

The effect of transport velocity upon spin torque

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Abstract. The paper analysis the effect produced by superposition of a rotation motion from contact area on a transport translational motion, having the same value for all contact points. A theoretical model is proposed for the calculus of the total friction torque. The deduced expression shows that the friction torque has a maximum value when the translational velocity is zero and rapidly decreases with increasing the translational velocity. The diminishing is not uniform, but is more evident at the beginning, when the ratio between the transport velocity and angular velocity is comparable to the radius of contact area and, afterwards, the decline of the friction torque is slower. The last part of the paper proposes a solution for an experimental validation of the theoretical results.

1. Introduction

The Hertzian point contact assumes that, between the boundary surfaces of two bodies, a theoretical contact point exists. Under normal force loading, in the region of the initial contact point, a contact surface occurs of elliptical shape and having the dimensions as found by Hertz in 1883 [1]. In the same paper, Hertz proves that on the contact area, a contact pressure occurs, having the maximum value in the centre of the contact ellipse and being zero on the boundary of the ellipse. The point contact between two bodies supposes that a class 1 kinematical joint is formed, namely, if one of the bodies is maintained motionless, the other one, from the six elementary motions theoretically possible, has a suppressed motion, the translation along common normal. The other simple motions determine, for an actual case, an extremely complex relative motion on the contact area.

The dynamic study of a body contacting a surface becomes substantially intricate when the hypothesis of smooth bodies is neglected and friction forces are considered between connections. Between the two surfaces, dry or fluid friction can happen. The dry friction case makes the dynamical study of a system complex because the characterisation is made using inequalities and, thus, working with unilateral links [2],[3]. Even under the hypothesis of perfect rigid bodies, the dynamical study with dry friction is difficult. To support this affirmation, the famous problem of the rolling coin can be mentioned [4],[5]. Considering that the relative motion between the two contacting bodies is described by the sliding velocity, positioned in a common tangent plane and by the relative angular velocity. The angular velocity is composed by the spin angular velocity, parallel to the common normal and by the rolling velocity, positioned in the common tangent plane. After setting equal to zero one or the other of the two component of angular velocity, with the hypothesis that the linear velocity is different from zero, there can be examined the rolling motion with sliding and the spinning motion with sliding.



Next, the axi-symmetric contact with spinning and sliding motion is considered, aiming to emphasise the influence of the transport velocity upon the spinning torque.

2. Theoretical considerations

As regarding an axi-symmetric contact, the contact area is a circle of radius r_{max} , figure 1. A current point is considered on the contact area, as the centre of elementary surface dA . The position of a current point from the contact area is set by the vector of position $\mathbf{r}$ of point M . The position of a current point M can be established in Cartesian coordinates, $M(x, y)$, or in polar coordinates, $M(\rho, \theta)$.

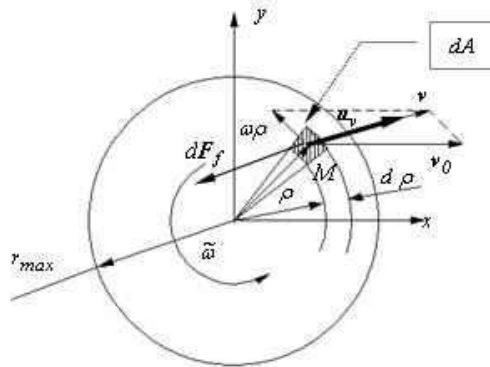


Figure 1. Relative motion on contact area.

The absolute velocity of a point from the contact area, $M(x, y) = M(\rho, \theta)$ is, according to [6],[7]:

$$\mathbf{v} = \mathbf{v}_0 + \omega \times \mathbf{r} = \begin{bmatrix} v_0 - \omega y \\ \omega x \\ 0 \end{bmatrix} \quad (1)$$

where ω is the angular velocity directed, in this case, along the common normal, and $\mathbf{v}_0$ is the transport velocity, identical for any contact surface point, with the direction of the horizontal. The unit vector of velocity $\mathbf{v}$ is:

$$\mathbf{u}_v = \frac{\mathbf{u}_v}{|\mathbf{u}_v|} \quad (2)$$

By calculating the absolute value of velocity (1) it results that, in any point of contact area, it is different from zero and as a consequence, the elementary friction force from the elementary surface is:

$$d\mathbf{F}_f = -\mu p(x, y) dA \mathbf{u}_v \quad (3)$$

where $p(x, y)$ is the pressure acting in point $M(x, y)$ and μ is the dynamic coefficient of friction. Between all contacting points, except for origin, relative motion exists. The elementary friction torque is:

$$d\mathbf{M}_f = \mathbf{r} \times d\mathbf{F}_f \quad (4)$$

By substituting in relations (1), (2), (3) and (4), after a series of calculus one obtains:

$$d\mathbf{M}_f = -\mu p(x, y) \frac{\omega \rho^2 - v_0 \rho \sin \varphi}{\sqrt{v_0^2 - 2v_0 \omega \rho \sin \varphi + \omega^2 \rho^2}} dA \mathbf{k} \quad (5)$$

where $\mathbf{k}$ is the versor normal to the contact surface in the theoretical contact point. To estimate the total torque, the integration of relation (5) on the entire contact area is required. Assuming a Hertzian pressure distribution, the integration can be made straightforwardly if the transition to polar coordinates is done:

$$p(\rho) = p_{max} \sqrt{1 - \frac{\rho^2}{r_{max}^2}} \quad (6)$$

where p_{max} and r_{max} are the maximum contact pressure and the maximum contact radius, respectively.

For the spinning torque the final expression is obtained:

$$M_f = -\mu p_{max} \int_0^{2\pi} \int_0^{r_{max}} \left(1 - \frac{\rho^2}{r_{max}^2}\right)^{1/2} \left(\rho^2 - \frac{v_0}{\omega} \rho \sin \varphi\right) \left(\frac{v_0^2}{\omega^2} - 2 \frac{v_0}{\omega} \rho \sin \varphi + \rho^2\right)^{-1/2} \rho d\rho d\varphi \quad (7)$$

Relation (7) shows that the friction torque does not depend separately on angular velocity and on transport velocity v_0 but depends on the ratio v_0 / ω , fraction that has the dimension of a length. For the integral from the right side it cannot be given an analytical expression.

By denoting the characteristic dimension:

$$r_0 = \frac{v_0}{\omega} \quad (8)$$

the relation (7) can be given under the final form:

$$M_f = -\mu r_{max}^3 p_{max} \int_0^1 t^2 \sqrt{1-t^2} \int_0^{2\pi} \left(t - \frac{r_0}{r_{max}} \sin \varphi\right) \left(t^2 - 2t \frac{r_0}{r_{max}} \sin \varphi + \frac{r_0^2}{r_{max}^2}\right)^{-1/2} d\varphi dt \quad (9)$$

To exemplify the effect of transport velocity upon spin velocity, it was considered the contact between a bearing ball 20mm in diameter placed on an horizontal steel plate. One considers the rotational speed of the ball 200rot/min to which corresponds an angular spin velocity. Although in literature there are models for the description of the many complex phenomenon of dry friction, below it is considered that the entire contact area that is constant. So, $\omega = 20.944 \text{ rad/s}$. the friction coefficient is thought to have the constant value $\mu = 0.2$ [8]. For the considered contact case, using the relations from the contact mechanics theory [9] for the normal force $Q = 100 \text{ N}$ it resulted the maximum contact radius $r_{max} = 1.866 \cdot 10^{-4} \text{ m}$ and the maximum contact pressure $p_{max} = 1.371 \cdot 10^9 \text{ Pa}$.

Applying relation (9), the variation of friction torque with respect to transport velocity was traced as seen in figure 2. The abscissa of the plot is dimensionless by dividing to relative rotational speed of a point from the boundary of contact area, ωr_{max} . It can be noticed that, for the ratio $(v / \omega \cdot r_{max}) < 1.5$ there is a strong dependency. External to this interval, the influence is much more diminished. For very high transport velocities, the total friction torque tends asymptotically to zero.

To obtain an intuitive representation, it was considered the ratio between friction torque and normal load, resulting a term with length dimension, represented in the ordinates of next figures.

In figure 3 there are traced, for several values of transport velocity, the plots of total friction torque versus normal load force. From these graphs it can be remarked that the influence of normal force upon spinning torque is much stronger for reduced values of the transport velocity.

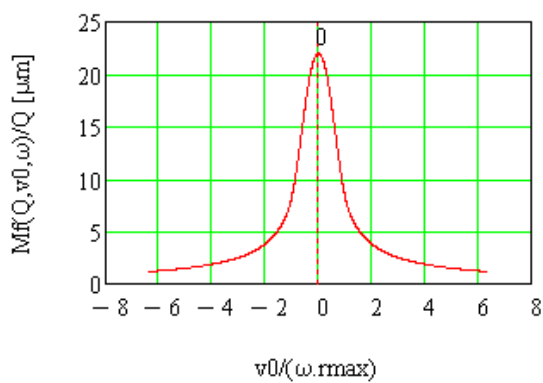


Figure 2. Total friction torque variation with transport velocity.

