

Numerical Simulation Bidirectional Chaotic Synchronization of Spiegel-Moore Circuit and Its Application for Secure Communication

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Abstract. Spiegel-Moore is a dynamical chaotic system which shows irregular variability in the luminosity of stars. In this paper present the performed the design and numerical simulation of the synchronization Spiegel-Moore circuit and applied to security system for communication. The initial study in this paper is to analyze the eigenvalue structures, various attractors, Bifurcation diagram, and Lyapunov exponent analysis. We have studied the dynamic behavior of the system in the case of the bidirectional coupling via a linear resistor. Both experimental and simulation results have shown that chaotic synchronization is possible. Finally, the effectiveness of the bidirectional coupling scheme between two identical Spiegel-Moore circuits in a secure communication system is presented in details. Integration of theoretical electronic circuit, the numerical simulation by using MATLAB®, as well as the implementation of circuit simulations by using Multisim® has been performed in this study.

1. Introduction

Chaos explain the behavior of certain dynamical nonlinear systems, i.e., systems which state variables evolve with time, exhibiting complex dynamics that are highly sensitive on initial conditions. Sensitivity to initial conditions of chaotic systems is familiarly known as the butterfly effect. Small changes in an initial state will make a larger difference in the behavior of the system at future states (e.g., [1]-[2]). Chaos behavior have been discovered in physical [3], ecology [4], neuroscience [5], chemical reaction [6], psychology [7], and economics [8]. In many implementation of engineering and computer science such as robotic system [9], text encryption [10], image encryption [11], image encryption [12], speech encryption [13] and other. One of most important engineering implementation



is secure communication because of the properties of random behaviors and sensitivity to initial conditions of chaotic systems (e.g., [14]-[17]).

Christiaan Huijgens (1665) the Dutch scientist noted the synchronizing behavior of pendulum clocks. Many scientists have been investigate the synchronization of several dynamical systems. When Pecora and Carroll published their observations of synchronization in unidirectionally coupled chaotic systems, synchronization of chaotic oscillators in particular became popular [18]. Many researchers simulated the chaos can be synchronized and applied to secure communication schemes (e.g., [14]-[20]).

Generally, this research focus on the development of chaos and non-linear dynamical system behavior in chaotic electrical oscillator. We investigate and analyze some basic properties to study the non-linear dynamics and chaotic behavior, such as eigenvalues structure, phase plane, Lyapunov exponent, and diagram bifurcation analysis, while the analysis of the synchronization in the case of bidirectional coupling between two identical generated chaotic systems. Moreover, some appropriate comparisons are made to contrast some of the existing results. And presented the effectiveness of the bidirectional coupling between two identical Spiegel-Moore [21] chaotic circuits in a secure communication system.

The paper is organized as follows. In section 2, the details of the proposed autonomous Spiegel-Moore circuit's simulation using MATLAB®. In section 3, build an analog circuit using Multisim®. In Section 4, the bidirectional coupling method is applied in order to synchronize two identical autonomous Spiegel-Moore chaotic circuits. The chaotic masking communication scheme by using the above mentioned synchronization technique is presented in Section 5. Finally, in Section 6, the concluding remarks are given.

2. Mathematical Model of Spiegel-Moore Circuit

Moore-Spiegel (1966) found a model the irregular variability in the luminosity of stars [21]. This is a three-dimensional autonomous nonlinear system that is described by the following system of ordinary differential equations:

$$\left. \begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -z + ay - x^2y - bx \end{aligned} \right\} \quad (1)$$

The system has one cubic non-linearities term and two positive real constants a and b . The parameters and initial conditions of the Moore-Spiegel system (1) are chosen as: $a = 9$, $b = 5$ and $(x_0; y_0; z_0) = (2, 7, 4)$, so that the system shows the expected chaotic behavior.

