

Radiative heat transfer estimation in pipes with various wall emissivities

Robin Langebach, Christoph Haberstroh

Technische Universitaet Dresden, Bitzer-Professur fuer Kaelte-, Kryo- und Kompressorentchnik, 01062 Dresden, Germany

E-mail: robin.langebach@tu-dresden.de

Abstract. Radiative heat transfer is usually of substantial importance in cryogenics when systems are designed and thermal budgeting is carried out. However, the contribution of pipes is commonly assumed to be comparably low since the warm and cold ends as well as their cross section are fairly small. Nevertheless, for a first assessment of each pipe rough estimates are always appreciated. In order to estimate the radiative heat transfer with traditional “paper and pencil“ methods there is only one analytical case available in literature – the case of plane-parallel plates. This case can only be used to calculate the theoretical lower and the upper asymptotic values of the radiative heat transfer, since pipe wall radiation properties are not taken into account. For this paper we investigated the radiative heat transfer estimation in pipes with various wall emissivities with the help of numerical simulations. Out of a number of calculation series we could gain an empirical extension for the used approach of plane-parallel plates. The model equation can be used to carry out enhanced paper and pencil estimations for the radiative heat transfer through pipes without demanding numerical simulations.

1. Introduction

The heat transfer through inclined pipes has been investigated by our group for a couple of years. The main driver for these investigations was the fact that the interconnecting pipework between the warm and cold temperature level can contribute to the heat intake of a cryogenic storage system, especially under critical inclinations where the warm end is placed below the cold end [1].

In these critical cases the total heat transfer mainly caused by natural convection can increase to several watts, while the pure conductive heat transfer is in the magnitude of several milliwatts [1].

In all of these previous considerations the radiative heat transfer was neglected since the order of magnitude was assumed to be very small in comparison to the convection. Nevertheless, a certain contribution has to be taken into account, especially when the pipe is placed in a way that almost no natural convection occurs inside.

In the literature only a number of investigations are available which deal with the radiative heat transfer in closed cavities. The paper of Hieke et al. [2] determined a negligible contribution of the radiative heat transfer in case of a pipe with large aspect ratio L/D . Hollands et al. [3] investigated the coupling of conductive and radiative heat transfer in honey comb cavities for solar panels with the help of numerical simulation. They could show that the radiative heat transfer under vacuum conditions in the cavity has a certain contribution to the total heat



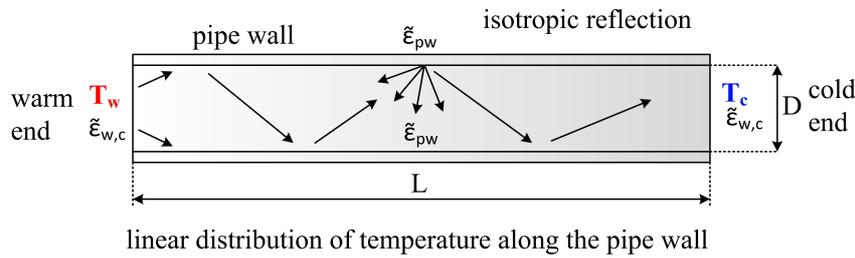


Figure 1. The pipe geometry and definitions for the calculation model.

transfer beside the thermal conduction. The study further revealed a rather low influence of the temperature distribution along the wall from the warm to the cold side. Slight changes of the temperature distribution only involved negligible changes in the radiative heat transfer. As a result, Hollands et al. [3] suggested a decoupled calculation of conduction and radiation for typical cases. This finding is in accordance with remarks in other publications, e.g. Rohsenow [4]. Tien and Yuen [5] extended the investigations of Hollands et al. [3] and introduced a larger number of different model geometries to be investigated – but in general with the same findings. They also determined that a decoupled calculation of the thermal conduction and the radiation is sufficient with respect to acceptable accuracies.

Easy-to-use analytical models or correlations for lumped heat transfer estimation through a pipe are only available for the case of pure heat conduction and for the case of coupled conduction, radiation and convection [6]. For the pure radiative heat transfer a calculation is only possible by using the analytically solved case of radiation between plane-parallel surfaces in VDI Heat Atlas [7]. Within this model no pipe wall influence - especially the emissivity - is taken into account here, but obviously should.

Hence, the main aim of our investigations is to study the radiative heat transfer through pipes including the effect of various pipe wall emissivities in order to generate an enhanced model for “paper and pencil“ heat transfer estimations.

2. Geometry, definitions and assumptions

The considered geometry is a straight pipe closed at both ends by end plates. The inner diameter of the pipe is D , the total inner length is L . The aspect ratio is $AR = L/D$.

The warm end of the pipe has a constant temperature of T_w , the cold end that of T_c . Along the pipe wall we assume a linear temperature distribution from T_w to T_c . Temperature changes due to radiative, convective or conductive heat transfer will be neglected in accordance with the mentioned references.

