

Application of iterative hill climbing to the sound speed profile inversion in underwater acoustics

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Abstract. An application of the iterative hill climbing algorithm to the solution of inverse problems of underwater acoustics is discussed. Modal dispersion data extracted from a recording of a pulse acoustical signal is used as the input for the geoacoustic inversion procedure. The mismatch of the dispersion curves extracted from experimental data and computed from a given set of waveguide parameters is minimized over the parameters space. The solution of two test inversion problems is considered. It is shown that the iterative launches of the hill climbing algorithm allow to locate either the global minimum or a local minimum which is reasonably close to the former. In the latter case a reasonable estimate of the waveguide parameters is obtained.

1. Introduction

Global optimization problems emerge in many areas of research in present-day science including, for instance, electrical engineering, operations research and molecular modelling. In many practical cases the cost function of such problems exhibits very complicated behaviour and has many local extrema that render classical local optimization algorithms (e.g. Hessian-based and gradient-based minimization) inapplicable. Usually the solution of those problems requires certain metaheuristics such as tabu search, simulated annealing and iterated local search [1]. Iterated local search was shown to be highly efficient in the solution of the vehicle routing problems [2], the problem of solvers parameters tuning [3], etc. At the same time, this algorithm is quite easy to implement.

In this study we apply the iterated local search based on hill climbing [4] to the problem of the acoustical parameters estimation in a shallow-sea geoacoustic waveguide and show that it can be successfully used for this purpose. The paper is organized as follows. In Section 2 we outline the idea of dispersion-based geoacoustic inversion, in Section 3 we describe the iterative hill climbing algorithm and its implementation in the case of the inversion problem. The study is concluded by Section 4 where the computational examples are presented.

2. Geoacoustic inversion based on the dispersion data matching

The term geoacoustic inversion in the field of underwater acoustics designates a specific class of problems related to the reconstruction of the media properties using some data on the sound



propagation through these media (see e.g. [5, 6] and references therein). A typical geoacoustic waveguide in a shallow sea is comprised by a water column and several layers of bottom sediments where the acoustical parameters, the sound speed $c=c(z)$ and the density $\rho=\rho(z)$, vary with the depth z . By contrast to the direct problems where the media parameters are known and acoustic pressure field has to be computed, in the inverse problems some acoustic data (e.g. the acoustic field at several points) is known, while the media parameters are inverted for.

Recently it was found that dispersion data could be successfully used as the input for the inversion of the environment parameters [7, 8]. An important advantage of this technique is that the dispersion data can be obtained from a single-hydrophone measurements [7]. By contrast, other inversion methods usually rely on the measurements performed using arrays of receivers that are very expensive and complicated in deployment [5, 6].

By the dispersion data we mean hereafter the dispersion curves (DCs), i.e. the functions $\tau=\tau(f,m)$, where f is the sound frequency and m is the mode number [6]. In shallow-water waveguides acoustic modes with different numbers m and at different frequencies f arrive to the receiver (which is located sufficiently far from the source) at different moments τ [6, 9]. The delays of one mode with respect to another depend on the properties of the media, i.e. the sound speeds and densities in the water column and the bottom [9]. Measuring such delays and using the DCs as the input for the inversion algorithms we may therefore recover the media parameters [7, 10, 11, 12, 13].

Usually an inversion algorithm is based on the matching of the DCs obtained from the experimental data and DCs computed using certain model of the waveguide. The experimental DCs (EDCs) are obtained from a recording of a pulse signal by a single hydrophone (the receiver) which is located several kilometres away from the source that emits the signal. The extraction of the DCs proposed in [7] is based on the time-frequency analysis of the received signal and the use of the so-called warping operators that greatly enhance the possibility of the modes separation. The theoretical DCs (TDCs) are obtained by the formula (see e.g. [11])

$$\tau(f,m) = \frac{R}{g_v(\bar{A}, f, m)} + \tau_0, \quad (1)$$

where R is the source-receiver range, g_v is the modal group velocity [9], τ_0 is the time offset correction. The waveguide is described by the set of parameters \bar{A} (usually \bar{A} contains the values of sound speed and density in various layers of the sea water and the bottom), and from these parameters one can compute the group velocities $g_v(\bar{A}, f, m)$ by solving acoustical spectral problem [8, 9, 11].

