Numerical and experimental investigations of transient behaviour of compact plate fin heat exchanger

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Abstract. In this paper, models for the transient simulation of a 2 stream plate fin heat exchanger (PFHE) with offset strip fins (OSF) have been formulated based on unsteady mass and energy conservation equations. The behaviour of a PFHE during start up, cool down and during a ramp change in the inlet temperature of one of the fluids have been studied with the help of these models. Initially, a simplified model considering constant helium properties is developed. This model is further evolved to take into account the temperature dependence of helium properties in the cryogenic temperature domain, along with axial heat conduction (AHC) and thermal capacity of the metal separating plate. Computer codes based on the models have been developed to simulate PFHE behaviour. Experiments at room temperature have been carried out for code validation. It has been found that the model based on constant helium properties is sufficient to predict behaviour of the heat exchanger under room temperature conditions.

1. Introduction

Compact PFHEs are the main static components of modern helium liquefaction and refrigeration systems. Transient analysis of PFHEs is useful for studying the various modes of operation such as cool down, warm up and the response to abrupt disturbances during normal operations. The larger goal is to develop suitable control logic for plant operations.

Transient analysis of compact PFHEs can be tackled in two ways; lumped and distributed parameter approach. Distributed parameter approach involves spatially varying temperature fields as well as the secondary effects leading to partial differential form of the governing equations; mass and energy conservation. The importance of the secondary effects on high effectiveness heat exchanger performance has been discussed by Julio et al [1].

Some authors [2-3] have used the lumped parameter method for their analysis, assumed constant thermal properties of fluids and absence of AHC [3]. A few of the authors have chosen finite difference techniques for solution of distributed parameter based models [4-5]. Rodriguez et al [6] have adopted method of lines approach for converting PDEs into ODEs, while analyzing a cryogenic shell and tube heat exchanger.

A study of the existing literature have revealed that although AHC has been dealt with [1] in both lumped and distributed parameter models, the effects of variable properties of helium at different temperatures, which typically a cryogenic heat exchanger is subjected to, have not been studied. In order to bridge this gap, a model of a PFHE, with OSFs, working at cryogenic temperature domain based on constant and variable fluid properties with helium as process gas, has been developed and presented in

this paper. Thus, a more generalized model for a PFHE is proposed which is validated experimentally at room temperature.

2. Governing Equations and Solution Procedure

The computational model has been described in Figure 1:

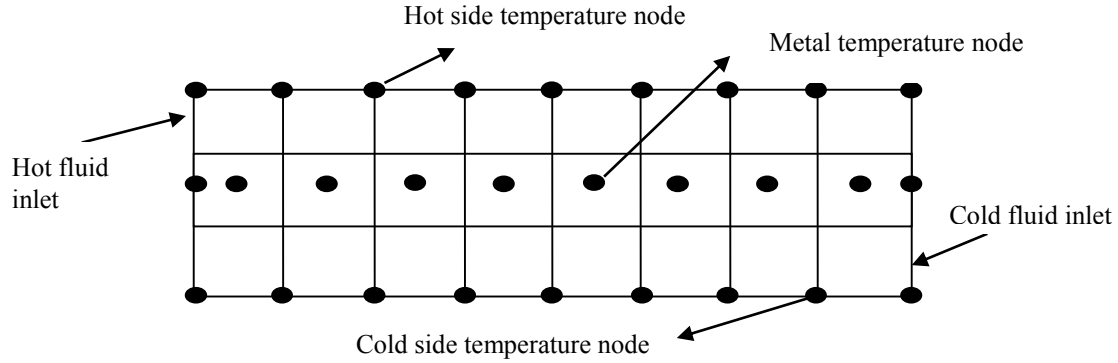Figure 1 Computational domain

The heat exchanger has been divided into n elements or cells, with the circles representing the node points. There are *n+ 1* nodes for each of the fluids and *n+2* nodes for the wall, including the boundary nodes. Following assumptions are considered while deriving the discretized governing equations:

i. No variation in temperature normal to the flow direction, considering smaller flow cross sections.

ii. Significant AHC in the metal matrix, considering the high thermal conductivity of Aluminium, which is the material of construction.

iii. There is no heat exchange with the surroundings, owing to vacuum and multilayer super insulation used in actual practice.

iv. Effect of pressure drop is negligible, since the prototype PFHE has been designed to inflict little pressure drops on the streams.

