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The effect of friction in coulombian damper

H S Wahad[1], A Tudor[1], M Vlase[2], N Cerbu[1] and K A Subhi[1,3]

[1] University Politehnica of Bucharest, Spl. Independentei 313, 060042, Bucharest, Romania
[2] Technical University of Civil Engineering of Bucharest, Romania
[3] AL-Furat AL-Awsat Technical University of Iraq

E-mail: haider.shahad@gmail.com

Abstract: The study aimed to analyze the damping phenomenon in a system with variable friction, Stribeck type. Shock absorbers with limit and dry friction, is called coulombian shock-absorbers. The physical damping vibration phenomenon, in equipment, is based on friction between the cushioning gasket and the output regulator of the shock-absorber. Friction between them can be dry, limit, mixture or fluid. The friction is depending on the contact pressure and lubricant presence. It is defined dimensionless form for the Striebeck curve (μ friction coefficient - sliding speed v). The friction may damp a vibratory movement or can maintain it (self-vibration), depending on the μ with v (it can increase / decrease or it can be relative constant). The solutions of differential equation of movement are obtained for some work condition of one damper for automatic washing machine. The friction force can transfer partial or total energy or generates excitation energy in damper. The damping efficiency is defined and is determined analytical for the constant friction coefficient and for the parabolic friction coefficient.

1. Introduction

In the shock-absorbers with dry and boundary friction regimes, also named coulombian shock-absorbers, the functional key element is the cushioning gasket (ring). This cushioning gasket is made of rubber, soaked with lubricant till saturation (oil or grease). These shock-absorbers are frequently used in machine building, with applications in construction equipments for different mixing materials, washing machines, and so on [1- 3].
Depending on the method of gasket radial fixing, the shock-absorbers are classified in direct absorbers (fixed in the outer shell), or reverse absorbers (attached to the rod).

The physical phenomenon of damping vibration, in equipment, is based on friction between the cushioning gasket and the output regulator of the shock-absorber. The output regulator is the steel rod of the direct shock-absorber or the shock-absorber housing for the "reverse" shock-absorber. In figure 1it is represented a shock-absorber with friction, (coulombian shock-absorber), used in automatic washing machines construction [4].

It is known that, in a friction coupling, when the sliding speed is variable (the case of starting, stopping or operating conditions with variable speed of rotation), the friction can be dry, limit, mixed or fluid, also depending on the contact pressure and lubricant presence. In these conditions, the friction is characterized by a Striebeck curve (friction coefficient μ - sliding speed v) [4], [5]. And also it is known that friction may damp a vibratory movement or can maintain it (self-vibration),depending on the friction coefficient variation with speed (increase, decrease or it can be relative constant)[6-11].

In all cases, the friction force is a disturbing force which is, partial or total, transferred through the shock-absorber or it can maintain the vibratory movement.

The paper aim is to analyze the damping phenomenon in a system with variable friction, Striebeck type, and to determine the work conditions of the direct coulombian shock-absorber when the effect of self - vibrations appears. It is defined and determined the damping efficiency of shock-absorber.

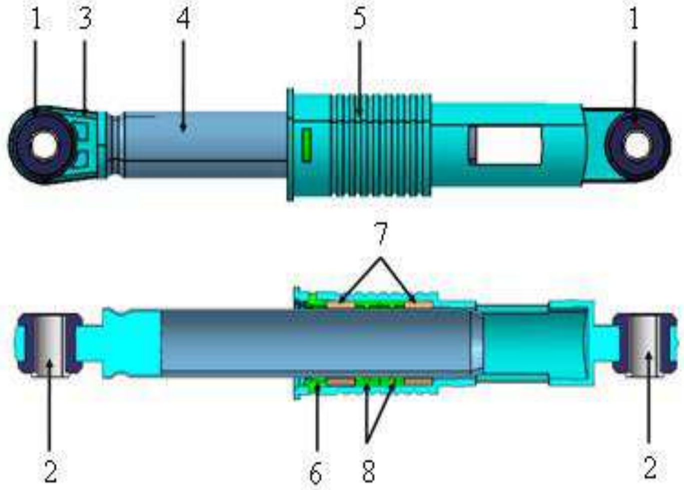


Figure 1a. Shock-absorber with friction [4]. 1 - Sealing member made of rubber; 2 - Plastic inner lining; 3 - Clamping sleeve of the shock-absorber; 4 - Steel rod; 5 - Body shock-absorber equipped with clamping sleeve; 6 - Limiting sleeve;
7 - Shock-absorber rings; 8 – Spacers.