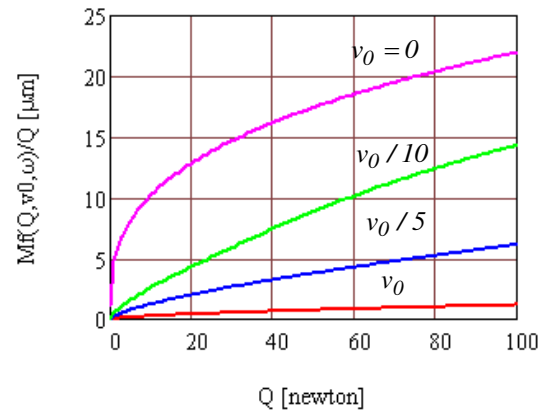


Figure 3. Normal force effect upon friction torque for different values of transport velocity.

The expression of the elementary friction torque can be obtained considering the integrant from surface integral (7):

$$M_{elemf} = -\mu p_{max} \left(1 - \frac{\rho^2}{r_{max}^2} \right)^{1/2} \left(\rho^2 - \frac{v_0}{\omega} \rho \sin \varphi \right) \left(\frac{v_0^2}{\omega^2} - 2 \frac{v_0}{\omega} \rho \sin \varphi + \rho^2 \right)^{-1/2} \quad (10)$$

Another interesting dependency is presented in figure 4. The contact area was divided into concentric circular rings having the same surface area.

$$M_{rad_i} = - \int_{R_{i-1}}^{R_i} \int_0^{2\pi} M_{elemf} dA \quad (11)$$

The friction torque corresponding to each ring was found and it was represented the friction torque as function of external radius of each ring.

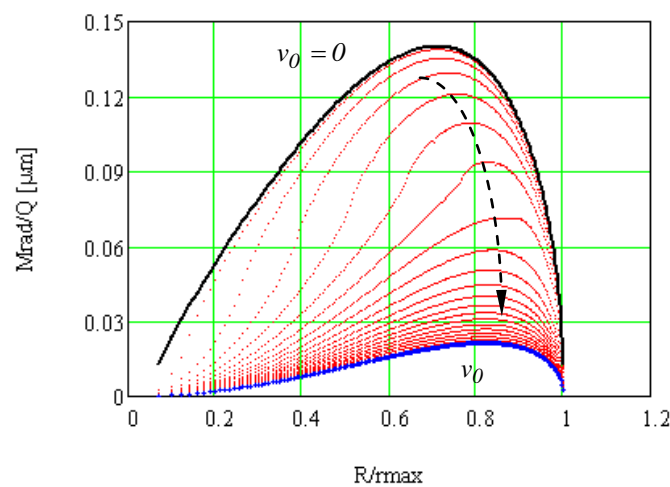
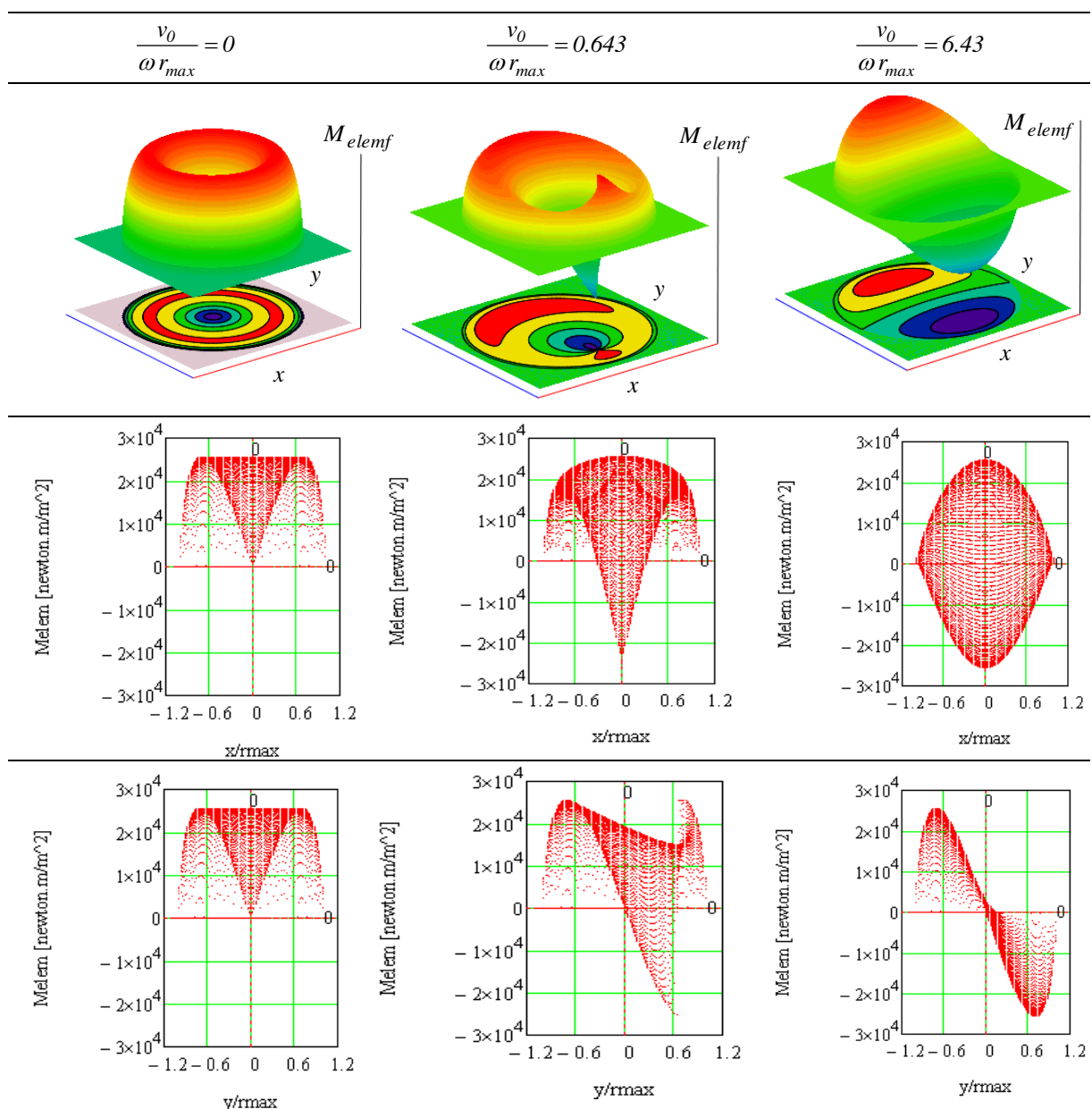