2.2 Model based on average (constant) helium properties

In this model, the physical properties of helium are considered to be constant and calculated at bulk mean temperature. Energy conservation equation is written for *i-1th* element. The temperature of that element is obtained as a function of transport properties and neighbouring temperatures. Forward time marching scheme [7] is used to approximate the time differential term.

The nodal temperatures for the hot side of *i-1th* element is written as:

$$T_{h,i+1}^{n} = T_{h,i+1}^{n}\left(1 - \frac{\Delta t u_{h}}{\Delta x} - \frac{\eta h A\mid_{h}}{\rho C p A c\mid_{h}}\right) + T_{h,i}^{n}\frac{\Delta t u_{h}}{\Delta x} + T_{w,i+1}^{n}\frac{\eta h A\mid_{h}}{\rho C p A c\mid_{h}} \tag{1}$$

Where,

ρ	=	Density of fluid
Cp	=	Specific heat of fluid
Ac	=	Cross section area of the PFHE channel
u	=	Velocity of fluid
T	=	Temperature of fluid and wall
η	=	Fin efficiency
h	=	Heat transfer coefficient
A	=	Heat transfer area per unit length
Δx	=	Length of one element
Δt	=	Time step

Subscripts h, c and w represent hot fluid, cold fluid and the wall respectively. Subscripts n and i represent temporal and nodal positions respectively.

The inputs to the model in order to obtain temperature profiles are the inlet conditions of both the streams, viz. mass flow rates, temperatures and pressures, the expected steady state exit conditions and heat exchanger dimensions along with the details of its core. The flow velocities are obtained using the mass flow rates, available flow areas and properties at the average of inlet and exit conditions.

The heat transfer coefficients are calculated as a function of fin parameters, Reynolds number and Prandtl number using the correlations for Colburn j factor provided by Manglik and Bergles [8].

Similar procedure carried out for the cold side gives rise to the following:

$$T_{c,i}^{n+1} = T_{c,i}^{n+1}\left(1 - \frac{\Delta t u_c}{\Delta x} - \frac{\eta hA\,|_c}{\rho CpAc\,|_c}\right) + T_{c,i+1}^{n}\frac{\Delta t u_c}{\Delta x} + T_{w,i+1}^{n}\frac{\eta hA\,|_c}{\rho CpAc\,|_c} \tag{2}$$

For the wall, energy balance equation can be expressed as:

$$T_{w,i+1}^{n+1} = T_{w,i+1}^{n+1}\left(1 - \frac{2kAc\,|_w\,\Delta t}{\Delta x^2} - \eta hA\,|_h\,\Delta t - \eta hA\,|_c\,\Delta t\right) + T_{h,i+1}^{n}\eta hA\,|_h\,\Delta t + T_{c,i}^{n}\eta hA\,|_c\,\Delta t$$
$$+ \left(T_{w,i+2}^{n} - T_{w,i}^{n}\right)\left(\frac{kAc\,|_w\,\Delta t}{\Delta x^2}\right) \tag{3}$$

The properties of fluids are calculated using HePak® [9]. The thermal conductivity, k and specific heat, Cp of the metal wall is calculated using the correlations by NIST [10].

2.2 Model based on real (variable) helium properties
Features of the model:
  i.   Physical and transport properties are evaluated iteratively at the nodes since their variation with temperature is now accounted for, to better represent the actual situation at cryogenic temperatures.
  ii.   The model incorporates the changes of fluid density across the element by invoking the continuity equation.
  iii. Implicit formulation schemes are used in case of fluids and for the separating plate the formulations are explicit.