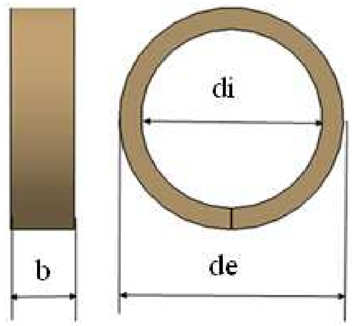


Figure 1b. The geometry of the shock-absorber ring [4].

2. Model for Striebeck friction analysis

When a friction coupling is in a state of film lubrication, the friction coefficient has a linear dependence with velocity for fluids with threshold flow

$$\mu_n = c_h v + \mu_o \tag{1}$$

where: c_h parameter depends on lubricant viscosity, the medium contact pressure and the couple geometry; μ_0 – coefficient, dependent on the lubricant rheology (threshold flow).
In the area of low and very low sliding speed, even if lubricant exists, it can`t be formed a continuous film bearing of lubricant. In this case, at zero velocity, there is static friction characterized by μ_S coefficient, which is dependent on the couple material, real contact pressure, and so on. It is accepted that in this zone, the friction coefficient is maintained constant until a certain velocity v_0:

$$\mu_0 = \mu_S \quad \text{for} \quad 0 \le v \le v_0 . \tag{2}$$

When velocity increases over v_0 limit, in contact, the normal force is transmitted through the real area which increases by velocity and friction coefficient decreases curvilinear by velocity until it reaches a limit value μ_m. Analyzing the results for different Striebeck curves, [4], [5], it is accepted in the zone of dry, limit and mixed friction a parabolic variation as:

$$\mu_u = av^2 + bv + c , \tag{3}$$

with a, b, c determinable constants for certain known points (v_0, μ_S), (v_m, μ_m) – minimum point.

For velocities higher than v_m a fluid friction begins, with a friction coefficient having an analytical form as equation (1). To resolve the analytical equation of vibratory movement in the shock-absorber it is accepted the existence of a point on the friction coefficient curve (3) for which line (1) is tangent.

The coordinates of this point (v_{cr}, μ_{cr}) define the transition from dry, limit and mixed friction to fluid friction. These coordinates satisfy the conditions of continuity and derivability of friction coefficient.

The following notations are made: $\mu_a=\mu/\mu_S$; $\mu_{ma}=\mu_m/\mu_S$; $v_{am}=v/v_m$; $v_{0m}=v_0/v_m$; $v_{0a}=v_0/v_m$; $v_{a\,cr}=v_{cr}/v_m$; $c_{ha}=c_n \cdot v_m/\mu_S$. Putting the continuity conditions everywhere and derivability in the zone of limit and mixed friction of friction coefficient function, Striebeck type, the constants a, b, c, μ_0 can be determined.

Thus, the relative friction-sliding coefficient for static friction ($\mu_a=\mu/\mu_S$) has the expression

$$\mu_a = \begin{cases} 1 \quad if \quad 0 \le v_{am} \le v_{om} \\ a_a v_{am}^2 + b_a v_{am} + c_a \quad if \quad v_{0m} < v_{am} < v_{a\,cr} \\ c_{ha} v_{am} + \mu_o \quad if \quad v_{am} \ge v_{a\,cr} \end{cases} . \tag{4}$$

The Striebeck curves are three kinds, depending on lubricant viscosity (c_{ha}), minimum sliding velocity (v_{0m}) and the minimum value of the friction coefficient μ_{ma}. As an example, the figure 2 shows the theoretical curve for some minimum value of the friction coefficient.

