

# The digital algorithm for fast detecting and identifying the asymmetry of voltages in three-phase electric grids of mechanical engineering facilities

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**Abstract.** The paper considers a new technique for the fast method of extracting symmetrical components of unbalanced voltages caused by the faults in electric grids of mechanical engineering facilities. The proposed approach is based on the iterative algorithm that checks if the set of at least three voltage discrete measurements belongs to a specific ellipse trajectory of the voltage space vector. Using classification of unbalanced faults in the grid and results of decomposing the voltages into symmetrical components, the algorithm is capable to discriminate between one-phase, two-phase and three-phase voltage sags. The paper concludes that results of simulation in Simulink environment have proved the correctness of the proposed algorithm for detecting and identifying the unbalanced voltage sags in the electrical grid under condition that it is free from high order harmonics.

## 1. Introduction

Unbalanced grid conditions often emerge because of asymmetrical voltage sags due to faults in transmission lines and in power transformers. The problem of voltage sags is the most acute for electric grids of mechanical engineering facilities with equipment of several megawatts such as arc furnace, cold/hot roll mills, blast furnace etc., which are sensitive to the electrical energy quality. Voltage sags may lead to disconnection of such equipment from the grid, interruption of the production process and consequent severe economic losses for the enterprise.

Fast and accurate identification of unbalanced grid conditions provide benefits for a number of power equipment. For the variable frequency drive, it can improve its ride-through capability via timely reconfiguration of the control system for implementing an auxiliary mode of operation in order to maintain the controllability of the drive. For protection devices of transmission systems, it may lead to faster isolation of faulted grid segments, therefore limiting voltage sag consequences for other parts of the power system. As to compensation devices in distribution networks, it may provide their faster respond to the advent of asymmetry and, therefore, proper and timely correction of the disturbances via putting into operation additional devices to secure nominal voltage for end-user devices.

The unbalanced state of the grid is usually described in terms of Fortescue's symmetrical components theory [1], which claims that any unbalanced three-phase system in a steady state can be represented as three balanced three-phase systems called sequences. Parameters of phasors of symmetrical components normally are estimated by applying the Fast Fourier Transform (FFT) to a set



of measurements. Extracted harmonics of each phase are analyzed to determine magnitudes and angles of positive and negative sequences [2].

An index of asymmetry for the unbalanced three-phase system usually is estimated through  $\alpha$  and  $\beta$  components of the voltage space vector by extracting basic and second harmonics with consequent calculation of the ratio of their magnitudes [3]. The performance speed of algorithms based on FFT is limited by the necessity to have a period of being processed signal in order to obtain the correct estimation of the phasors parameters - otherwise application of FFT to a part of sinusoid leads to the spectrum leakage and false computations [4].

In case of dynamical signals, Lyon's theory of generalized symmetrical components is used [5]. This theory allows transforming the dynamic phasors to time domain signals by putting into correspondence the phase shift between phasors to a respective time delay. Delayed signals are possible to obtain via direct accumulation of measurement samplings [6] or by digital filters, which provide additional benefits such as attenuating the harmonics and adaptive tuning. The typical examples of these circuits are second-order low-pass filters [7] and dual second order generalized integrators [8]. However, the performance speed of such circuits is bounded by the necessity to implement time delay of quarter of a period.

Frontier industrial controllers have processors that provide calculating sophisticated math functions during a cycle of computation. For real-time control systems in this case, the basic purpose is developing algorithms, which effectiveness mainly depends on computational performance of the controller instead of amount of data collected. The aim of the paper is to present this algorithm applied to fast and reliable detection of the grid voltage parameters under steady state unbalanced conditions.

## 2. Theoretical basis

Fault events in the power systems affect both magnitude and phase angle of voltages. During the sags, a voltage magnitude goes down to less than 90% of nominal value. Changes in phase angles are in the form of a phase angle jump, which is defined as a shift of zero crossing of instantaneous voltage in comparison with pre-event observations. Depending on different drops of RMS voltages and a phase angle between phases, the sags are classified into seven categories according to the ABC classification [9]. It is useful to find relations between these categories of sags and parameters of a rotating space vector, representing a three-phase system in different frames.