By invoking the continuity equation and using the backward time marching scheme (FTBS) [7] in the ith element of the hot side, the following equation is obtained:

$$\rho_{i+1,h}^{n+1} = \frac{\rho_{i+1,h}^{n} + \frac{\Delta t}{\Delta x}\rho u\,|_{i,h}^{n+1}}{1 + \frac{\Delta t}{\Delta x}u_{i+1,h}^{n+1}} \tag{4}$$

Equation of energy conservation and backward time marching scheme [7] gives:

$$u_{i+1,h}^{n+1} = \frac{\Delta x}{\Delta t}\left(\frac{\rho H\,|_{i+1,h}^{n}}{\rho H\,|_{i+1,h}^{n+1}} - 1\right) + \frac{\rho u H\,|_{i,h}^{n+1}}{\rho H\,|_{i+1,h}^{n+1}} - \frac{0.5\left(\eta h\,|_{i+1,h}^{n} + \eta h\,|_{i,h}^{n}\right)A_h\Delta x\left(0.5\left(T_{i+1,h}^{n+1} + T_{i+1,h}^{n}\right) - T_{i+1,w}^{n}\right)}{\rho H\,|_{i+1,h}^{n+1}\,Ac_h} \tag{5}$$

Where, H stands for fluid enthalpy.

This value is compared with the guess value considered while solving the continuity equation for convergence.

Equations for cold side:

From the continuity equation, the following can be arrived at:

$$\rho_{i,c}^{n+1} = \frac{\dfrac{\Delta t}{\Delta x}\rho u\mid_{i+1,c}^{n+1} - \rho_{i,c}^n}{1 + \dfrac{\Delta t}{\Delta x}u_{i,c}^{n+1}} \tag{6}$$

Using the energy equation, the following can be expressed:

$$u_{i,c}^{n+1} = \frac{\Delta x}{\Delta t}\left(\frac{\rho H\mid_{i,c}^n}{\rho H\mid_{i,c}^{n+1}} - 1\right) + \frac{\rho u H\mid_{i+1,c}^{n+1}}{\rho H\mid_{i,c}^{n+1}} + \frac{0.5\left(\eta h\mid_{i+1,c}^n + \eta h\mid_{i,c}^n\right)A_c\Delta x\left(0.5\left(T_{i,c}^{n+1}+T_{i,c}^n\right)-T_{i+1,w}^n\right)}{\rho H\mid_{i,c}^{n+1}Ac_c} \tag{7}$$

In case of the separating plate, explicit form of the discretized equation has been considered for the sake of simplicity.

$$\rho CpT\mid_{i+1,w}^{n+1} = T_{i+1,w}^n\left\{\rho Cp\mid_{i+1,w}^n - \frac{0.5(\eta h\mid_i^n + \eta h\mid_i^n).\Delta t A_c}{(A)_w} - \frac{0.5(\eta h\mid_{i+1}^n + \eta h\mid_i^n)A_h\Delta t}{(A)_w} - \frac{k_{i+1}\Delta t}{\Delta x^2}\right\} +$$
$$T_{i+2,w}^n\frac{(k_{i+1}+k_{i+2})0.5\Delta t}{\Delta x^2} + T_{i,w}^n\frac{(k_{i+1}+k_i)0.5\Delta t}{\Delta x^2} + \frac{0.5(\eta h\mid_i^n + \eta h\mid_i^n).\Delta t A_c\, 0.5(T_{i,c}^{n+1}+T_{i,c}^n)}{(A)_w} +$$
$$\frac{0.5(\eta h\mid_{i+1}^n + \eta h\mid_i^n)A_h\, 0.5(T_{i+1,h}^{n+1}+T_{i+1,h}^n)}{(A)_w} \tag{8}$$

Final temperature is obtained as,

$$T_{i+1,w}^{n+1} = \frac{\rho CpT\mid_{i+1,w}^{n+1}}{\rho Cp\mid_{i+1,w}^{n+1}} \tag{9}$$

3. Numerical studies with a 2 layer PFHE

Based on the mathematical model and discretized equations developed, computer programs are written for both the cases, i.e. analysis based on average fluid properties and that based on actual (nodal) fluid properties. Response of a prototype PFHE is studied for a ramp change in the temperatures of the inlet fluids. Table 1 gives the construction details of the sample PFHE core.