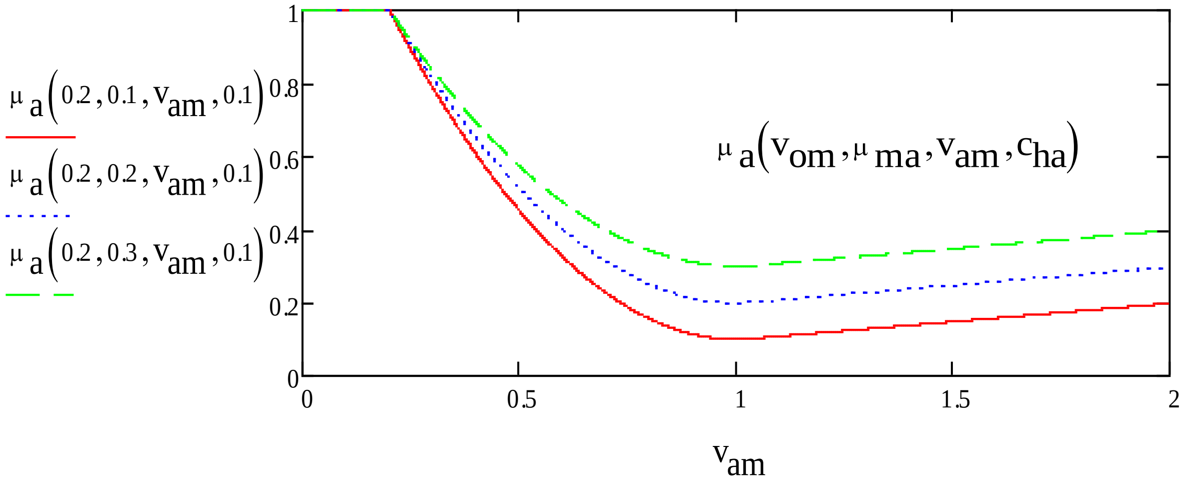


Figure 2. Striebeck curves for some minimum friction coefficients.

3. Analytical model for vibratory movement in the coulombian damper

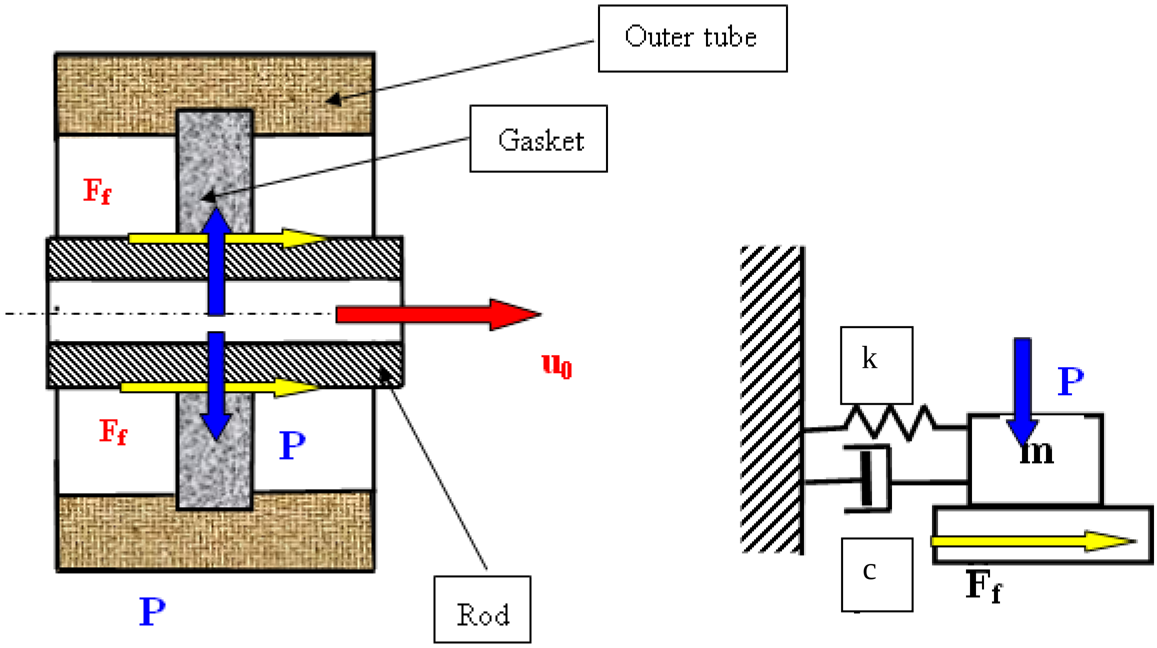
To analyze the outer shell shock-absorber movement witch is in contact with the two damping gaskets, it is considered as known: the outer shell mass (m), the axial rigidity of the outer shell along with the drive support (k), the viscous damping in the drive support joint (c), the gasket geometry (inner diameter D_g, width b_g and friction curve, Striebeck type ($\mu_a - v_{am}$) (equation 4).