According to the Clark transform [1], coordinates of such vector in a stationary  $\alpha\beta$  plane can be obtained from instantaneous voltage measurements in  $abc$  system as follows:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (1)$$

Using complex vector notation, we can write down:

$$\vec{v} = v_\alpha + jv_\beta \quad (2)$$

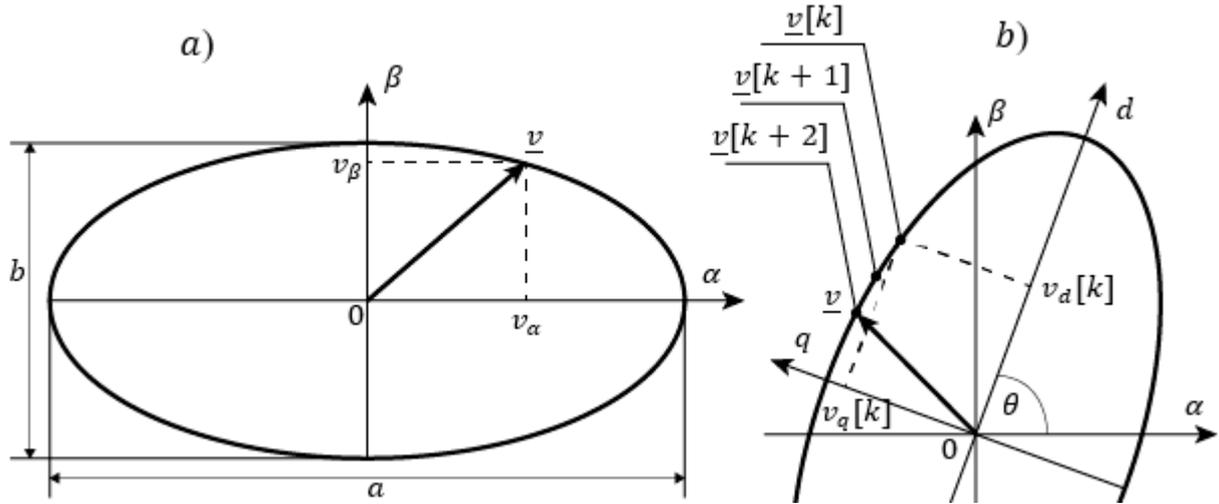
According to the Fortesque's theory [1], each voltage vector of the unbalanced system can be represented as sum of positive and negative sequence components:

$$\vec{V}_a = \vec{V}_a^+ + \vec{V}_a^-; \quad \vec{V}_b = \vec{V}_b^+ + \vec{V}_b^-; \quad \vec{V}_c = \vec{V}_c^+ + \vec{V}_c^- \quad (3)$$

Combining equations (1), (2) and (3) gives representation of space vectors in terms of positive and negative sequences:

$$\vec{v} = \vec{v}^+ + \vec{v}^- \quad (4)$$

Vectors  $\vec{v}^+$  and  $\vec{v}^-$  rotate in opposite directions. For unbalanced systems, the vector has an elliptical locus with parameters  $a, b, \theta$  indicated in Figure 1.



**Figure 1.** Ellipse locus of a grid voltage vector during unbalanced operation in:  $\alpha\beta$  coordinates – a) and  $dq$  coordinates – b)

The ellipse equation is as follows:

$$v_{\alpha}^2 \cdot a^{-2} + v_{\beta}^2 \cdot b^{-2} = 1 \quad (5)$$

Here  $a, b$  – major and minor axes of ellipse.

According to equation (4), these quantities are the maximum and minimum sum of magnitudes of positive and negative components:

$$a = V^+ + V^-; \quad b = |V^+ - V^-| \quad (6)$$

There are three types of voltage sags according to the classification based on symmetrical components. During one-phase voltage sags only axe  $a$  remains unaffected while  $b$  decreases. Two-phase sags lead to unequal decrease in both axes  $a$  and  $b$ . Three-phase voltage sags lead to an equal decrease in axes  $a$  and  $b$ .