Table 1 Construction details of sample PFHE core

Length	1.2 m	End Plate Thickness	3 mm
Width	184 mm	Side Bar Width	8 mm
No. Of Layers	2	Fin Density	714.1732 / m
Separating Plate Thickness	0.8 mm	Fin Length	3 mm
Plate Spacing	6.5 mm	Fin Metal Thickness	0.2 mm

Steady state outlet temperatures of the sample PFHE, for given inlet temperatures, were obtained from the code developed by Goyal et al. [11] and results are shown in the Table 2.

Table 2 Steady state parameters

	Hot	Cold
Inlet conditions	10 K, 13 bar(a)	6 K, 1.3 bar(a)
Outlet conditions	7.18 K, 13 bar(a)	9.41 K, 1.3 bar(a)
Mass flow rates	10 g/s	10g/s

The temperature profiles obtained from the code developed by Goyal et al. [11] and the parameters as shown in Table 2 are used as initial conditions for transient simulations, using the constant and variable properties code separately. The boundary inputs for transient simulation are shown in Table 3.

Table 3 Transient inputs for sample PFHE

Hot side inlet	Cold side inlet	Wall side
Linear decrease to 9 K in 60s, then constant at 9 K.	Fixed at 6 K.	Adiabatic ends.

As expected of counter flow, high effectiveness heat exchangers, the cold side outlet temperatures follow the trend of hot side inlet temperature. Any variation in hot side inlet temperature tends to have a more prominent effect on the cold side outlet temperature than its own outlet temperature. The variation in cold side exit temperatures with time, as obtained from constant properties and variable properties transient models with varying hot side inlet temperature have been presented in Figure 2 and in Table 4. The time taken by the PFHE to arrive at the new steady, as predicted by the models is given in Table 4.
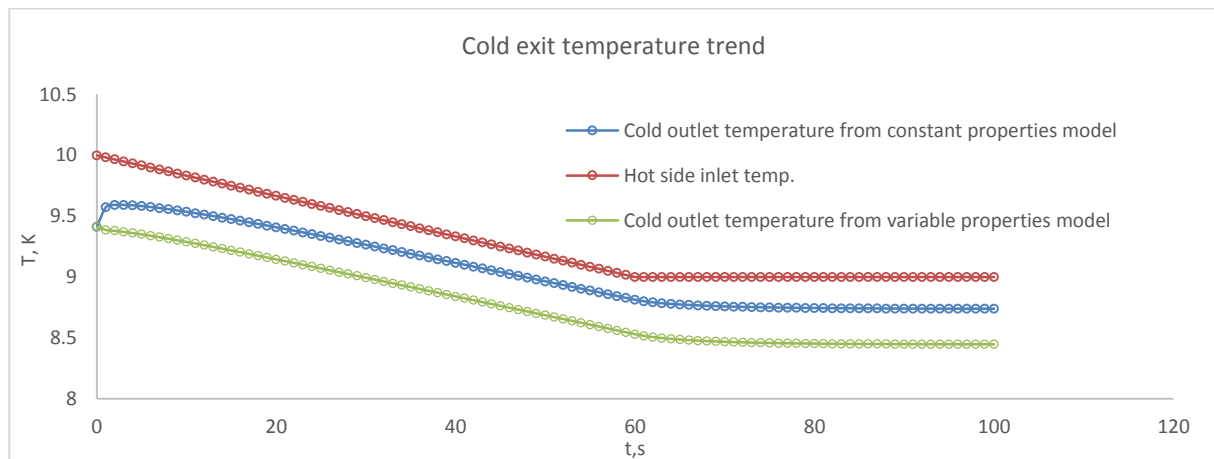

Figure 2 Transient cases with constant and variable properties model

Table 4 Outlet temperatures for case study of Table 4

	Hot side	Cold side
Steady state temp. and time elapsed, constant properties	6.84 K, 100 s	8.74 K, 100 s
Steady state temp. and time elapsed, variable properties	6.76 K, 100 s	8.45 K, 100 s

In Figure 2, initial sudden rise in the cold exit temperature is due to the fact that the input initial temperature profile of the PFHE is based on the steady state profile based on variable properties model [11].