As a model, it is proposed the system from figure 3, consisted of a material point having "m" mass, supported with Striebeck friction (F_f) on a body. This body moves with variable speed: $u = u_0 \cos(\omega t)$, where u_0 – is the drive speed amplitude; ω - pulsation; t – time (figure 3).

The friction force between the material point and the rigid support depends on the relative velocity $v = u$ - dx/dt. If the "u" velocity is high enough (u > dx/dt), then the friction force will act in the positive direction of Ox axis, otherwise it will act in the negative direction of the axis. Thus, upon the material point will act the forces: – c (dx/dt), kx and F_f (figure 3b). The friction force depends on the friction coefficient and the normal force, generated by the contact pressure, (p_m), between the rod and the gasket:

$$F_f = \mu F_n = \mu_S \mu_a 2\pi D_g b_g \cdot p_m = k_a \mu_a \quad , \tag{5}$$

where parameter $k_a = \mu_S 2\pi D_g b_g p_m$ depends on the static friction characteristics of the gasket material (μ_S), gasket geometry (D_g, b_g) and the radial contact pressure (p_m), determined by the fitting assemblage and the elastic characteristics of the gasket material.



a) b)

Figure 3. Scheme of forces (a) and the dynamic model (b) in the coulombian shock-absorber.

The sliding friction coefficient relative to the static one, μ_a, has the expression (4), in which the sliding velocity is

$$v = u - dx/dt \qquad \text{or} \qquad v_{am} = \frac{v}{v_m} = \omega_{mi}\cos\omega t - dx_m/dt \tag{6}$$

where $u_m = u_0/v_m$ and $dx_m/dt = (dx/dt)/v_m$ are the relative driving velocity, respectively, the relative oscillation velocity.

The differential equation of motion will be:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_f = k_a\mu_a \, . \tag{7}$$

Taking into account μ_a (equation (4)), the solution of equation (7) has different forms, depending on the driving speed, u, and Striebeck friction characteristics.

Case 1 – constant friction coefficient (first part of Striebeck curve).
For the first part of Striebeck curve ($v < v_0$) for which the friction coefficient is constant ($\mu_a = \mu_S$), equation (7) can be written in the classic mode [3], [12]:

$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + p^2x = q\,, \tag{8}$$

where: $2\alpha = c/m$; $p^2 = k/m$; $q = k_a\mu_S/m$.

The values of α and p parameters lead to different forms of solutions for the equation (8). Thus, noting $c_0 = 2\sqrt{km}$ the critical coefficient of damping, which corresponds to the case $\alpha = p$, $\xi = c/c_0$ – the damping factor, there are three different situations: $c < c_0$, $c > c_0$ şi $c = c_0$.
a) Sub case $c < c_0$ $(\alpha < p)$ (small damping). In this case, the homogenous equation roots attached to it (8) ($r^2+2\alpha r+p^2=0$) are complex and the solution of the (8) equation is

$$x_1 = ae^{-\alpha t} \cdot \cos(\beta t - \varphi) \tag{9}$$

$\beta^2 = p^2 - a^2$, a and φ are the integration constants that can be determined from the initial conditions:

For the initial moment $t = 0$, $x = 0$ and $\frac{dx}{dt} = v_{x_0} = \max; \left(\frac{d^2x}{dt^2} = 0\right)$. It results $\varphi = \frac{\pi}{2}$; $a = v_{x_0} / \beta$.