2.1. Equilibrium Point Analysis

Spiegel-Moore system has one equilibrium points $E_0 (0, 0, 0)$. The dynamical behavior of equilibrium points can be studied by computing the eigenvalues of the Jacobian matrix J of system (1) where:

$$J(\bar{x}, \bar{y}, \bar{z}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2xy - b & a - x^2 & -1 \end{bmatrix} \quad (2)$$

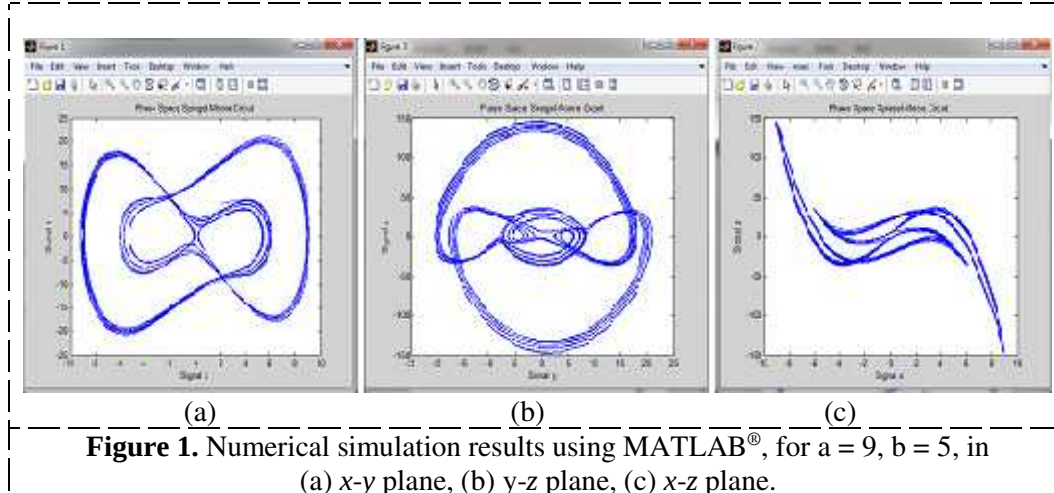
For equilibrium points $E_0 (0, 0, 0)$ and $a = 9$, $b = 5$, the eigenvalues are obtained by solving the characteristic equation, $\det[\lambda I - J_1] = 0$ which is:

$$\lambda^3 + \lambda^2 - 9\lambda + 5 = 0 \quad (3)$$

Yielding eigenvalues of $\lambda_1 = 2.126552154$, $\lambda_2 = -3.753037679 - 8.660254040i$, $\lambda_3 = 0.626485525 + 8.660254040i$. The above eigenvalues show that the system has unstable spiral behavior. In this case, the phenomenon of chaos is presented.

2.2. Numerical Simulation

In this section, software MATLAB® used for numerical simulations. To solve the system of differential equations (1) used the fourth-order Runge-Kutta method. Figure 1 (a)-(c) show the projections of the phase space orbit on to the xy plane, the yz plane and the xz plane, respectively. As it is shown, for the chosen set of parameters and initial conditions, the Spiegel-Moore system presents chaotic attractors.

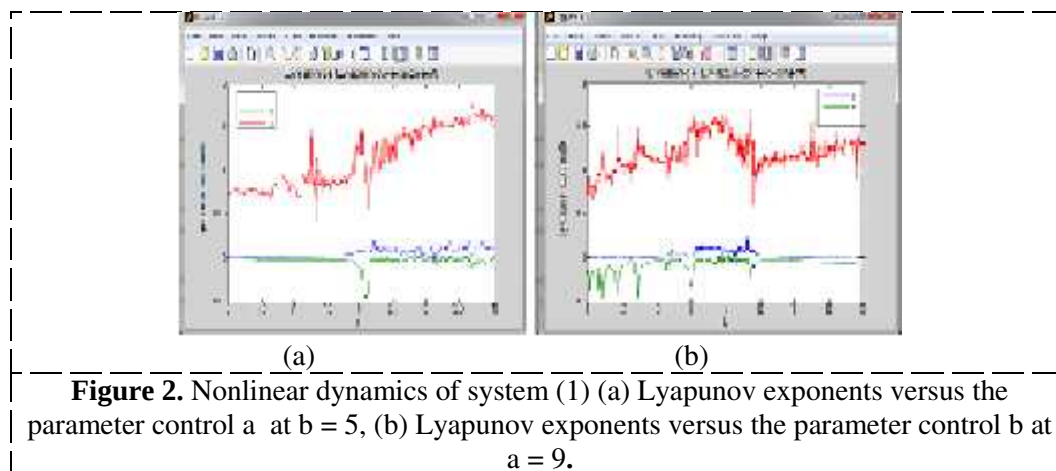


2.3. Lyapunov Exponent Analysis

Three Lyapunov exponents ($\lambda_1, \lambda_2, \lambda_3$) it is also known from the theory of nonlinear dynamics that for a three dimensional system (1). In more details, for a 3D continuous dissipative system the values of the Lyapunov exponents are useful for distinguishing among the various types of orbits. So, the possible spectra of attractors, of this class of dynamical systems, can be classified in four groups, based on Lyapunov exponents (e.g., [16], [22]).