Provided that EDCs $\tau_e(f,m)$ and TDCs $\tau_t(f,m)$ are obtained for certain vector of frequencies f_1, f_2, \dots, f_N , we can compute the root mean square mismatch $E(\bar{A}, \tau_0)$ of the experimental and theoretical dispersion data using by formula

$$E(\bar{A}, \tau_0) = \sqrt{\frac{\sum_{i=1}^N \sum_{m=1}^M |\tau_t(f_i, m) - \tau_e(f_i, m)|^2}{NM}}, \quad (2)$$

where M is the number of modes detected on the spectrogram of the received signal. We can therefore estimate the waveguide parameters \bar{A} and the scalar τ_0 by the minimization of error function $E(\bar{A}, \tau_0)$:

$$[\bar{A}^*, \tau_0^*] = \min_{\bar{A}, \tau_0} E(\bar{A}, \tau_0) \quad (3)$$

where the values \bar{A}^* and τ_0^* constitute the global minimum point for the function $E(\bar{A}, \tau_0)$. Note that parameter space in this problem can have large number of dimensions [10], while the cost function usually has many local minima that deny the use of the simple local search algorithms.

In the previous work a variety of global minimization algorithms was used to carry out the solution of (3). In the papers [8, 14] a brute-force global search was used, in [7] the genetic algorithm was employed, and in the recent work [10] a trans-D Bayesian inversion method was developed.

In the present study we restricted our attention to the discrete (finite) search space with linearly-spaced trial values for each parameter in vector \bar{A} and for τ_0 .

3. The application of iterative hill climbing to minimization of the error function

According to the previous section, our goal was to minimize the objective function $E(\bar{A}, \tau_0)$ in a finite search space. Following the authors of ParamILS (Iterated Local Search in Parameter Configuration Space [3]), for this purpose the simple hill climbing algorithm [4] was chosen. ParamILS is a state-of-the-art tool for tuning the parameters of solvers (SAT, SMT, etc.). With the help of ParamILS good speedups were achieved in the different scientific areas (protein folding, AI planning, etc.). The relative simplicity of the hill climbing algorithm was another reason why we chose it. Hill climbing algorithm was also successfully used for the environmentally adaptive optimization of the sonar settings [15].

According to the basic idea of simple hill climbing we developed the following algorithm:

1. start with some randomly chosen point $E(\bar{A}, \tau_0)$ from a finite search space;
2. iteratively, adjust a single parameter of the point (here by parameters we mean τ_0 and the elements of \bar{A}), keeping the modification if the value of $E(\bar{A}, \tau_0)$ improves and undoing it otherwise;
3. terminate when no single element adjustment yields an improvement of $E(\bar{A}, \tau_0)$, or when the time limit was exceeded.

Note that this algorithm will typically terminate in a local minimum, in which changing value of any single element will not achieve any improvement. In order to jump from a local minimum an algorithm based on hill climbing is usually expanded by some metaheuristic: tabu search, simulated annealing or iterated local search. Following the authors of ParamILS algorithm [3] we chose iterated local search (ILS) [1] for this purpose.

The workflow of the iterative hill climbing algorithm can be described as follows. Starting from some initial point, ILS first performs simple hill climbing search until a local minimum $E(\bar{A}, \tau_0)$ is reached, and then it cycles through the following steps:

1. apply perturbation to (\bar{A}, τ_0) (in the form of multiple random changes of the parameters values);
2. perform simple hill climbing search until a new local minimum $(\bar{A}', \tau_{0'})$ is reached;
3. accept the better (with respect to the value of E) of the two points (\bar{A}, τ_0) and $(\bar{A}', \tau_{0'})$ as the starting point of the next cycle.