The period of movement is $\mathrm{T} = 2\pi / \beta$

The particular solution of the equation (8), (x_2), has the right member form and replacing it in equation (8) it results:

$$x_2 = q / p^2 \tag{10}$$

From the equations (9) and (10) it results:

$$x = x_1 + x_2 = \frac{v_{x_0}}{\beta} e^{-\alpha t} \cos\left(\beta t - \frac{\pi}{2}\right) + \frac{q}{p^2} \tag{11}$$

or $\quad x_a = xp^2 / q$.

By differentiating with respect to time, it results the vibration speed (v_x) and the acceleration of the vibration (a_x):

$$v_x = \frac{dx}{dt} = \frac{v_{x_0}}{\beta} e^{-\alpha t}\left[\beta \cos\beta t - \alpha \sin \beta t\right]; a_x = \frac{dv_x}{dt} = \frac{v_{x_0}}{\beta} e^{-\alpha t}\left[\left(\alpha^2 - \beta^2\right)\sin \beta t - 2\alpha\beta \cos\beta t\right] \tag{12}$$

Putting the condition that for the initial moment $t=0$, $v=0$, it results $v_{x0} = u_0$.
The relative sliding velocity between the gasket and the rod of the shock-absorber, in the presence of which friction occur, is

$$v_{am} = \frac{u - v_x}{v_m} = u_{0m}\left[\cos\omega t - \frac{1}{\beta} e^{-\alpha t}\left(\beta \cos\beta t - \alpha \sin \beta t\right)\right] \tag{13}$$

.
where $u_{0m}=v_0/v_m$, $v_{am}=v/v_m$.

If it is considered the initial hypothesis $\mu_a = 1 = constant$ and this coefficient is maintained constant when the instantaneous sliding velocity $v_{am} < v_{0m}$, it can be deduced the time range from a period when the shock-absorber dissipates energy and the time range when the shock-absorber becomes vibrator, thus it generates self-vibrations.

This phenomenon is illustrated in figure 4, where the instantaneous sliding velocity (v_{am}) is a parameter compared with the specific velocity from the Striebeck curve (v_{om}). This relative velocity (v_{a1}) is an important work parameter for analyze efficiency of damper:

$$v_{a_1} = v_{am} - v_{om} \quad . \tag{14}$$

Thus, for $\alpha = 1$, $p = 2$, $q = 5$, $u_{0m} = 2$, $v_{0m} = 0{,}1$, $\omega = 2$, it results that during a certain period $T = 2\pi/\beta = 3{,}68$ *s*. In the time elements $t = 0.26...0.867$ *s*, $t = 2.37...3.68$ *s* „the damper" generates vibrations, having the role of a vibrator (excitatory). Its basic function (damping) is accomplished in the time ranges $t = 0...0.26$ *s* and *0.867. ...2.37 s.*

The area situated under the line $v_{a1} = 0$ is an indicator of the damping, and the area situated above the line $v_{a1} = 0$ is an indicator of the self-vibrations. It is defined the *damping efficiency* as the ratio between the dissipated energy due to friction and the total energy during a certain time element

$$\eta_a = \frac{W_a}{W_t} - \frac{\frac{1}{2} m \frac{1}{T} \left[\int_0^{tcr1} v_{a_1}^2 dt + \int_{tcr2}^{tcr3} v_{a_1} dt + \ldots \right]}{\frac{1}{2} m \frac{1}{T} \int_0^T v_{a_1}^2 dt} = \frac{\sum_{i=1}^{n} I_{ai}}{I_t}, \tag{15}$$

