Analysis of progressive cavity pumps specific wear processes using Bitter and Hutchings models

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Abstract. Progressive cavitation pumps are designed to work in aggressive environments thus their wear is inevitable. The specific tribological coupling of these pumps is composed of a helical rotor with a single outward helix and a stator with a double inside helix. These elements are in relative motion and direct contact to each other and also in direct contact with the pumped oil. Therefore the main forms of wear of rotor-stator coupling elements are the abrasive wear and the erosion wear. In this paper is presented the analysis of erosion with Bitter and Hutchings models. The results are useful for estimation of progressive cavity pumps specific abrasive-erosive wear.

1. Introduction

The researchers attention was focused on the study of the influence factors from the abrasive erosion models relations, to specify the share of their influence, the precision in describing the erosive processes, the generalization degree and the limits of applicability.

The main assumptions on which erosion models were developed are as following [1-3]:

a – the erosive wear particle occurs due to the mechanical action of abrasive particles on the surfaces that cause microcracking and microcutting of the incident material layer (the microcutting component of erosion);

b - the kinetic energy of the abrasive particles is generated by their impact with the target surface causing the deformation of the material, initiation and propagation of crack and detachement of wear particle when the cracks have reached a critical length (the localized plastic deformation component associated with the mechanical fatigue process).

The first case is typical to ductile material and the second one to fragile material. The assumptions made are based on Johnson, Cook and Holmquist studies [4-5] and analyzed by Wang [6].

2. Bitter and Hutchings analitical models of abrasive erosion wear

2.1. Bitter model

In 1963 Bitter [7] analyzed the abrasive erosion regarding two components:

- the microcutting component
- the component of plastic deformation of erodent material surface.

Bitter considers that the microcutting process is attend by particle sliding on the surface.

The sliding speed is distinguished when:

- tangential component different from zero at the time of detachement of the incident surface;
- tangential component is zero at a given time of the impact.

Thus, Bitter proposed relations for calculating the adimensional rate of wear, differentiated according to the size of the speed tangential component.

The equation for the tangential speed component different from zero is:

$$E_{B,C_1} = \frac{2\rho C_1 \left(U_i \sin\alpha - U_{el}\right)^2}{\left(U_i \sin\alpha\right)^{0.5}} \cdot \left(U_i \cos\alpha - \frac{\varepsilon_C C_1 \left(U_i \sin\alpha - U_{el}\right)^2}{\left(U_i \sin\alpha\right)^{0.5}} \right) \tag{1}$$

where:

$$C_1 = 0.361 \frac{\sigma^{0.25}}{H_S^{1.25}} \tag{2}$$

and for the tangential speed component is zero, the equation is:

$$E_{B,C2} = \frac{\rho\left(U_i^2 \cos\alpha - C_2 \left(U_i \sin\alpha - U_{el}\right)^{1.5}\right)}{2\varepsilon_C} \tag{3}$$

where:

$$C_2 = 0.547 \frac{H_S^{2.25}}{\sigma^{0.25} E_c^2} \tag{4}$$

For repeated surface deformation component, Bitter suggest the following relation for calculating the rate of wear:

$$E_{B,DW} = \frac{\rho\left(U_i \sin\alpha - U_{el}\right)^2}{2\varepsilon_D} \tag{5}$$

where:

$$U_{el} = 1.25 \frac{H_S^{2.5}}{\sigma^{0.5} E_e^2} \tag{6}$$

The terms used in previous equations have the following meanings:

$E_{B,C1}$, $E_{B,C21}$ and $E_{B,DW}$ – erosion wear rate

v_1 - impact speed of the particle;

v_{el} - the impact speed that is reached at the elastic limit of the material;

ε_D - the specific energy of deformation;

ρ_{me} - density of the erosive particle material;

E_c- modulus of elasticity on impact;

ε_c - the specific wear energy for microcutting;

C_1 and C_2- constants with specified relations.

The overall erosive rate of wear is calculated as sum of the specified components, differentiated by the angle of incidence

$$E_B = E_{B,DW} + E_{B,C_1} \text{ - for } \quad \alpha \le \alpha_0 \tag{7}$$

$$E_B = E_{B,DW} + E_{B,C_2} \text{ - for } \quad \alpha > \alpha_0 \tag{8}$$

where α_0 corresponds to the angle of impact at which the tangential component of speed is zero, which occurs when the particle leaves the incident surface.