### 3. Proposed algorithm

The proposed algorithm is based on verification of belonging of at least 3 measured samplings to one of the ellipse-like trajectories of the rotating space vector. For simplicity, the proposed algorithm will be discussed in detail for the set of two of samplings.

In this case, equation (5) goes to the system of two equations in respect to two variables:

$$v_{\alpha}[k]^2 \cdot a^{-2} + v_{\beta}[k]^2 \cdot b^{-2} = 1; \quad v_{\alpha}[k+1]^2 \cdot a^{-2} + v_{\beta}[k+1]^2 \cdot b^{-2} = 1 \quad (7)$$

Solution of system (7) for axes  $a$  and  $b$  can be written as follows:

$$a = \left( \left| v_{\alpha}^2[k+1] \cdot v_{\beta}^2[k] - v_{\alpha}^2[k] \cdot v_{\beta}^2[k+1] \right| \right)^{\frac{1}{2}} \cdot \left( \left| v_{\beta}^2[k]^2 - v_{\beta}^2[k+1]^2 \right| \right)^{-\frac{1}{2}}; \quad (1)$$

$$b = \left( \left| v_{\alpha}^2[k] \cdot v_{\beta}^2[k+1]^2 - v_{\alpha}^2[k+1] \cdot v_{\beta}^2[k]^2 \right| \right)^{\frac{1}{2}} \cdot \left( \left| v_{\alpha}^2[k]^2 - v_{\alpha}^2[k+1]^2 \right| \right)^{-\frac{1}{2}}$$

Equation (8) is valid for the case axe  $a$  of the ellipse is aligned along with the alpha-axis. With asymmetrical faults, the ellipse-like trajectory may be rotated at arbitrary angle  $\theta$  depending on the parameters of the unbalanced system. Therefore, all projections should be also rotated at this angle, which may be implemented by using the synchronous reference frame (Figure 1b). Coordinates of a vector in the orthogonal reference frame rotated at angle  $\theta$  are calculated via the Park transform [10]:

$$v_d = v_{\alpha} \cdot \cos \theta + v_{\beta} \cdot \sin \theta; \quad v_q = v_{\beta} \cdot \cos \theta - v_{\alpha} \cdot \sin \theta \quad (9)$$

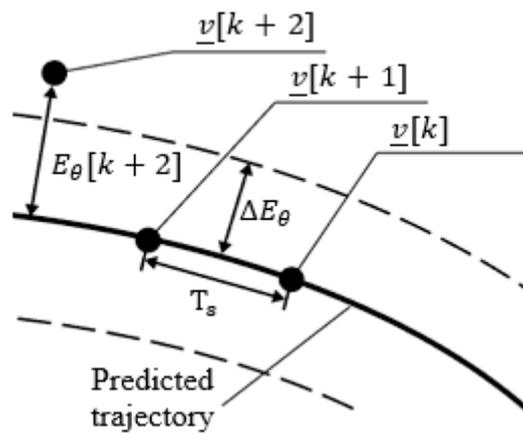
In a new coordinates system, the ellipse axes  $a$  and  $b$  are defined as follows; (2)

$$a = \left( \left| v_d^2[k+1] \cdot v_q[k]^2 - v_d^2[k] \cdot v_q^2[k+1] \right| \right)^{\frac{1}{2}} \cdot \left( \left| v_q[k]^2 - v_q^2[k+1] \right| \right)^{\frac{1}{2}}; \quad (10)$$

$$b = \left( \left| v_d^2[k] \cdot v_q[k+1]^2 - v_d^2[k+1] \cdot v_q^2[k] \right| \right)^{\frac{1}{2}} \cdot \left( \left| v_d[k]^2 - v_d^2[k+1] \right| \right)^{\frac{1}{2}}$$

Solving equation (5) for the third sampling under condition that axes  $a$  and  $b$  are found from equations (10), we obtain formula for assessment of deviation  $E_\theta$  of sampling  $\underline{v}[k+2]$  from the ellipse trajectory:

$$E_\theta = \left( v_d^2[k+2] \right) \cdot a^{-2} + v_q^2[k+2] \cdot b^{-2} - 1 \quad (3)$$



**Figure 2.** Possible placement of samplings in respect to the ellipse trajectory

One of the easiest ways to estimate the angle  $\theta$  is to solve equation (11) for a predefined set of angles in the range:  $\theta = 0 - 90^\circ$ . The quantization of angles affects accuracy of the algorithm and is dependent on the performance of the controller.