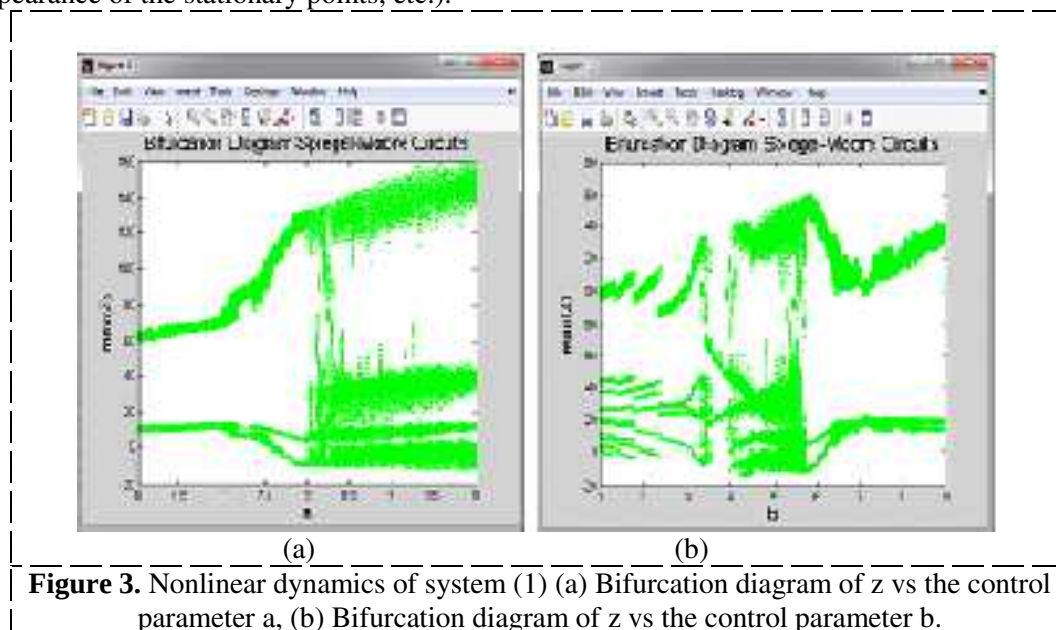
- $(\lambda_1, \lambda_2, \lambda_3) \rightarrow (-, -, -)$: a fixed point
- $(\lambda_1, \lambda_2, \lambda_3) \rightarrow (0, -, -)$: a limit point
- $(\lambda_1, \lambda_2, \lambda_3) \rightarrow (0, 0, -)$: a 2-torus
- $(\lambda_1, \lambda_2, \lambda_3) \rightarrow (+, 0, -)$: a strange attractor

Therefore, the last configuration just possible third-order chaotic system. In this case, a positive Lyapunov exponent reflects a “direction” of stretching and folding and determines chaos in the system. So, in figure 2 (a) and (b) the dynamics of the proposed system’s Lyapunov exponents for the variation of the parameter $a \in [6 - 10]$ and $b \in [1 - 9]$. For $8 \leq a \leq 10$ and $4.2 \leq b \leq 5.7$ a strange attractor is displayed as the system has one positive Lyapunov exponent, while for values of $6 < a < 8$ and $4.2 > b > 5.7$ is a transition to limit point behavior as the system has two negative Lyapunov exponents.



2.4. Bifurcation Diagram Analysis

Bifurcation indicate a situation in which the solutions of a nonlinear system of differential equations alter their character with a change of a parameter on which the solutions depend [16]. Bifurcation theory studies these changes (e.g. dependence of their stability on the parameter, appearance and disappearance of the stationary points, etc.).



Spiegel-Moore circuit of figure 3 (a) and (b), was written to result the bifurcation diagrams by MATLAB® program. In this diagram a possible bifurcation diagram for system (1), in the range of $6 \leq a \leq 10$. For the chosen value of $8 \leq a < 10$ and $4.2 \leq b \leq 5.7$ the system displays the expected chaotic behavior. Also, for $6 \leq a < 8$ and $4.2 > b > 5.7$, a reverse period doubling route is presented.

