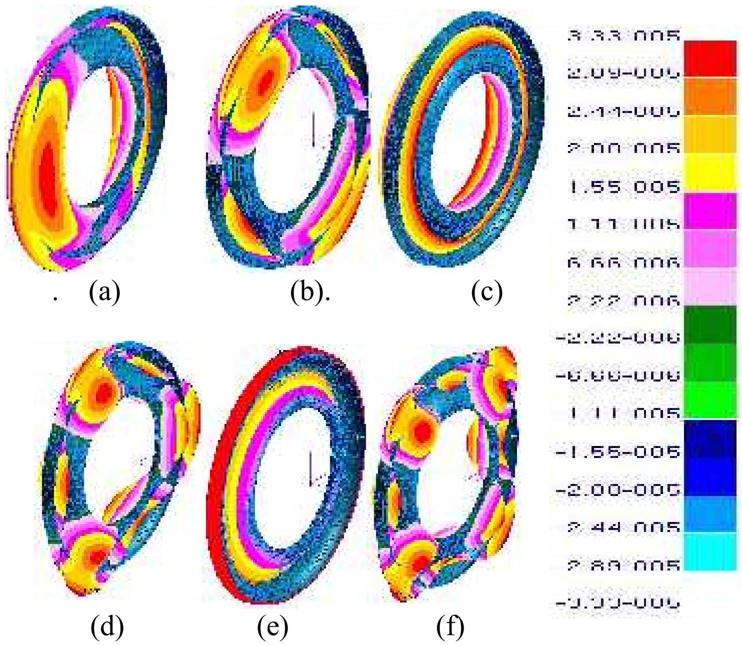

Figure 6. The tyre sidewall displacement propagate in natural frequency

Table 3 presents the sidewall wave numbers and displacement behaviour of natural frequency. The variety of displacement propagation generate the variation numbers of wave in surface and space direction. In case of large deflection on tyre sidewall, it can be destroyed and initiate sidewall crack when this deflection generated during tyre application in the range of this frequency. It can be avoided through a certain stiffness of sidewall that maintained in certain tyre pneumatic pressure.

Table 3. Sidewall behaviour

Frequency (Hz)	Wave Number	Max. Dicpl. (mm)	Displacement Direction
267,59	1	3,28	x, y
267,8	1	3,3	x, z
273,05	2	3,5	x, z
273,33	2	3,43	x, y, z
313,88	0,5	2,42	x, y, z
329,47	3	3,38	x, y, z
329,52	3	3,38	x, y, z
412,14	0,5	2,27	x, y, z
424,46	3	3,32	x, y, z
424,55	4	3,33	x, y, z

The tyre stiffness behaviour after combined two sidewalls and tyre tread should be better. The types of tyre structure displacement deflect diagonally and bended refer to the axis. It make the robustness of tyre rigidity. The sidewalls of tyre after couple to the tyre tread are in phase for the symmetric mode shapes and out of phase for the asymmetric mode shapes. They showed that the modes related to the sidewall stiffness appear first, followed by the radial and Fore/Aft modes. Its mean the variety of sidewalls wave propagation mode reduced by sidewall-tread Mountain. Figure 7 shows the displacement of smooth tyre after the tread and sidewall combined together.

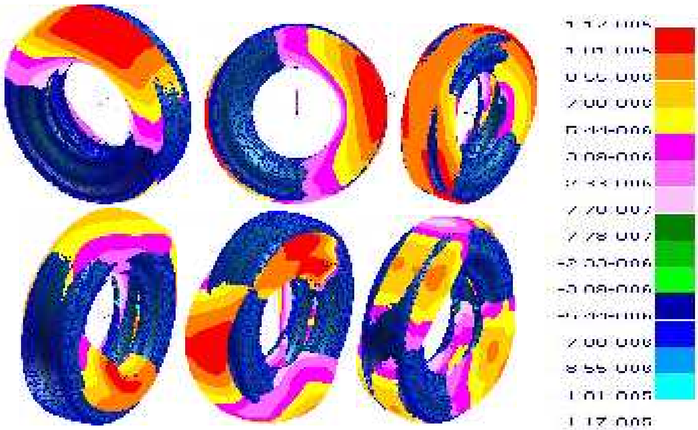

Figure 7. Tyre displacement of natural frequency

Table 4 present the tyre wave numbers and displacement behaviour of natural frequency. This is verified using simulation results, show that the magnitude of tyre displacements least and the flexibility of tyre more stiff than the separate body of tread and sidewall. It will change the mode and pattern of wave propagation.

Table 4. Whole Tyre structure behaviour

Frequency (Hz)	Wave Number	Max. Dicplacement (mm)	Displacement Direction
38,72	0,5	0,931	azimut y
38,96	0,5	0,95	azimut z
57,23	0,5	0,68	x & bending x
124,46	1	0,938	bending x
128,51	2	1,17	bending x
135,68	0,5	0,903	tran yz & bending x
136,85	0,5	0,983	bending x
168,14	1	1,61	yz & x
174,77	y 2 x 0.5	0,981	yz & x

The motion of displacements propagate the waves in azimuth y and z ways, also bended refer to x axis. These results show that the natural frequencies of tyre model are slightly lower than those of tread band and sidewall model. This simulation indicated that the mode of wave propagation was performed by static analysis of the non-inflated tyre condition. The eigenvalues were extracted and perform the displacement due to wave's propagation variety of whole tyre structure, tread and sidewall rigidity.

5. Conclusion

From the point of view of natural frequency, flexural wave propagation in the tyre has been analysed using simulation models. Vibrations of the tyre sourced by the combination of several different wave types that generated by displacements variety, which appear at different natural frequencies. It has been found that two wave types propagate in the tyre tread and the membrane effect is dominant due to sidewall radial stiffness. As a result, tyre structure constructed by tread and sidewall. The waves of tyre propagated by shorter displacement compare to the tread and sidewall. The number modes of tyre structure circumferential wave are less than tread and sidewall, this is the reason why the tyre structure has a higher stiffness. The combination of wave modes will be one of the tyre acoustic sources.

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