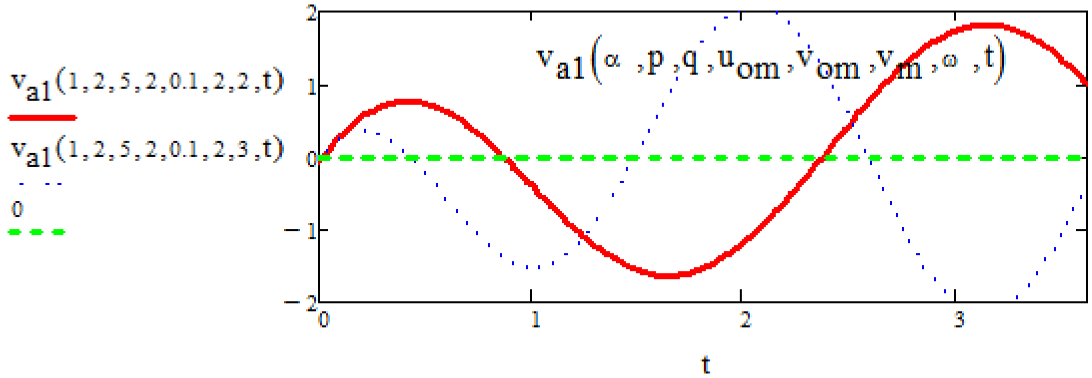


Figure 4. The critical speed for functioning as a damper.

where $t_{cr_1}, t_{cr_2}, t_{cr_3}, \ldots, t_{cr_m}$ are the time elements when the instantaneous velocity is null [roots of equation (20)]; I_{ai} – is the integral of the specific energy of damping due to friction; I_t – is the integral of the specific total energy. Thus, for the two curves in figure 4, the *damping efficiency* is $\eta_a = \eta_{a3} = 0.433$ for the curve with $\omega = 2\, rad/s$ and $\eta_a = \eta_{a4} = 0.624$ for the curve with $\omega = 3\, rad/s$.
Curve 1 ($\omega = 2\ rad/s$) intersect the axis in three points, and curve 2 ($\omega = 3\ rad/s$) in four points.

For a real shock-absorber, used in washing machines, $\alpha = 790; p = 880; q = 2; u_{om} = 2; v_{om} = 0{,}1; v_m = 2$ (SI measurement units), it results that the critical time is $t_{cr1} = 32.21\ \mu s$ and the *damping efficiency* is only $1{,}78 \cdot 10^{-6}$ (negligible), when the working pulsation is $\omega = 2\ rad/s$.
In the case of the shock-absorber housing that is attached to equipment frame through an aluminum socket, the housing movement (distance, velocity, acceleration, damping efficiency) is obtained by particularizing the expressions (9) – (15) for $\alpha = 0$. It results $\beta = p$,

- the deflection $x = \frac{v_{x_0}}{p} \sin pt + \frac{q}{p^2}$; velocity $v_x = v_{x_0} \cos pt$ - acceleration $a_x = -p v_{x_0} \sin pt$

-damping efficiency $\eta_{a2} = 0.51$ (for $p = 2; q = 5; u_{0m} = 0.1; v_{0m} = 0.1; v_m = 2; \omega = 3$ - SI measurement units); $\eta_{a4} = 0.376$ (for $p = 2; q = 5; u_{0m} = 0.1; v_{0m} = 0.1; v_m = 2; \omega = 4$).
It can be noted that for this particular case, the damping efficiency is obtained by constant friction ($\mu = \mu_s$) for small instantaneous velocities.

b. Sub case $c > c_0$ ($\alpha > p$) (high damping)
The homogenous equation roots attached to equation (7) are real and both negative. The solutions of the homogenous equation (natural vibration) (equation 11) is [12],

$$x_1 = b\, e^{-\alpha t} \sinh(\gamma t), \tag{16}$$

where $\gamma = \sqrt{\alpha^2 - p^2}$, b – the determinable amplitude from the initial conditions:

- at zero moment $t = 0, x_1 = 0$ and $\frac{dx_1}{dt} = v_{x0} = \max\left(\frac{d^2 x_1}{dt^2} = 0\right)$. It results $b = v_{x0}/\gamma$.

The particular solution of the equation (11), (x_2), has the same form as the right term, $x_2 = q/p^2$. Hence it results that:

-deflection (space) $x = x_1 + x_2 = \frac{v_{x_0}}{\gamma} e^{-\alpha t} \sin h(\gamma t) + \frac{q}{p^2}$; - velocity $v_x = \frac{dx}{dt}$; acceleration $a_x = \frac{dv_x}{dt}$.