As the next step, we choose the solution with minimal absolute value of  $E_\theta$  and compare it with the predefined threshold  $\Delta E_\theta$ , defining the tolerance curve (Figure 2). If the solution error is greater than tolerance value  $\Delta E_\theta$ , then we assume that the transients exists and therefore wrong extraction of symmetrical components takes place. Otherwise, we detect the unbalanced steady-state grid operation with positive and negative sequence components, which magnitudes can be calculated by substitution of estimated values of axes  $a$  and  $b$  to equation (6):

$$V^- = |a - b| \cdot 2^{-1}; \quad V^+ = a - V^- \quad (4)$$

The balanced system has the circle trajectory of the voltage vector and, therefore, will be represented as a special case of the ellipse with  $a=b$ . The type of unbalanced voltage sag can be determined by deviation of quantity  $a$  from the pre-event value.

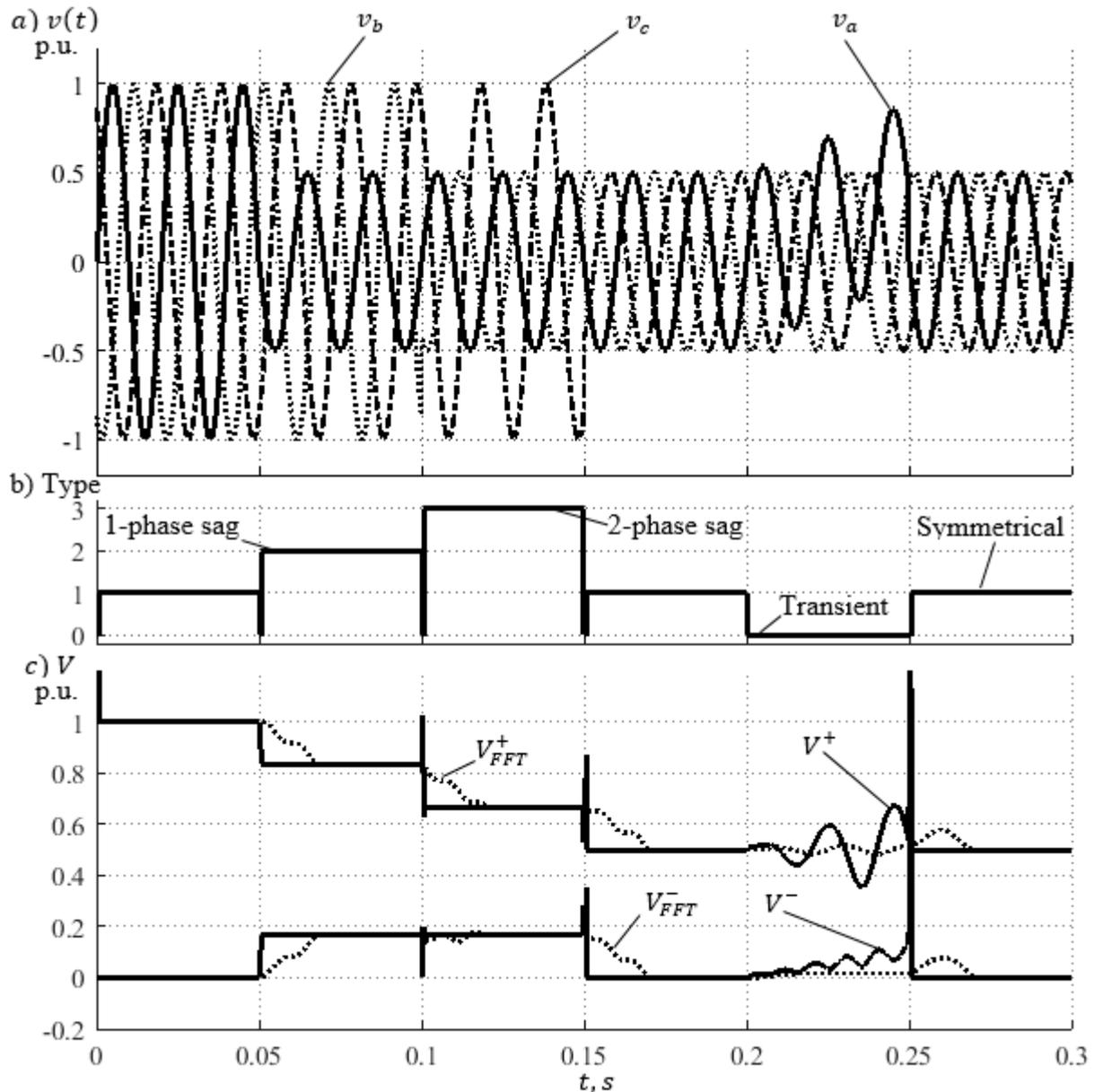
The proposed algorithm is valid for the case of ideal asymmetry of the system without distortions of the voltage waveforms. In case of distortions, the algorithm should be modified for the instance by extension of measurements and filtering the signals.