3. Analog Circuit Simulation using Multisim®

A simple electronic circuit was designed using Multisim® software, and that can be used to study chaotic phenomena. The circuit employs simple electronic elements, such as resistors, capacitors, multiplier and operational amplifiers. Figure 4, the voltages of C_1 , C_2 , C_3 are used as x , y and z , respectively. The nonlinear term of system (1) are implemented with the analog multiplier. The corresponding circuit equation can be described as:

$$\left. \begin{aligned} \dot{x} &= \frac{1}{C_1 R_6} y \\ \dot{y} &= \frac{1}{C_2 R_2} z \\ \dot{z} &= -\frac{1}{C_3 R_7} z + \frac{1}{C_3 R_8} y - \frac{1}{100 C_3 R_9} x^2 y - \frac{1}{C_3 R_{10}} x \end{aligned} \right\} \quad (4)$$

We assume $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 100 \text{ k}\Omega$, $R_8 = 11.1 \text{ k}\Omega$, $R_9 = 1 \text{ k}\Omega$, $R_{10} = 20 \text{ k}\Omega$. $C_1 = C_2 = C_3 = 1 \text{ nF}$. The circuit has three integrators (by using Op-amp TL082CD) in a feedback loop and a multiplier (IC AD633). The supplies of all active devices are $\pm 15 \text{ V}$. With Multisim[®], we obtain the simulation results of system (1) as shown in figure 5. Compared with figure 1, a good qualitative agreement between the numerical simulation and the Multisim[®] results of the Spiegel-Moore circuit is confirmed. The parameter variable a of system (1) is changed by adjusting the resistor R_8 , and obeys the following relation:

$$a = \frac{1}{C_3 R_8} \quad (5)$$

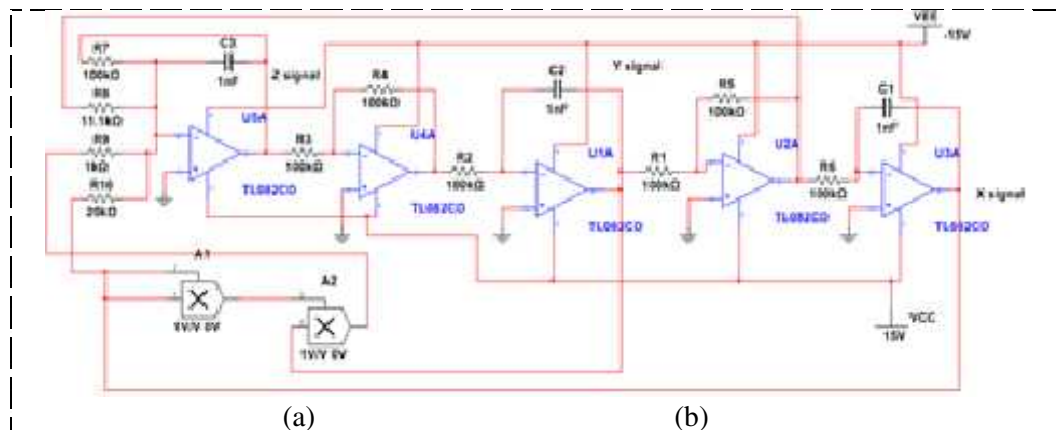


Figure 4. Schematic of the proposed Spiegel-Moore circuit by using Multisim[®].

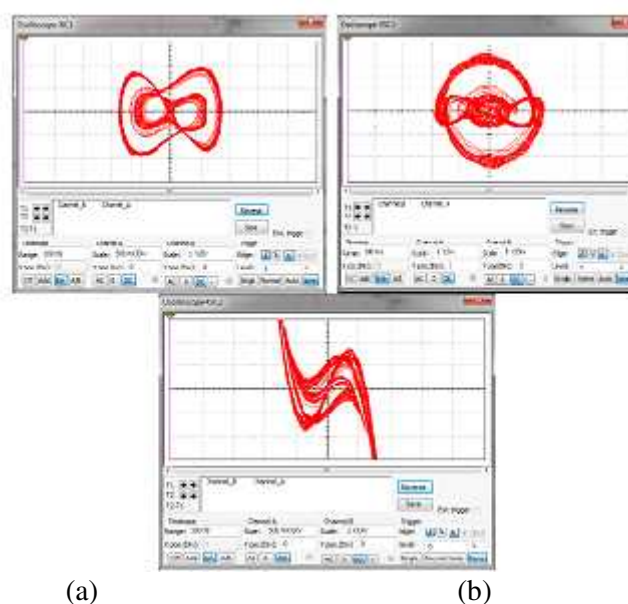


Figure 5. Various projections of the chaotic attractor using Multisim[®], for $a = 9$, $b = 5$
(a) x-y plane (b) x-z plane (c) y-z plane.