4. Computational experiments

In previous work (see [14]) we developed a simple normal mode code capable of computing the modal group velocities $v_g(f, \bar{A})$ from a given waveguide parameters \bar{A} . This routine was used as a part of brute-force search algorithm SSPEMDD [16] for the minimization of the error function E implemented using the Message Passing Interface (MPI). For the numerical experiments described below the iterative hill climbing algorithm suggested in the previous section was implemented using C++ and added to SSPEMDD toolbox. It employs the same normal mode evaluation routine based on the finite differences (see [14] for the details).

4.1. Test case 1: inversion of the sound speed profile in the water

In the first series of experiments we used the same synthetic dispersion data as in [14]. We estimated the sound speed profile (SSP) in the water using the dispersion curves in the frequency range from 150

Hz to 250 Hz at $R = 3500$ m (see [14] for the detailed description). We used five nodes at fixed depths z_0, \dots, z_4 for the approximation of SSP by a vector c_0, \dots, c_4 ($c(z_i) = c_i$ is the sound speed c at depth z_i) with the fixed (known) value of the sound speed c_0 at the sea surface $z_0 = 0$. The search space for this case is presented in table 1.

Table 1. Search space for the test case 1.

parameter	min. value	max. value	step
R	3400 m	3600 m	5 m
$c_j (j = 1, 2, 3, 4)$	1450 m/s	1500 m/s	2.5 m/s

Note that in this test we set τ_0 to zero and estimate the correction to the source-receiver distance R instead. Also note that due to the processing errors the DCs extracted from a pulse signal do not match their theoretical counterparts exactly. More details on the DCs extraction technique can be found in [7] and references therein.

In [14] the brute-force search for this test case was carried out on 15 nodes of the “Academician V.M. Matrosov” computing cluster (Irkutsk Supercomputing Center of SB RAS). Each computing node of this cluster consists of two 16-core CPU AMD Opteron 6276, and therefore 480 CPU cores in total were used. All 7973721 points of the search space (see table 1) were processed in 8 hours and 35 minutes, as a result the global minimum point with $E = 0.0069710$ s was found.

We performed 20 separate launches of the iterative hill climbing algorithm from different randomly chosen start points, including 10 launches with 100 iterations and 10 launches with 1000 iterations. Each of these runs was executed on a single core of CPU AMD Opteron 6276. The characteristics of the obtained 20 local minima are shown in table 2 and in figure 1. One launch took 44 minutes on average in the case of 100 iterations and 8 hours 2 minutes on average in the case of 1000 iterations.

Table 2. The local minima found in the test case 1. The local minima that coincide with the global one are marked in bold.

serial number	local minimum, 10 iterations	local minimum, 100 iterations
1	0.0105368	0.0071304
2	0.0105368	0.0071304
3	0.0090778	0.0099037
4	0.0071304	0.0069710
5	0.0071304	0.0071304
6	0.0100120	0.0069710
7	0.0099037	0.0069710
8	0.0103995	0.0099037
9	0.0071304	0.0071304
10	0.0112283	0.0069710

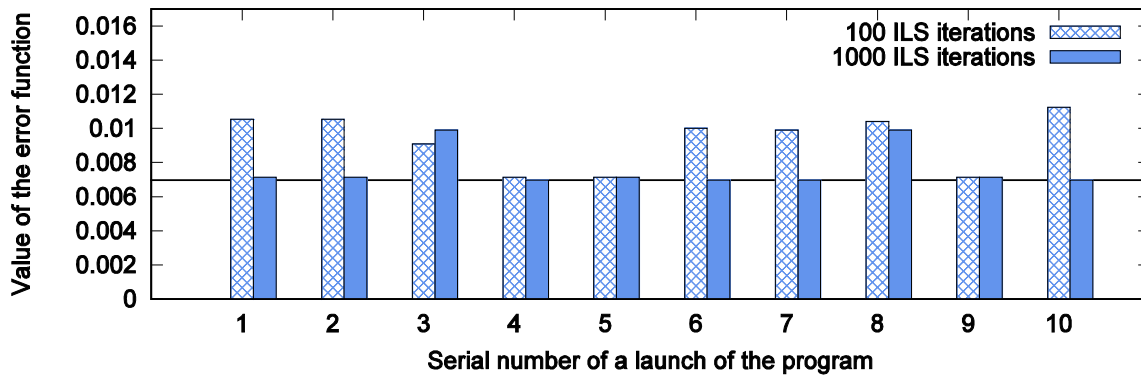


Figure 1. Results of 10 separate launches (with different random starting points) in the test case 1. The global minimum level $E = 0.0069710$ s is shown by a horizontal line.