#### 4. Results of simulation

Simulation has been carried out in the Simulink environment with assistance of a processing unit built up in accordance with the proposed algorithm and controlled three-phase voltage source with the predetermined type of the voltage asymmetry in accordance with the symmetrical components classification. The controller is assumed to be fast enough to compute the whole cycle of operations

between measurements at the sampling frequency of 10kHz. A step of discrete variation of angle  $\theta$  was adopted as one degree.

Results of the simulation in per units are shown in Figure 4.



**Figure 3.** Results of the proposed algorithm study under different conditions. Top-to-down: phase voltages waveform – a); fault types identified by the algorithm: transients, symmetrical state, one-phase sag, two-phase sag – b); positive  $V^+$  and negative  $V^-$  sequence magnitudes extracted by the proposed algorithm in comparison with magnitudes  $V_{FFT}^+$ ,  $V_{FFT}^-$  obtained with the FFT-based algorithm – c).

Figure 4a demonstrates the modelled voltage waveform during asymmetrical operation of the system. Figure 4b shows the type of asymmetry detected by the algorithm. Figures 4c-d show positive and negative sequence components extracted by the algorithm and for comparison the same

components extracted by FFT-based algorithm are shown as well. The electrical grid operates in a normal mode in the interval of time  $0 \leq t \leq 0.05s$ .

During the one-phase sag at time  $t=0.05 - 0.1s$  the voltage in phase A decreases to 50% of nominal value. It can be seen that proposed algorithm is possible to detect one-phase sags and extract the magnitudes of positive and negative sequences nearly instantly. The same is fair for two-phase voltage sags taking place at time  $t=0.1 - 0.15s$ . It is evident that the algorithm distinguishes between one-phase and two-phase faults. During a three-phase voltage sag, at time  $t=0.15 - 0.2s$ , the algorithm detects the circle form of the voltage vector trajectory and classifies the system as a symmetrical one.

To study a respond of the algorithm to transients, the voltage in phase A was changing at the span of  $t=0.2 - 0.25s$ . Because the minimal error of evaluating the ellipse parameters proved to be higher than tolerance threshold, the algorithm detects transients, and therefore identification of positive and negative components is not possible.

## 5. Conclusions

Paper presents a novel digital algorithm that may be effectively used for fast extraction of the positive and negative sequences from asymmetrical voltage systems. In comparison with other similar algorithms, no spectrum analysis or filtering is needed. The proposed algorithm iteratively solves a set of equations to align the synchronous reference frame along the major or minor axes of the elliptical locus formed by a grid voltage space vector. To detect asymmetrical conditions in the system, at least 3 measured samplings are required.

In the presence of additional harmonics in the system, the algorithm is to be modified by the increase in the number of samplings to calculate the deviation error or by using filtered signals.

The algorithm performance has been validated by computer simulation of processing unbalanced signals of different origins in order to detect and identify one-phase, two-phase and three-phase voltage sags on the basis of symmetrical components classification. Results of simulation have revealed nearly instantaneous extraction of positive and negative sequences and clear detection of every type of faults.

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