4. Bidirectional Chaotic Synchronization

4.1 Mathematical Model of Bidirectional Coupling

The case of bidirectional coupling two systems interact and coupled with each other creating a mutual synchronization. Following bidirectional coupling configuration (e.g., [14]-[20]), described:

$$\left. \begin{aligned} \dot{x}_1 &= y_1 \\ \dot{y}_1 &= z_1 + g_c(y_2 - y_1) \\ \dot{z}_1 &= -z_1 + a_1 y_1 - x_1^2 y_1 - b x_1 \\ \dot{x}_2 &= y_2 \\ \dot{y}_2 &= z_2 + g_c(y_1 - y_2) \\ \dot{z}_2 &= -z_2 + a_2 y_2 - x_2^2 y_2 - b x_2 \end{aligned} \right\} \quad (6)$$

The coupling coefficient g_c is present in the equations of both systems, since the coupling between them is mutual. Numerical simulations of system (6) using the 4th-order Runge-Kutta method, are used to describe the dynamics of chaotic synchronization of bidirectionally coupled Spiegel-Moore circuits.

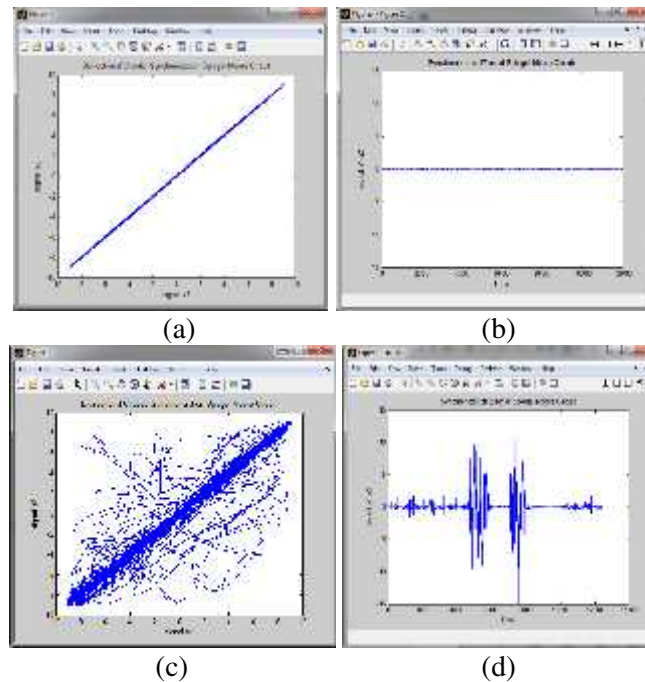


Figure 6. Phase portrait of x_2 vs x_1 and error $x_2 - x_1$ in the case of bidirectionally coupled Spiegel-Moore circuits, for (a) $g_c = 1.47$ (full synchronization) and (c) $g_c = 0.5$ (full desynchronization), for $a_1 = 9$, $a_2 = 8.9285$, $b = 5$.

In bidirectional coupling, the coupled systems are connected in such a way that they mutually influence each other's behavior. Synchronization numerically appears for a coupling factor $g_c \geq 1.47$ as shown in figure 6 (a)-(b), with error $e_x = x_1 - x_2 \rightarrow 0$, which implies the complete synchronization.

4.2 Analog Circuit Simulation using Multisim®

Synchronization of chaotic motions among the coupled dynamical systems is an important generalization for the phenomenon of synchronization of linear system, which is useful and indispensable in communications. Simulation results show that two systems synchronize well. Figure 7 shows the circuit schematic for implementing the bidirectional synchronization of coupled Spiegel-

Moore systems. Chaotic synchronization appears for a coupling strength $R_{21} \leq 680 \text{ m}\Omega$, as shown in figure 8(a). For different initial conditions or resistance coupling strength $R_{21} > 680 \text{ m}\Omega$, the synchronization cannot occur as shown in figure 8(b).

