IOP Conf. Series: Materials Science and Engineering **173** (2017) 012002 doi:10.1088/1757-899X/173/1/012002

Computer simulation of the surface heating process by the movable laser

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Abstract. The model of heating the target surface in air environment with moving laser beam is created. The equation was solved for the case of dynamically changing laser beam energy and constant parameters of the air flow.

1. Introduction

The task of heating the surface of the target moving laser beam is relevant, as is used in many industrial processes.

Practical implementation of this operation to ensure optimal heating and cooling parameters of the target is impossible without building a mathematical model that takes into account the specific conditions of the heating laser beam and cooling the surface by convection. The complexity and urgency of this problem increases when the beam heats the surface is moving.

The mechanism of target surface heating with a pulsating-periodic laser beam, moves over the surface of the plate in a predetermined manneris considered. [1-3].

The target is a flat plate with predetermined thickness δ. Spatial coordinate system, in which the problem is solved, is stationary. The laser beam moves over the surface of the plate in a predetermined manner with the speed $v(x,y,t)$ (fig.1).

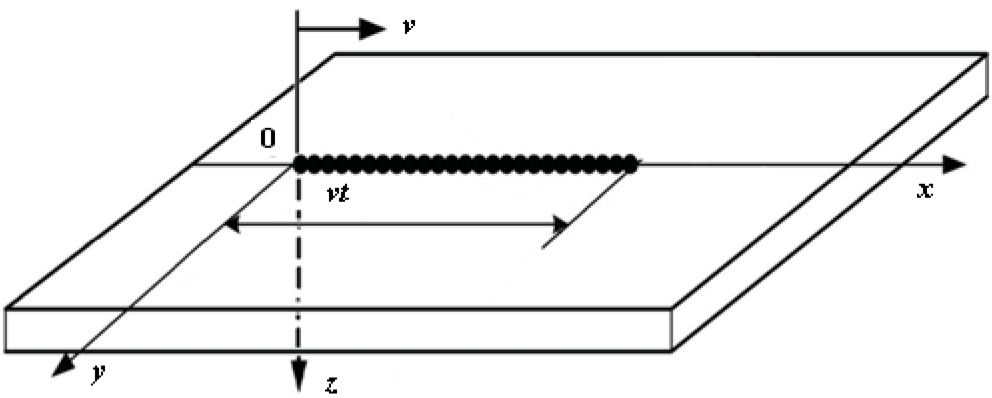

Figure 1. The scheme of the concentrated heat source movement on the plate surface.

2. Mathematical model

The system of equations describing the process of heating-cooling the target surface is as follows:

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T_i}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right), \quad \forall\, x, y, z \in V,\ t > 0, \tag{1}$$

$$q_l = -\lambda_z \frac{\partial T}{\partial z}, \quad \forall\, x, y \in S,\ t > 0\,, \tag{2}$$

$$\alpha(T_g - T) = -\lambda_z \frac{\partial T}{\partial z}, \quad \forall\, x, y \in S,\ t > 0\,, \tag{3}$$

$$T(x, y, z)\big|_{t=0} = T_0 = \text{const}\,,\ \forall\, x, y, z \in V\,, \tag{4}$$

where (1) – is the differential equation of the heat conductivity; (2) – is the boundary conditions on the laser hot spot surface *S*are of follows; (3) – is the boundary conditions of heat exchange with ambience on the surface *S*are the next; (4) – is the initial conditions for $t = 0$; T – is the target surface temperature; t – is the time; x, y, z – are spatial coordinates; λ_x, λ_y ,λ_z– are heat conductivity coefficientsin the direction of axis$x$, $y$and z; V – target volume; $q_l = f(x,y,t,e)$ – theheat flux from the laser beam; α – is the heat exchange coefficient; T_g – ambient temperature; S – target surface; T_0– initial temperature.

To solve this problem a method of finite elements [4,5] is used. The decision comes down to the definition of steady-state value of the next functional:

$$\Phi[T(r,z)] = \int_x \left\{ \frac{1}{2}\left[\lambda_x^*\left(\frac{\partial T}{\partial r}\right)^2 + \lambda_y^*\left(\frac{\partial T}{\partial y}\right)^2 + \lambda_z^*\left(\frac{\partial T}{\partial z}\right)^2 \right] + q_C^*\, T \right\} dxdydz - \\ - \int_S \left[q_l T + \alpha\left(T_c - \frac{1}{2}T\right) T \right] dS, \tag{5}$$

where $q_C^* = c^* \rho^* \frac{\partial T^*}{\partial t}$ – the heat flux, conditioned by the heat capacity of the material, index “ * ” corresponds to a temperature distribution in the space fixed at the moment.