Analog to case a) ($\alpha < p$), it is determined the instantaneous sliding velocity (v_{a1}) and it is presented in figure 5. It is noticed the existence of certain zones with normal operation as a damper ($v_{a1} < 0$) and zones with self-vibrations [vibrator, ($v_{a1} > 0$)].

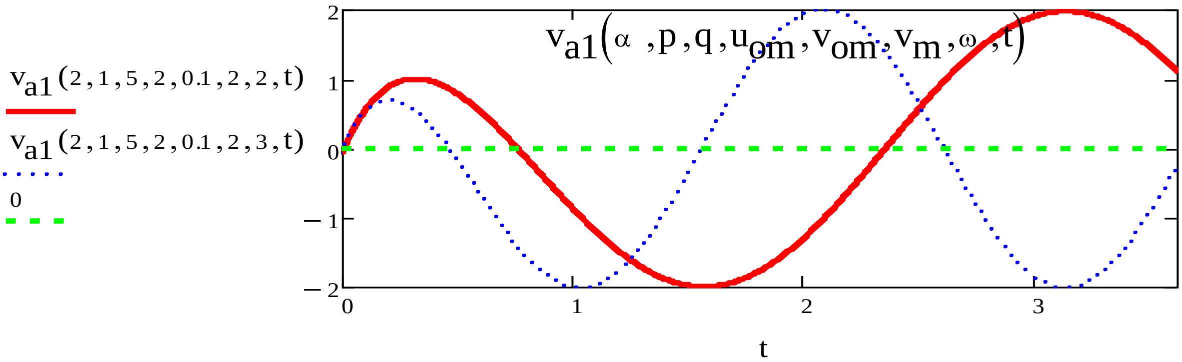


Figure 5. The instantaneous sliding velocity for high damping.

Thus, for $\alpha = 2, p = 1, q = 5, u_{0m} = 2, v_{0m} = 0.1$, it results that, during a certain period $T = 2\pi/\beta = 3.68\ s$, in the time range $t = 0.013...0.76\ s$, $t = 2.36...3.68\ s$ „the damper" generates vibrations, having the role of a vibrator. Its basic function (damping) is accomplished in the time ranges $t = 0...0.013\ s$ and $0.76....2.36\ s$.

For $\alpha = 2$ and $p = 1$ (the numerical situation reversed as in case *a)* ($\alpha = 1, p = 2$), the damping efficiency is net superior $\eta_{a2} = 0.941$ ($\omega = 2$) and $\eta_{a4} = 0.665$ ($\omega = 3$) comparing with $\eta_{a2} = 0.443$ ($\omega = 2$) and $\eta_{a4} = 0.624$ ($\omega = 3$).

c) Sub case $c = c_0$ ($\alpha = p$) (critical damping)

The characteristic equation roots are real and equal to - α. In these conditions, the movement can be characterized by

deflection $x = x_1 + x_2 = v_{x_0} t\, e^{-\alpha t} + \frac{q}{\alpha^2}$; - velocity $v_x = \frac{dx}{dt}$ and acceleration $a_x = \frac{dv_x}{dt}$.

The *damping efficiency* can be evaluating similarly. At example, for $\alpha = 1, p = 1, q=5, u_{0m} = 2$, $v_{0m} = 0.1$ is $\eta_a = 0.908$ ($\omega = 2$) and $\eta_a = 0.918$ ($\omega = 3$).

Case 2. The friction coefficient variable by speed as a parabolic law (second part of Striebeck curve).

In this case, the friction coefficient depends on the instantaneous sliding speed (7):

The differential equation of movement (8) becomes

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = k_a\left(a_a v_{am}^2 + b_a v_{am} + c_a\right) \quad \text{where} \quad v_{am} = \frac{1}{v_m}\left(u_0 \cos\omega t - \frac{dx}{dt}\right). \tag{17}$$