2.2. Hutchings model

Hutchings [1] consider two models for the erosion wear: the microcutting erosion model and the plastic deformation erosion model. For microcutting erosion Hutchings suggests two equations for calculating wear rate:

- for the normal angle of incidence $\alpha_0 = 90^{\circ}$, the relation is:

$$E_{H,n} = K_{er} \cdot \frac{\rho_m \cdot v^2}{H} \tag{9}$$

where: K_{er} – erosive wear coefficient (the ratio between the volume of detached material by wear and the total volume of material affected by the microcutting process).

- for oblique impact, $\alpha_0 < 90^{\circ}$, the relation is:

$$E_{H,n0} = K_{er} \cdot \frac{\rho \cdot v^n}{H} f(\alpha) \tag{10}$$

where: n – speed exponent with values in the range 2÷2.5;
$f(\alpha)$ - depending on the angle of incidence.

When the erosion is caused by plastic deformation due to spherical abrasive particles that strikes the target surface for incidence angle less than 90°, the relation is:

$$E_H = 0.033 \cdot \alpha_p \cdot \frac{\rho_m \cdot \rho_a^{0.5} \cdot v^3}{\varepsilon_{crt}^2 \cdot H_d^{1.5}} \tag{11}$$

The α coefficient is defined by the ratio between the volume of plastically deformed material and the indentation volume. This coefficient depends on the geometry of the indentation, impact speed and the characteristics of the target material.

Hutchings consider the Coffin-Manson equation to calculate the ε_{crt} (fatigue model for normal impact):

$$\varepsilon_{crt} = \Delta\varepsilon_p \cdot N_c^{1/2} \tag{12}$$

where: N_c – the number of stress cycles at which occurs the particle of wear.
$\Delta\varepsilon_p$ (plastic deformation) determined with the relations [8]:

$$\Delta\varepsilon_p = 0.2 \cdot \frac{a}{r} \tag{13}$$

$$a = r\sqrt{2 \cdot v} \cdot \sqrt[4]{2 \cdot \sigma / 3 \cdot H} \tag{14}$$

where: r –particle radius; a – indentation radius.

3. Results and analysis

3.1. Bitter model

Bitter [7] describes the mechanism of abrasive erosion considering two components that occurrs simultaneously: the microcutting and repeated deformation of the target surface.

Bitter parameters involved in the model are listed in table 1.

Table 1. Bitter model parameters.

parameter	calculation value	
	elastic deformation	microcutting
ρ_m – density of the targe material [kg/m^3]	1.19	
H_s – surface hardness [N/m^2]	stator I: 80; stator II: 75; stator III: 70	
E_m – equivalent elastic modulus [N/m^2]	stator I: 4.2; stator II: 3.6; stator III: 1.7	
ρ_a – density of the particle material [kg/m^3]	2590÷2670	
v – particle impact speed [m/s]	4.71	
c_r – coefficient of restitution	0.5	
α – the angle of incidence [°]	30 °	
α_0 – the angle at which the speed is zero	0°	
$\varepsilon_D, \varepsilon_C$ – the specific energy for wear [J/m^3]	$\varepsilon_D = 4.7\cdot10^{10}$	$\varepsilon_C = 2.2\cdot10^{10}$

The analysis of wear rate was achived for two cases:

1. $E_B = f(\alpha)$ - the variation depending on the angle of incidence, for the three analyzed stators (H_s – surface hardness: stator I: 80 N/m^2; stator II: 75 N/m^2; stator III: 70 N/m^2);
2. $E_B = f(v)$ - the variation depending on the impact speed, for the three analyzed stators (H_s – surface hardness: stator I: 80 N/m^2; stator II: 75 N/m^2; stator III: 70 N/m^2).

The results are shown in figure 1 and figure 2.

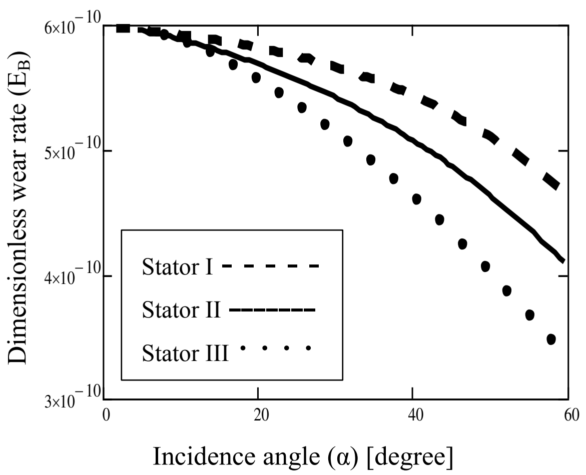

Figure 1. Wear rate variation depending on the angle of incidence.