The problem reduces to the definition of change of the temperature distribution over the volume of the target as a function of time:

$$\delta\Phi\left[T\left(x, y, z, t^*\right)\right] = 0\,. \tag{6}$$

As a finite element we use the cubic hybrid element, which is formed by the union of several tetrahedrons. A function of the temperature distribution is a linear function and the heat capacity of the element is concentrated in the nodes [4,5,6] (fig.2):

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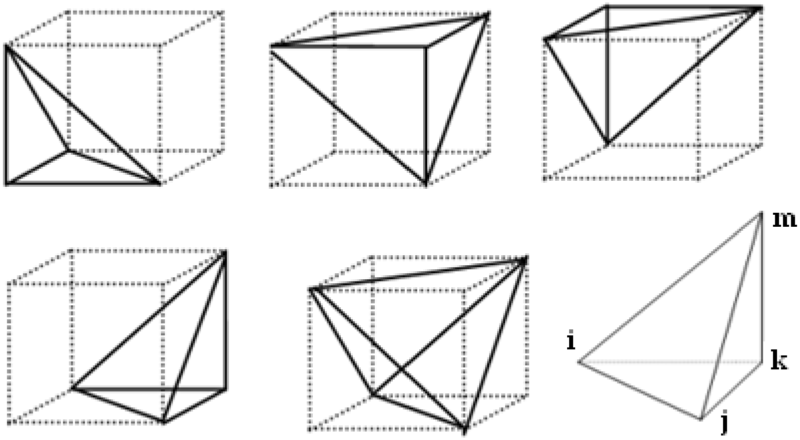


Figure 2. Scheme of the hybrid and tetrahedral finite element

The temperature at any point of finite element is determined by nodal temperatures quantity linearly:

$$T(x, y, z) = \left(N_1T_1 + N_2T_2 + N_3T_3 + N_4T_4\right), \tag{7}$$

where N_l – shape coefficient of tetrahedral finite element; T_l – the temperature of the nodal points of a finite element; l =1, 2, 3, 4.

The linear function of a temperature approximation (7) in this element is as follows:

$$T(x,y,z) = [1, x, y, z]\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}; \tag{8}$$

where α_1, α_2, α_3, α_4 – are the coefficients of the approximating function.

Vector temperature values in nodes is as follows:

$$\boldsymbol{T} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i & z_i \\ 1 & x_j & y_j & z_j \\ 1 & x_k & y_k & z_k \\ 1 & x_m & y_m & z_m \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \boldsymbol{C\alpha}.$$

Inversion of matrix ***C***permits to determine the coefficients of the tetrahedron N_i :

$$N_i = \frac{1}{6V_T}\sum_{i=1}^{4}\left(a_i + b_i x + c_i y + d_i z\right);$$

$$\frac{1}{6V_T}\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} = \boldsymbol{C}^{-1};\ V_T = \frac{1}{6}\det\boldsymbol{C},$$

where V_T – volume of the tetrahedron; x_l, y_l, z_l – coordinates of the nodal points; indexes a_l, b_l, c_l, d_l – the coefficients of the approximating function of temperature in the amount of finite element; $l \in \{i, j, k, m\}$; $i, j, k, m = 1, 2, 3, 4$ are obtained by cyclic permutation in the sequence.

The heat flow, which is absorbed by the finite element due to the heat capacity of the material expressed through heat flows in nodes:

$$q_c = \int_v c\frac{\partial T}{\partial t}dv = \int_v c\left[\sum_{l=1}^{4} N_l \frac{\partial T_l}{\partial t}\right]dv = \sum_{l=1}^{4}\frac{\partial T_l}{\partial t}\int_v cN_l dv = \sum_{l=1}^{4} q_{cl},$$
$$q_{cl} = \frac{\partial T_l}{\partial t}C_l, \qquad C_l = \int_v cN_l dv, \tag{9}$$

where C_l – reduced heat capacity of the node; v – the volume of finite element; c – the specific heat capacity of the target material.