For validation of developed transient models, the steady state results obtained from the case study of Table 4 are matched with the published code for steady state analysis [11]. As evident from Table 3, the inlet temperature of the hot side fluid will be at 9 K after the ramped change in the hot inlet temperature. The cold side fluid inlet temperature is always fixed at 6 K. These two inlet temperatures, with the rest of parameters as mentioned in Table 2, are used to generate new steady state profile and exit

temperatures using the code developed by Goyal et al [11]. The profile and temperatures so obtained are compared with the temperature profiles obtained from the case study of Table 4 using constant properties and the variable properties transient models. The results are shown in Figure 3 and Table 5.

Table 5 Steady state outlet temperatures

	Hot side	Cold side
Steady state temp., constant properties	6.84 K	8.74 K
Steady state temp., variable properties	6.76 K	8.45 K
Steady state temp., published model[11]	6.74 K	8.46 K

Figure 3 Comparison of steady state temperature profiles

4. Experimental Investigations

The same 2 stream PFHE, which was considered for numerical studies, is fabricated and developed for experimental validation of code. Experiments have been carried out at room temperature range using Helium as working fluid. The process flow diagram is shown in Figure 4.
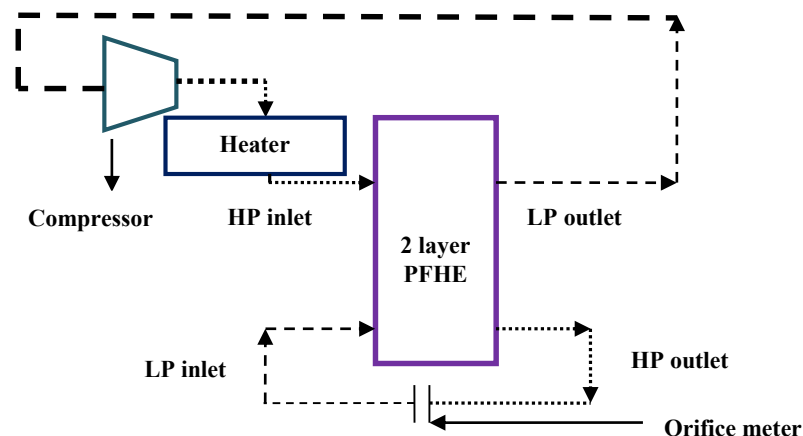

Figure 4 Schematic of the experimental set up

The experimental set up consists of a two layer PFHE. Other equipment/components that are required to carry out the experiment are Pt-100 RTDs, absolute pressure transmitters, orifice meters with differential pressure transmitters. For insulation of the heat exchanger and associated piping, foam and aluminium sheets have been used. A 1 kW heater is wrapped on the inlet HP pipe line for controlling the inlet temperature of fluid. Pt-100 RTDs are used along the length of the PFHE and on the inlet-exit nozzles. Pressure transmitters are used to measure the inlet-exit pressures of the streams.

In this experiment, the effect of any changes in the inlet temperature of the hot side fluid on the other fluid's exit temperature is studied. The exit temperatures of the hot and cold side fluids are plotted against time in Figure 5. The model based on the constant properties was validated using the experimental data. The heat exchanger exhibited an effectiveness value of about 0.95 with helium flow rate of 5 g/s.
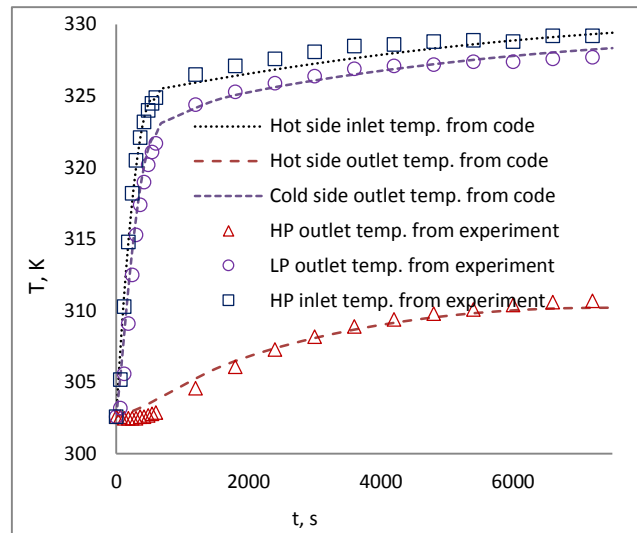