Figure 4. Friction torque (divided by normal load) variation with relative radius.

In table 1 there are presented the spatial plots, level curves and sections on the direction of transport motion and normal to it, for different values of transport velocity. Here, x y are the coordinates of a moving point which is placed inside a rectangle which envelopes the contact area. It can be noticed that, for zero transport velocity, the elementary friction torque is positive on the entire surface and has an axi-symmetric distribution. With increasing transport velocity, regions where the elementary friction torque is negative occur, and thus, the total friction torque decreases. For greater values of transport velocity, the spatial graph of elementary friction torque has the centre of contact area as anti-symmetry point, and therefore the elementary friction torque from a point would be compensated by the one from the symmetrical point with respect to the contact centre and, globally, the friction torque is zero.

Table 1. The effect of transport velocity upon elementary friction torque



3. Experimental validation

Next, an experimental qualitative justification of the influence of transport velocity upon spinning friction torque is presented. It is considered a bearing ball, 60 mm diameter, placed on the surface of a biconcave lens (figure 5). The ball is manually run into rotation motion and it is observed that when the velocity of the ball, after launching, exceeds a certain value, the motion of the ball is spatial and the contact point moves on the surface of the lens. The angular velocity of the ball decreases due to friction and finally, the contact point becomes immobile and the ball has a motion around the vertical axis passing through the contact point. The rotation motion carries on till resting is settled.

When following launching, the angular velocity of the ball was not high enough, the motion of the ball was from the beginning a rotation around a still axis. Applying a reflective stamp on the ball surface in the equator region, it can be found the rotation velocity of the ball using a non-contact tachometer. In figure 6 there are presented the plots of ball rotational speed for a series of successive launchings. For all launches one can notice the existence of two regions: a zone where a noise exists, corresponding to spatial motion and a region much smoother, corresponding to the motion around an immobile axis.



Figure 5. Experimental set-up.