Concerning the radiative properties we assume the whole geometry to obey the model of grey and diffusive surfaces with a constant total hemispherical emissivity¹ ε . The authors are fully aware of the complex characteristics of radiation with respect to wavelength, hemispherical emission and temperature dependencies. However, they are not specifically taken into account here.

The emissivity of both end plates is defined as $\varepsilon_{w,c}$, the emissivity of the pipe wall as ε_{pw} . The pipe geometry and corresponding definitions are shown in Figure 1. Due to these system definitions a net radiation exchange² q_{rad} between the warm and the cold end of the pipe is induced. The radiative heat transfer will be evaluated per area in W/m^2 .

¹ Within this paper we generally name the total hemispherical emissivity just emissivity for the sake of legibility.

² Within this paper we generally name the net heat exchange by the radiative heat transfer between the warm and the cold end.

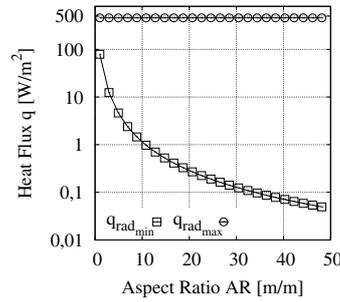


Figure 2. Calculation of the radiative heat transfer per area in a straight pipe with varying aspect ratio AR for constant temperatures $T_c = 4\text{ K}$ and $T_w = 300\text{ K}$ and an emissivity of $\varepsilon_{w,c} = 1$.

3. Available analytical approaches

The only available approach for estimating the radiative heat transfer along a straight pipe without complex numerical computations is that of two plane-parallel surfaces [7]. At first glance, two plane-parallel surfaces do not have too much in common with a straight pipe. But, on second view we can use this model to at least find asymptotic values for the radiative heat transfer along the straight pipe.

Theoretical minimum of radiative heat transfer In case the pipe wall has no reflecting properties, we can assume the two plane-parallel surfaces at the ends are the only parts of the pipe geometry contributing to the radiative heat transfer. The net amount of the absorbed heat at the cold plate is directly emitted by the warm plate. No further reflection of the pipe wall is taken into account. Hence, the minimum radiative heat transfer can be calculated with the following set of equations:

$$q_{rad_{min}} = \frac{C_s}{\frac{2}{\varepsilon_{w,c}} + \frac{1}{\phi} - 2} \left(\frac{T_w^4 - T_c^4}{100^4} \right) \quad (1)$$

$$\phi = 1 + 2 \cdot AR^2 \left(1 - \sqrt{\frac{1}{AR^2} + 1} \right) \quad (2)$$

In this set $C_s = 5,67 \frac{W}{m^2 K}$ is a constant, ϕ is the case specific view factor.

An exemplary calculation of q_{min} for varying values of AR is given in Figure 2.

Theoretical maximum radiative heat transfer On the basis of $q_{rad_{min}}$ it is also possible to derive the maximum radiative heat transfer $q_{rad_{max}}$ per area. With respect to our definitions the maximum case would occur by using an arbitrary emissivity $\varepsilon_{w,c}$ and a pipe wall with ideal reflectivity. This means, each reflected ray is transported to the opposite end. In equation (1) obviously no dependency to ε_{pw} is given. But, we can assume this case to be equivalent to the geometry condition $AR \rightarrow 0$. As a result, the ideal $q_{rad_{max}}$ can be calculated as follows:

$$q_{rad_{max}} = \frac{C_s}{\frac{2}{\varepsilon_{w,c}} - 1} \cdot \left(\frac{T_w^4 - T_c^4}{100^4} \right) \quad (3)$$

A calculation of $q_{rad_{max}}$ for varying values of AR is given in Figure 2. The calculation emphasizes that $q_{rad_{max}}$ is an extension of the maximum case of $q_{rad_{min}}$ due to the independence of the length of the pipe.

4. Numerical investigation and empirical model approach

Even though the radiative heat flux is obviously low for typical pipes with higher aspect ratios L/D , the available approach with two plane-parallel surfaces only helps to localize the region of expectable q_{rad} . This was the motivation to start own numerical simulations in order to characterize the influence of the pipe wall more precisely.

For this we used the radiation module of Ansys CFX in order to evaluate the radiative heat transfer numerically. Within the radiation module we applied the *Discrete Transfer Model* (DTM)[8] in combination with the surface-to-surface option to numerically map the radiation field. Basically, this model approach simulates a discrete number of rays approaching and departing from any surface element. For each ray a transport equation is solved in order to model energy conservation during emission, transmission and reflection. The surface-to-surface option is a simplification with respect to the fluid inside the pipe. Surface-to-surface implies that any gas inside the pipe is fully transparent and no interaction with transmitted rays takes place. In order to achieve a mesh independent solution, we carefully carried out a number of mesh optimization calculations.

Based on this simulation background we carried out a series of simulations with the following changes of the boundary conditions:

- aspect ratio $AR = 1...50$
- emissivity $\varepsilon_{w,c} = 0...1$
- emissivity $\varepsilon_{pw} = 0...1$
- temperatures $T_c = 4 K$, $T_w = 300 K$

With these varying input values we carried out a calculation series with 433 single simulations. After each simulation we evaluated the radiative heat input to the cold end.