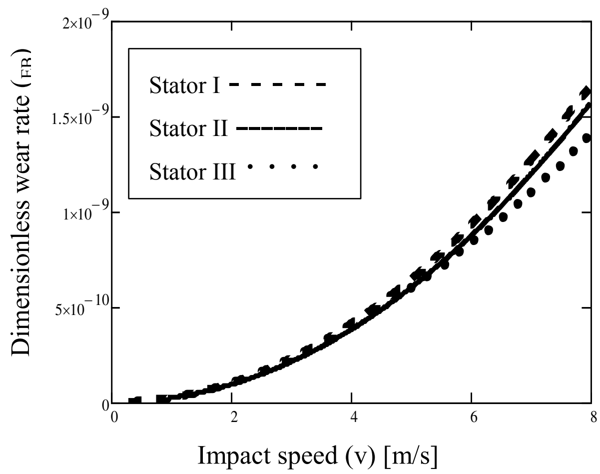

Figure 2. Wear rate variation depending on the impact speed.

3.2. Hutchings model

Hutchings [1] has developed two models for abrasive erosion wear differentiated by the erosion mechanism, as follows:

- microcutting erosion model (the first model);
- erosion by plastic deformation model (second model)

The involved parameters in the calculating equations, for both models, are listed in table 2.

Table 2. Hutchings model parameters.

parameter	calculation value	
	microcutting	plastic deformation
ρ_m – density of the targe material [kg/m^3]	1.19	
H_s – surface hardness [N/m^2]	stator I: 80; stator II: 75; stator III: 70	
v – particle impact speed [m/s]	4.71	
c_r – coefficient of restitution	0.5	
ρ_a – density of the particle material [kg/m^3]	--	0.33
Γ – rate of removed volume depending on the particle volume and the critical stress	--	0.7÷13

The analysis of wear rate was achived for the following cases:

1. $E_H = f(v)$ - the variation depending on the impact speed, for the three analyzed stators (H_s – surface hardness: stator I: 80 N/m^2; stator II: 75 N/m^2 ; stator III: 70 N/m^2);

The results are shown in figure 3 and figure 4.

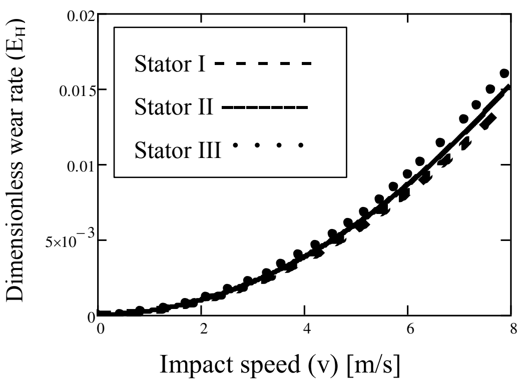

Figure 3. Wear rate variation depending on the impact speed for the microcutting model.

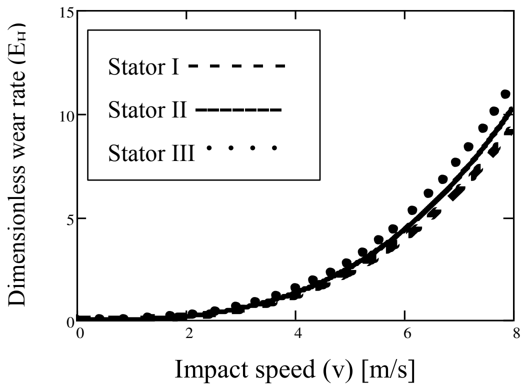

Figure 4. Wear rate variation depending on the impact speed for the plastic deformation model.

2. $E_H = f(\Gamma)$ - the variation depending on the ratio of removed volume depending on the particle volume and the critical stress, for the three analyzed stators (H_s – surface hardness: stator I: 80 N/m^2; stator II: 75 N/m^2 ; stator III: 70 N/m^2).

The results are shown in figure 5.

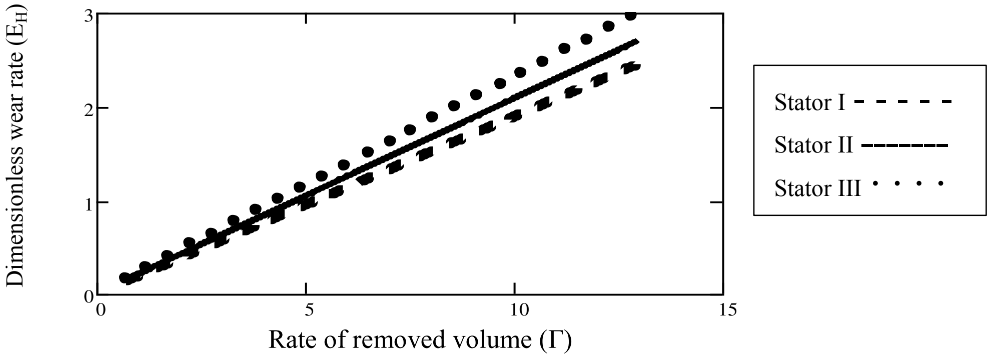

Figure 5. Wear rate variation depending on the rate of removed volume for the plastic deformation.