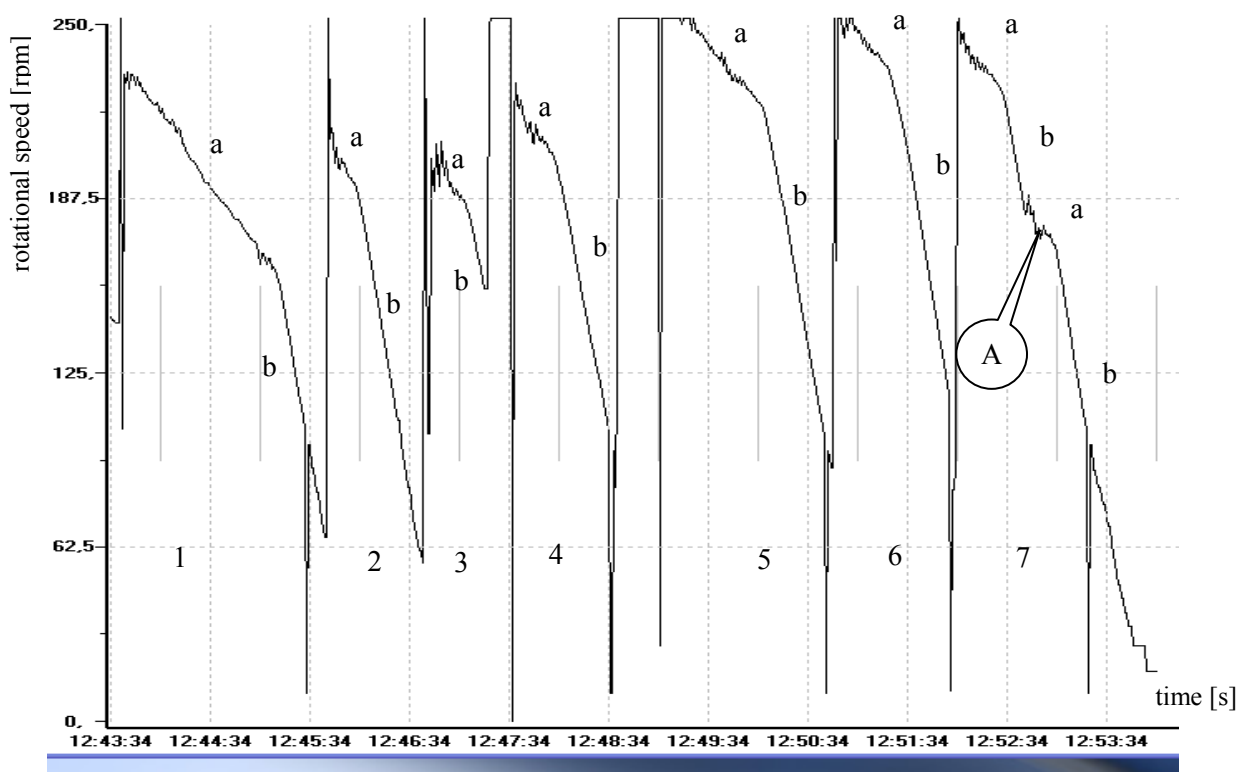


Figure 6. Experimental plots of rotational speed versus time.

The slope of the region with noise is smaller than the slope for the smooth region, fact that confirms that when the contact point is mobile the friction torque is smaller than for the case when it is immobile. Additionally, the two regions are connected, fact that confirms that a continuous variation with velocity of the friction torque. On the smooth regions the plots are straight lines and parallel to each others and, thus, they prove the presence of a constant torque, having the same value for all launches. In the A point from the curve corresponding to the last launching it can be noticed a region with noise intercalated within a smooth region. This region was produced by applying a small shock upon the lens after the rotation about the immobile axis was installed. After the shock was applied, the motion of the ball becomes spatial but quickly diminishes and the rotation about the axis restores.

4. Conclusions

The paper considers the Hertzian contact of two axi-symmetric bodies. In a first phase, one considers that the relative motion between the two bodies is rotation about common normal. The distribution of elementary friction forces on contact area generates a spinning friction torque. The work studies the effect upon relative motion from contact area of superposition of a relative translation having the same values in all points of contact area.

A theoretical model for the estimation of the total friction torque is proposed. The expression obtained shows that the friction torque reaches maximum for zero translation velocity and decreases rapidly with increasing the translation velocity. However the decrease is not uniform, being more evident at the beginning when the ratio between the transport velocity and angular velocity is comparable to the radius of contact area and afterwards, the decrease of friction torque is much slower.

The last part of the paper proposes a solution for the experimental corroboration of theoretical results. The contact between a ball and a spherical cavity is considered and after launching the ball into rotation motion, two stages can be identified: in a first phase, the ball has a spatial motion and the contact point is mobile while during the second phase the motion of the ball stabilizes and the direction of angular velocity remains still.

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