The surface heat flux on the boundary surfaces are also represented in the form of concentrated nodal values:

$$q_l = \int_S q_l dS_l + \int_s \alpha(T_g - T)dS_g = \sum_l q_l + \sum_l q_{gl} - \sum_l q_{\alpha l},$$

where summation is performed over the nodal points of the boundary side of the finite element.

Let us consider the original functional in the class of linear functions of temperature (9). Minimization of this functional can be carried out over the nodal values [4], as they uniquely define the temperature at any point in the field of research.

Let us introduce into the original functional (5) conditions (7) – (9) and differentiate it respect to the nodal temperature of the tetrahedron T_l for the field of the finite element:

$$\frac{\partial \Phi}{\partial T_l} = \iint_v \left[\lambda_x \frac{\partial}{\partial x}\frac{\partial}{\partial T_l}\left(\frac{\partial T}{\partial x}\right) + \lambda_y \frac{\partial}{\partial y}\frac{\partial}{\partial T_l}\left(\frac{\partial T}{\partial y}\right) + \lambda_z \frac{\partial}{\partial z}\frac{\partial}{\partial T_l}\left(\frac{\partial T}{\partial z}\right)\right]dxdydz + q_{cl}. \tag{10}$$

To get the matrix form one should combine equations:

$$\left[\frac{\partial \Phi}{\partial T}\right] = \boldsymbol{\lambda T} + {}^4\boldsymbol{C}\frac{\partial \boldsymbol{T}}{\partial t}, \tag{11}$$

where $\boldsymbol{\lambda}$ – is the heat conduction matrix of the finite elements; ${}^4\boldsymbol{C}$ – is the heat capacity diagonal matrix of size 4×4; $\boldsymbol{T}$ – vector of temperature in all nodal points [4].

The final equation of the process of the functional minimization on the temperature at the nodal points are obtained by combining all the derivatives (11) over all finite elements on which the field of research is sampled [4].

Heat conduction matrix, heat capacity matrix and vectors of the nodal heat flow for the structure can be obtained by adding the respective members of heat conduction matrix, heat capacity matrix and finite elements heat flows. Resulting equation of the finite element method for this case takes the form:

$$\boldsymbol{\Lambda T} = -{}^n\boldsymbol{C}\left[\frac{\partial T}{\partial t}\right] + \boldsymbol{Q_\alpha} + \boldsymbol{Q_L}, \tag{12}$$

where $\boldsymbol{\Lambda}$, ${}^n\boldsymbol{C}$ –is the global heat conductivity matrix and heat capacity matrix correspondingly of size $n \times n$; $\boldsymbol{Q_L}$, $\boldsymbol{Q_\alpha}$ –are the global vectors of the nodal heat flow of radiant and convective heat exchange correspondingly;

$$\boldsymbol{\Lambda}=\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & \dots & \Lambda_{1n}\\ \Lambda_{21} & \Lambda_{22} & \dots & \Lambda_{2n}\\ \dots & \dots & \dots & \dots\\ \Lambda_{n1} & \Lambda_{n2} & \dots & \Lambda_{nn}\end{bmatrix};\ {}^{n}\boldsymbol{C}=\begin{bmatrix}C_{11} & 0 & \dots & 0\\ 0 & C_{22} & \dots & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & C_{nn}\end{bmatrix};\boldsymbol{Q_L}=\begin{bmatrix}Q_{L1}\\ Q_{L2}\\ \dots\\ Q_{Ln}\end{bmatrix},\ \boldsymbol{Q_\alpha}=\begin{bmatrix}Q_{\alpha 1}\\ Q_{\alpha 2}\\ \dots\\ Q_{\alpha n}\end{bmatrix} \tag{13}$$

3. Numerical solution

For numerical calculations of the equation (12) the implicit difference scheme of solution was used.Usage of finite element with concentrated heat capacity shown on fig.2 allows to exclude oscillation of numerical values of temperature for sharply pronounced transient heating processes and get the calculating scheme for transient temperature field, suitable for constructing the method of numerical investigation of the heat state of the investigated target.