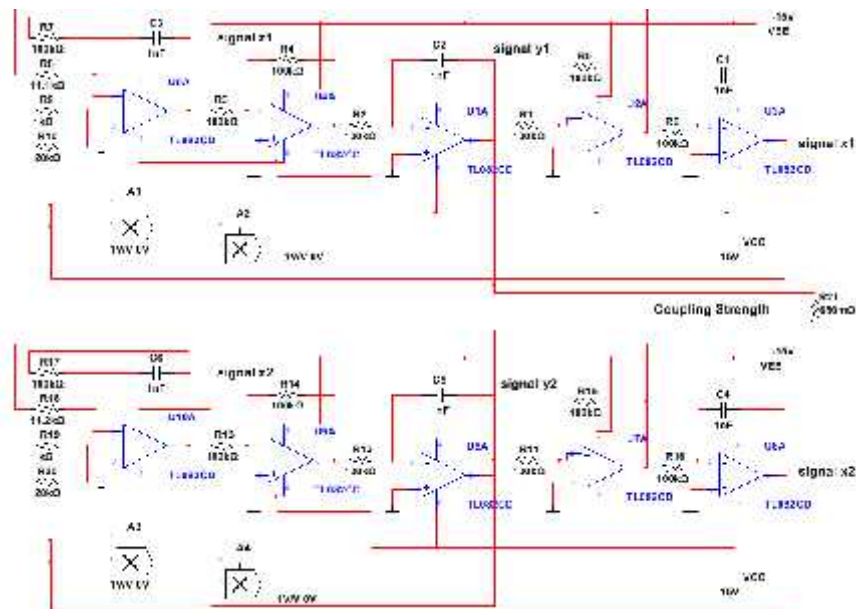


Figure 7. Bidirectional chaotic synchronization Spiegel-Moore circuit by using Multisim®

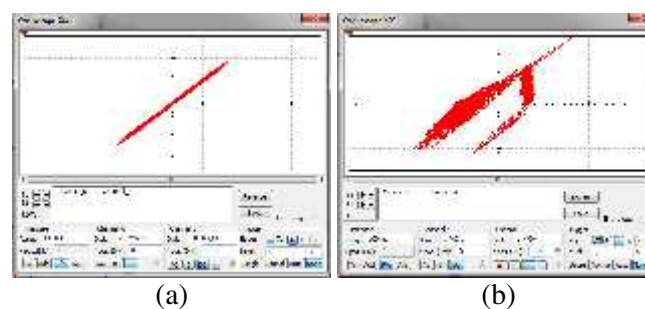


Figure 8. Synchronization phase portrait of x_2 vs x_1 , for (a) $R_{21} = 680 \text{ m}\Omega$ and (b) $R_{21} = 2 \text{ }\Omega$, with Multisim[®].

5. Applications in Secure Communication System

Caused the fact that output signal can recover input signal, it can be implement secure communication for a chaotic system. The attendance of the chaotic signal between the transmitter and receiver has proposed the use of chaos in secure communication systems. The system design depends as we described on the self synchronization property of the Spiegel-Moore circuits. Transmitter and receiver systems are identical except for their control value a as equation (5), in which the transmitter system R_8 is 11.1 kohm and the receiver system R_{18} is 11.2 kohm as shown in figure 7 and 9. In this masking scheme, a low-level message signal is include to the synchronizing driving chaotic signal in order to regenerate a clean driving signal at the receiver. So, the message has been perfectly recovered by using the signal masking approach through synchronization in the Spiegel-Moore circuits.

Sinusoidal wave is added to the generated chaotic x signal, and the $S(t) = x + i(t)$ is feed into the receiver. The chaotic x signal is regenerated allowing a single subtraction to retrieve the transmitted signal, $[x+i(t)]-x_r = i'(t)$. If $x = x_r$. The simulation results shows that Spiegel-Moore chaotic circuit is an

excellent for chaotic masking communication when the frequency information is at intervals of 0.8 kHz – 3 kHz. Otherwise, when the frequency information is more than 3 kHz or less than 0.8 kHz, the chaotic masking communication is not occur.