4.2. Test Case 2: bottom parameters inversion

In this example we estimated the bottom parameters in shallow sea (sound speed c_b and density ρ_b) together with the arrivals offset τ_0 (see equation (1)) using the dispersion data extracted from a simulated pulse signal. We used the waveform from [17] for the emitted signal. The DCs were extracted in a frequency range from 20 Hz to 200 Hz. The sound speed in the water column was considered constant $c_w = 1500$ m/s, and the bottom depth was set to $h = 90$ m. The forward propagation problem was solved using our normal mode code [16] with $c_b = 1800$ m/s, $\rho_b = 1.8$ g/cm³, and the signal in the receiver located at $R = 8000$ m from the source was recovered using the standard Fourier synthesis technique [9]. The computed signal is shown in figure 2, while the DCs are plotted in figure 3 against the spectrogram of the computed signal (see [7, 11] for the detailed explanation of the extraction technique).

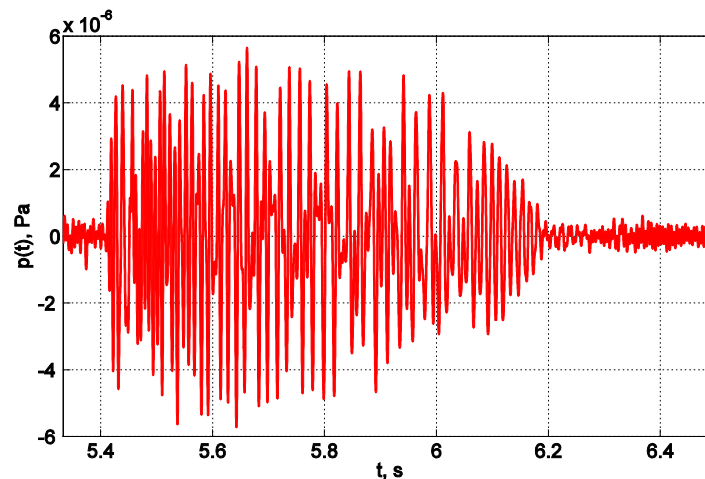


Figure 2. Simulated pulse signal at the range $R = 8000$ m. This signal was used for the extraction of DCs in the test case 2.

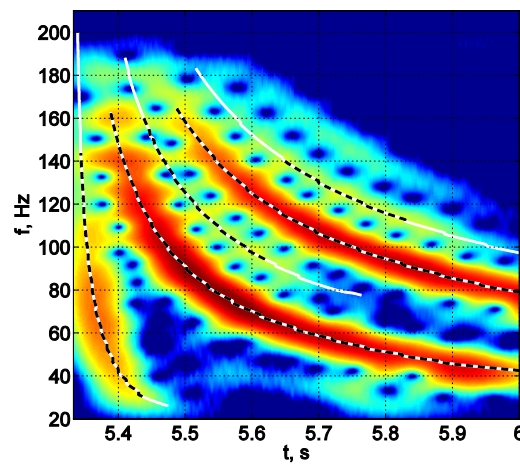


Figure 3. Spectrogram of the signal shown in figure 2 and the dispersion curves extracted from this spectrogram (plotted as black dashed lines). The DCs were used as the input for the inversion algorithm in the test case 2.

The search space for this example is described in table 3.