The analysis of the obtained results for Bitter and Hutchings models leads to the following findings:

- the shape of wear rate variation curves with the angle of incidence is the same for different values of the friction coefficient and for the same value of the impact speed;
- both Hutchings models consider exponentially increases of erosive wear rate with the increasing of the impact speed, indicating that the first model estimates higher values for erosive wear;
- the second Hutchings model shows a linear increasing variation of the erosive wear rate with the value of the ratio of removed material volume that depends on the particle volume (coefficient $\boldsymbol{\Gamma}$);
- Bitter model shows that the intensity of erosive wear increases linearly with the angle of incidence to its value when the speed is zero ($\alpha = \alpha_0$), then the variation of the intensity of erosion decreases exponentially;
- the impact speed also produces an exponential increase of the wear intensity up to $\alpha > \alpha_0$ and then it decreases exponentially for $\alpha < \alpha_0$;

4. Analysis and validation of the mathematical models

The calculated values of the wear rate, for the angle of incidence $\alpha = 20°$, for the speed $v = 4.71$ [m/s], are shown in table 3, for Finnie, Bitter and Hutchings model, compared to the values experimentally determined for the three analyzed stators.

Table 3. Wear rate calculated for the working conditions.

Stator	The calculation model of the rate of wear	Rate of wear	
		Calculated value	Experimentally determined value [9]
I	Finnie I	$6.454 \cdot 10^{-3}$	$5.356 \cdot 10^{-3}$
	Finnie II	$4.841 \cdot 10^{-3}$	
	Bitter	$5.533 \cdot 10^{-10}$	
	Hutchings (microcutting)	$4.928 \cdot 10^{-3}$	
	Hurchings (plastic deformation)	$1.887 \cdot 10^{-1}$	
II	Finnie I	$6.885 \cdot 10^{-3}$	$5.783 \cdot 10^{-3}$
	Finnie II	$5.163 \cdot 10^{-3}$	
	Bitter	$5.523 \cdot 10^{-10}$	
	Hutchings (microcutting)	$5.257 \cdot 10^{-3}$	
	Hurchings (plastic deformation)	$2.079 \cdot 10^{-1}$	
III	Finnie I	$7.376 \cdot 10^{-3}$	$6.175 \cdot 10^{-3}$
	Finnie II	$5.532 \cdot 10^{-3}$	
	Bitter	$5.577 \cdot 10^{-10}$	
	Hutchings (microcutting)	$5.633 \cdot 10^{-3}$	
	Hurchings (plastic deformation)	$2.306 \cdot 10^{-1}$	

The comparative analysis of calculated rate of wear with the one experimentally determined, shows the following:

- first Finnie model highlights the biggest difference between the calculated rate and experimentally determined rate of wear. The equation can be applied only in the rare cases in practice, where the content of the erosive solids particles reaches 60%;
- the results acchived with the second Finnie model and the Hutchings model (microcutting) are the closest to the experimentally determined values of wear rate. It is therefore reccomanded their use in the evaluation of erosive process of progression cavitation pumps. Hutchings model

(microcutting) is suitable for the cases where the microcutting is predominant, and the second Finnie model is suitable where the microcutting is accompanied by plastic deformation caused by the impact of abrasive particle on the target material;
- the values obtained for the erosive wear rate with Bitter model are practically invalide, so it can't be applyed in the present case;
- the results obtained with Hutchings model (plastic deformation) can not be used because the plastic deformation of stator materials (the base assumption for the Hutchings equations) is not specific to the dynamic of the erosive process for progressive cavitation pump.

5. Conclusions
The presented results of the erosion wear analysis using Finnie, Bitter and Hutchings models are useful for the followings:
- quantitative estimation of erosive wear;
- highlighting the erosion evolution;
- quantify the direct influence and in interaction of the most significant parameters of erosion.

Analysis of factors influence on abrasive erosion provides data reguarding: the share of their influence, their generalization degree and the limits of applicability.

The obtained results provides solutions necessary for the design and for constructive optimization of the progressive cavitation pump or for other mechanical systems that are functioning in a similar condition of erosive wear.

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