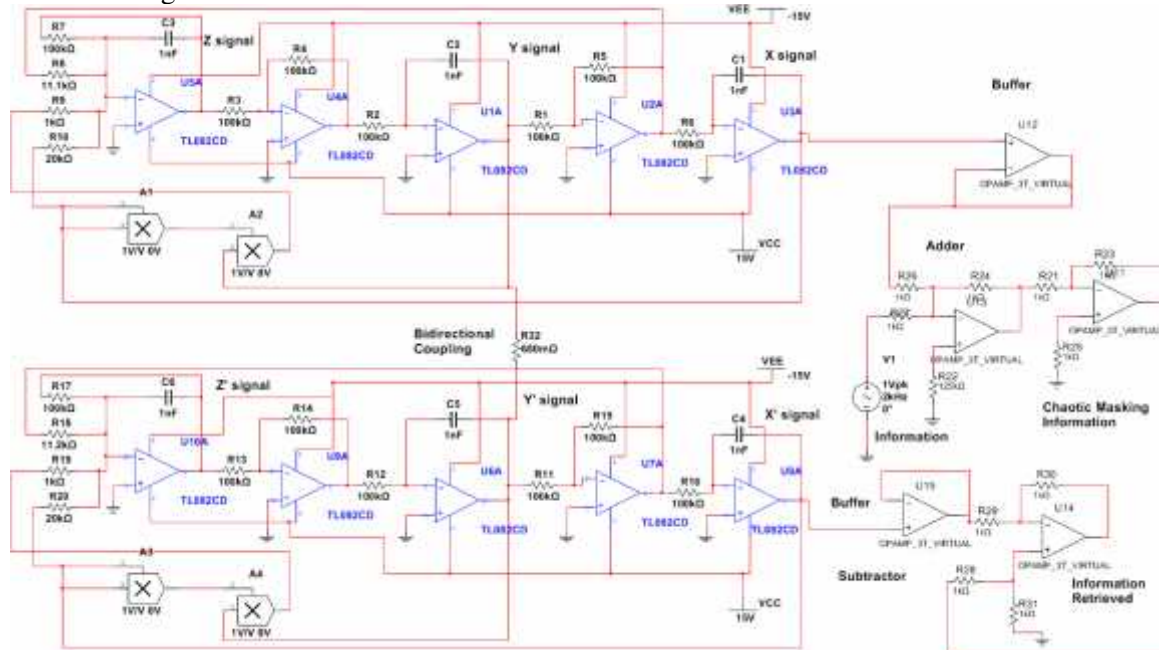


Figure 9. Spiegel-Moore circuit masking communication system for sinusoidal wave

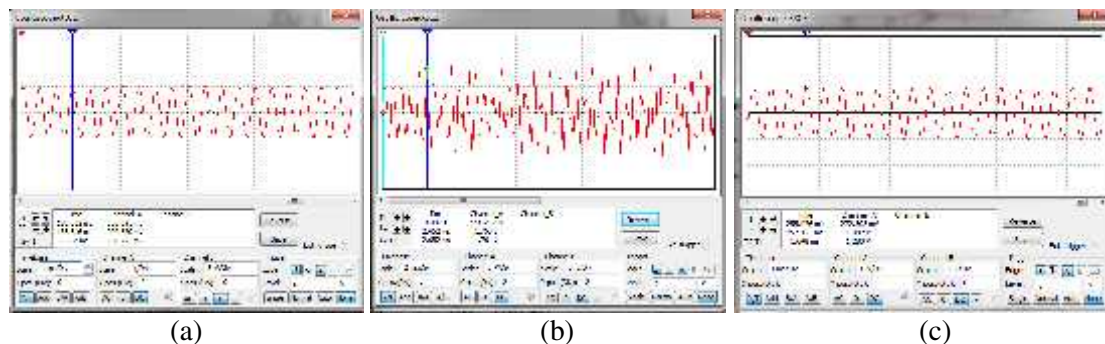


Figure 10. Multisim® outputs of Spiegel-Moore circuit masking communication system with input 1V and 2 KHz: (a) Information signal, (b) Chaotic masking transmitted signal, (c) Retrieved signal.

6. Conclusion

In this paper, Spiegel-Moore chaotic circuit system including chaotic motions, by means of Lyapunov exponent spectrum, diagram bifurcation analysis has been studied. Moreover, it is implemented via a designed circuit with Multisim® showing very good agreement with the numerical simulation result. The chaotic synchronization of two identical Spiegel-Moore circuits system has been investigated by implementing bidirectional method technique. Chaotic synchronization, realization circuit and chaos masking were realized by using MATLAB® and Multisim® programs. Finally, the comparison between MATLAB® and Multisim® simulation results demonstrate the effectiveness of the proposed secure communication scheme.

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