The finite-difference expression for the time derivative gives:

$$\boldsymbol{\Lambda T}^{p+1}=-\boldsymbol{C}\left(\boldsymbol{T}^{p+1}-\boldsymbol{T}^{p}\right)\frac{1}{\Delta t}+\boldsymbol{A}\left(\boldsymbol{T}_g^{p+1}-\boldsymbol{T}^{p+1}\right)+\boldsymbol{Q}_L^{p+1}, \tag{14}$$

where p – is the index corresponding to the time; $\boldsymbol{T}^{p+1}$, $\boldsymbol{T}^{p}$ –are vectors of nodal temperatures in current and subsequent time; T_g^p – ambient temperature.

To combine in the left side of an equation all terms containing unknown quantities we pick out from the matrix $\boldsymbol{Q}_\alpha$ unknown temperatures of structure nodal points:

$$\boldsymbol{Q}_\alpha=\boldsymbol{A}\left(\boldsymbol{T}_g-\boldsymbol{T}\right), \tag{15}$$

where $\boldsymbol{A}$ –is the diagonal matrix of convective heat exchange on surface of target;

$$\boldsymbol{A}=\begin{bmatrix}A_{11} & 0 & \dots & 0\\ 0 & A_{22} & \dots & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & A_{nn}\end{bmatrix}$$

Shifting to the left side of the equation (15) the terms containing the unknown $\boldsymbol{T}^{p+1}$, one can get:

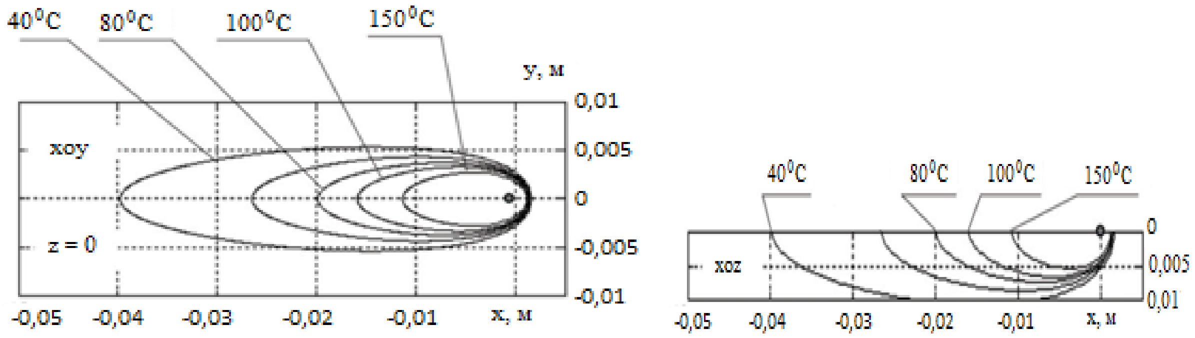
$$\left(\boldsymbol{\Lambda}+\frac{1}{\Delta t}\boldsymbol{C}+\boldsymbol{A}\right)\boldsymbol{T}^{p+1}=\frac{1}{\Delta t}\boldsymbol{CT}^{p}+\boldsymbol{AT}_g^{p+1}+\boldsymbol{Q}_L^{p+1}, \tag{16}$$

The vector $\boldsymbol{T}$ of size *n* at the initial time is determined from the initial conditions. The solution of resulting linear system of algebraic equations is carried out by the method of adjoint gradient [4,8].

The algorithm for solving system (1) – (4) was worked out.

The results are shown on fig.3 as isotherms.

Computer experiment was held for target of 10*mm* thick, the material has the conductivity 0.5 W/(mK), grid of size 300×100×20 vas used.



targetsurface target cross section

Figure 3. Isotherms in the plane *x-y* and *x-z* of the plate

Numerical simulation of the process is carried out by means of theoretical and computational complex of computer modeling and visualization of the heat exchange processes [9].

4. Conclusion

The developed method of numerical solution process of the surface heating by the movable laser beam allows to carry out a computer simulation of the target surface heating and cooling, and investigate the dynamics of target surface heating by pulsating-periodic laser.

Acknowledgments

This article was written with financial support of the Russian Ministry of Education under the Agreement of granting subsidies № 14.577.21.0084 on June 5, 2014. Unique identifier for Applied Scientific Research RFMEFI57714X0084.

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