After some algebraic calculus, it results:

$$\frac{d^2x}{dt^2} + \left(2\alpha + A_u \cos\omega t - B\frac{dx}{dt}\right)\frac{dx}{dt} + p^2 x = D\cos^2\omega t + E\cos\omega t + F_a \tag{18}$$

were $p^2 = k/m$, $2\alpha = c/m$ – the viscous damping parameter in the system itself; A_u, B, D, E, F_a – the geometrical and functional parameters of the shock-absorber:

$$A_u = \frac{2u_0 a_a k_a}{v_m^2} + \frac{u_0 b_a k_a}{v_m};\quad B = \frac{a_a k_a}{v_m^2};\quad D = \frac{u_0^2 b_a k_a}{v_m^2};\quad E = \frac{u_0 b_a k_a}{v_m};\quad F_a = \frac{F_a k_a}{m}.$$

The differential equation (18) is nonlinear and thus, it is proposed the MATHCAD 2000 program for its numerical solving („Odesolve" subroutine). The curves from figure 6 are obtained for the experiment conditions that define the parameters:

$$\mu_s = 1.4; \mu_m = 0.12; v_m = 0.07\ m/s; c_h = 0.08\ s/m; \omega = 2\ rad/s; v_o = 0.1 v_m; v_{cr} = 1.2 v_m.$$

Thus, it results the geometrical and functional parameters of the shock-absorber:

$$A_u = 1.8\cdot 10^3; B = 1.612\cdot 10^4; D = 58.05; E = -135.45; F_a = 2.32\cdot 10^3.$$

The instantaneous sliding velocities, $v_{a_1} = v_{am} - v_{om}$, v_{a11} for $\alpha = 1848$ and v_{a110} for $\alpha = 8758$ are presented in figure 6.

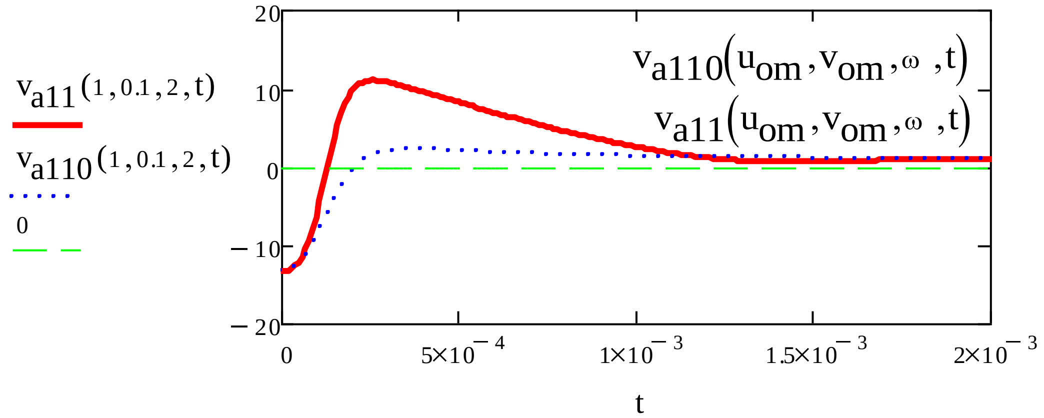


Figure 6. The instantaneous sliding speed in the shock-absorber that function in the parabolic zone.

For the two values of the damping (α), taken as example, it is observed that the shock-absorber works more like a „vibrator" by friction self-vibration.
The damping efficiency is small. Thus, for $\alpha = 1848$, $\eta_a = 0.126$, and for $\alpha = 8758$, $\eta_a = 0.144$.

3. Conclusions

The friction can be fluid, limit and mixed or conventionally dry in the coulombian damper with rubber rings, saturated with liquid lubricant. The relative sliding speed between the rod and the ring and the working period are principal parameters of friction type.

The friction curve, on a large range of working speed of the damper can be considered as a Striebeck type curve.

The curve of the friction coefficient dependent on the relative sliding speed is simulated as a parabola for the dry, limit and mixed friction regime, and being tangent to a line for the Newtonian fluid friction.

Any kind of coulombian damper with Stribeck friction works in vibratory regime with different damping, depending on the pressure between rubber rings and the working rod and also the state of friction. The theoretical model of movement can explain and determine the damping efficiency.

The coulombian dampers are sources of vibrations when the friction coefficient decreases with the increase of the relative speed between the rings and the rod.

The damping efficiency is one important parameter to characterize coulombian damper and depends to geometry, contact pressure and material of damper rings and rod.

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