Figure 5 Comparison of room temperature experiments with results of constant property code

Since RTDs have been mounted on the end plates of the heat exchanger along its length at definite intervals, it is possible to validate the steady state temperature profiles for the hot and cold fluids from the discretized average properties model. The comparison of the profiles from the model and the experiment is shown in Figures 6 and 7.
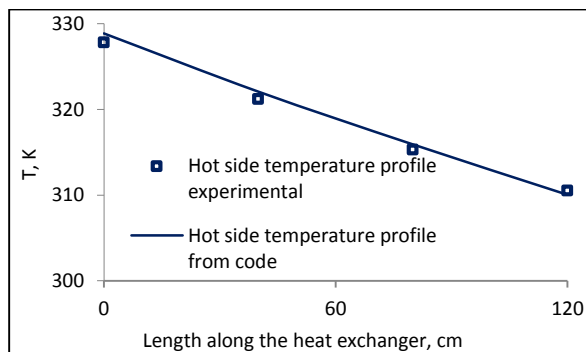

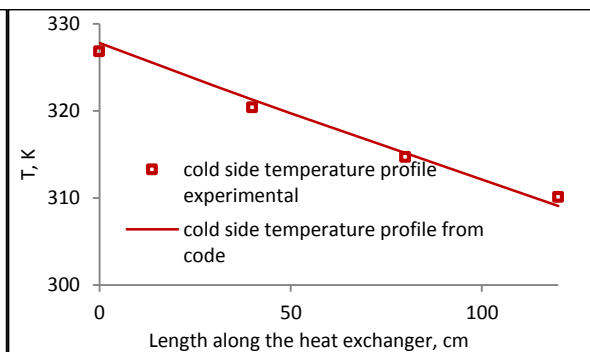Figure 6 Hot side temperature profile   Figure 7 Cold side temperature profile

5. Conclusion and future development prospects

In this paper, two models for transient analysis of compact PFHE have been proposed. Both the models are based on distributed parameter approach of analysis of heat exchangers. Governing equations for both the models have been obtained in discretized form by invoking the energy conservation equation, in case of constant properties model and equations for mass and energy conservation, in the variable properties model. Using the computer codes based on these models, simulations have been performed using sample boundary conditions and initial conditions obtained from the published code for steady state analysis [11]. Validation of both the transient models has also been done using the published code for steady state analysis [11].

The usefulness of the constant properties model is more at higher temperature domains, where the properties of helium are unaffected by temperature. Experiments at room temperature have been performed on an actual PFHE, the data from which have been used to validate the constant properties model for transient behaviour. The constant properties model has been satisfactorily validated with the help of experiments, as is seen from the temperature profile comparisons in Figures 6 and 7 as well as Figure 5, where change in exit temperatures with time for hot and cold streams have been compared. Validation of constant properties model with experiments at liquid nitrogen temperature or even lower temperatures can also prove to be useful. The constant properties model takes less execution time compared to the variable properties model in which the properties are to be updated at every time step.

The differences in the two transient models become evident from Table 4, where the predicted results of transient simulations have been presented. These differences arise because of the large variation in fluid and metal properties in the temperature domain of the analysis, a phenomenon not taken care of by the constant properties model. Both the models predict a similar time requirement by the PFHE to travel between two steady states. The variable properties model fares better than the constant properties model in prediction of the PFHE behaviour. There is a perfect match between the steady state behaviour predicted from transient code based on variable properties and steady state code developed by Goyal et al. [11], this is due to the fact that at steady state governing equations for both the models are same.

The variable properties transient model can be further validated with the experiments carried out at LHe temperatures. The code based on variable properties model can also be improved in terms of solution techniques to reduce the execution time.

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