Table 3. Search space for the test case 2.

parameter	min. value	max. value	step
c_b	1600 m/s	1900 m/s	5 m/s
ρ_b	1.4 g/cm ³	2.0 g/cm ³	0.05 g/cm ³
τ_0	-0.015 s	0.015 s	0.005 s

For this case SSPEMDD was launched on 10 computing nodes of the computing cluster, and therefore 320 CPU cores in total were used. All 48373 points of the search space (see Table 3) were processed in 52 minutes, and the global minimum point with $E = 0.00267443$ s was found. We also performed 20 separate launches of the iterative hill climbing algorithm with different randomly chosen start points: 10 launches with 10 iterations and 10 launches with 100 iterations. The Error function values for the obtained 20 local minima are shown in Table 4 and in figure 4. One launch took 48 minutes on average in the case of 10 iterations and 7 hours 48 minutes on average in the case of 100 iterations (again one CPU core was used for each launch).

Table 4. The local minima found in the test case 2. The local minima that coincide with the global one are marked in bold.

serial number	local minimum, 10 iterations	local minimum, 100 iterations
1	0.00299985	0.00267443
2	0.00291565	0.00269999
3	0.00277798	0.00268122
4	0.00291565	0.00269999
5	0.00269999	0.00269999
6	0.00271570	0.00267443
7	0.00269999	0.00267443
8	0.00274744	0.00268122
9	0.00267443	0.00269999
10	0.00279155	0.00269999

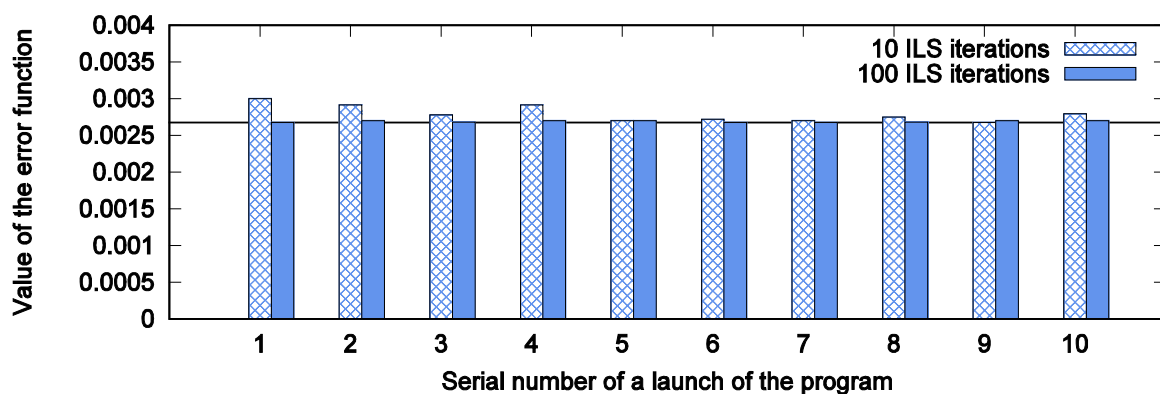


Figure 4. Results of 10 separate launches (with different random starting points) in the test case 2. The global minimum level $E = 0.00267443$ s is shown by a horizontal line.

It turned out that iterative hill climbing requires significantly less computational resources (in comparison with brute-force) to achieve a local minimum with the acceptable characteristics. In the test case 1 one launch of iterative hill climbing with 100 iterations took on average 5597 times less resources, and 513 times less resources for one launch with 1000 iterations. In the test case 2 one launch with 10 iterations took on average 346 times less resources, and 36 times less resources for one launch with 100 iterations.

5. Conclusion

In this study an application of the iterative hill climbing algorithm to the problem of geoacoustic inversion was considered. The results of the numerical experiments presented in the Section 4 show that it allows to obtain a reasonable estimate for the SSP or bottom parameters much faster than an exhaustive search algorithm even for a relatively low-dimensional search space (when the use of the latter is still feasible). Not that relatively small number of iterations is sufficient to achieve very good approximation for the set of waveguide parameters \bar{A} .

In future work we will apply our algorithm to the full-scale inversion problems that require higher dimensions of the search space (up to 20-30) and compare it with other methods in terms of accuracy and efficiency. We also plan to expand our hill climbing algorithm by another metaheuristics (e.g. simulated annealing and tabu search).

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