In order to correlate the results it was necessary to find a form function which matches all physical requirements. The requirements can be described as follows:

- $q_{rad_{min}}$ is the lower asymptotic solution. $q_{rad_{min}}$ is the result when either $AR \rightarrow \infty$ or $\varepsilon_{pw} \rightarrow 0$.
- $q_{rad_{max}}$ is the upper asymptotic solution. $q_{rad_{max}}$ is the result when either $AR \rightarrow 0$ or $\varepsilon_{pw} \rightarrow 1$.
- The correlation has to follow the approach $q_{rad_{mod}} = f(q_{rad_{min}}; q_{rad_{max}}; AR; \varepsilon_{pw})$.

The resulting form function is:

$$q_{rad_{mod}} = q_{rad_{min}} + (1 - \varepsilon_{pw})^{f_m \cdot AR} \cdot (q_{rad_{max}} - q_{rad_{min}}) \quad (4)$$

The terms for $q_{rad_{min}}$ and $q_{rad_{max}}$ are the same as in equation (1) and (3). The term f_m is an unknown function or a constant which has to be correlated to the results of the simulations.

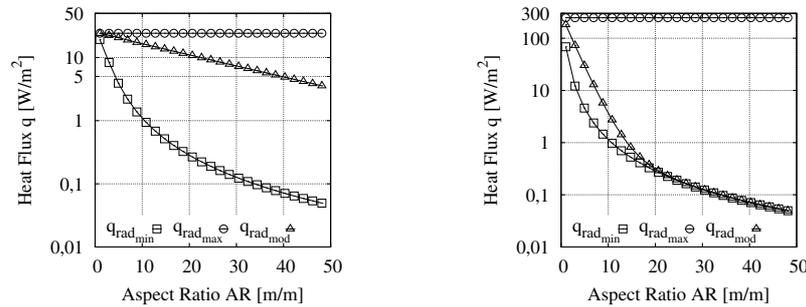
In our case we correlated f_m to a constant value. With the help of a least square fit we obtained $f_m = 0.38$. Hence, the final correlation is:

$$q_{rad_{mod}} = q_{rad_{min}} + (1 - \varepsilon_{pw})^{0.38 \cdot AR} \cdot (q_{rad_{max}} - q_{rad_{min}}) \quad (5)$$

An evaluation of the modelled radiative heat transfer is given in Figure 4 for two different cases. The results of the calculations will be discussed in detail in the following section.

5. Discussion of results

In order to interpret and discuss the results of our modelled radiative heat transfer it is helpful to shortly summarize what we have done so far. Firstly, we applied the available analogous model of radiation between plane-parallel surfaces. With this model only the two asymptotic cases



(a) q_{rad} with $\varepsilon_{w,c} = 0.1$ and $\varepsilon_{pw} = 0.1$. (b) q_{rad} with $\varepsilon_{w,c} = 0.7$ and $\varepsilon_{pw} = 0.7$.

Figure 3. Calculation of the radiative heat transfer for all three cases $q_{rad_{min}}$, $q_{rad_{max}}$ and $q_{rad_{mod}}$ in a straight pipe with varying aspect ratio AR for constant temperatures $T_c = 4\text{ K}$ and $T_w = 300\text{ K}$.

can be calculated due to the neglected influence of the pipe wall. One result is the theoretical minimum of q_{rad} since only direct transfer of rays between warm and cold end is considered. The other result is the theoretical maximum for q_{rad} which can occur in case the pipe wall has fully reflecting properties so that all rays coming from the end walls will be reflected to the opposite end. This is why we can observe the apparent independence of length. Both cases are not representative for typical construction materials but have been used as limiting parameters for the correlation. More common values for emissivity are e.g. $\varepsilon = 0.7$ for strongly corroded copper surfaces or $\varepsilon = 0.1$ for polished stainless steel. Both cases have been analysed in Figure 4(a) and (b) with the correlation.

A deeper look to the results shows some details about the characteristics to be expected for both materials.

In case of strongly corroded copper we can observe that the initial radiative heat transfer is higher and therefore also the maximum value $q_{rad_{max}}$ for comparably short pipes. But at the same time the absorption at the pipe wall is higher and therefore the decrease of $q_{rad_{mod}}$ towards higher aspect ratios is very large.

In case of stainless steel the initial radiative heat transfer is lower and therefore also the achievable maximum value of $q_{rad_{max}}$. The much better reflecting properties of the pipe wall involve a higher reflection of the rays inside the pipe. Hence, the decrease of $q_{rad_{mod}}$ to higher aspect ratios is comparably low with respect to corroded copper.

From this we can conclude that specifically stainless steel pipes are more critical in terms of radiative heat intake in comparison to copper. They can be expected to have a higher radiative heat transfer from the warm to the cold end (otherwise unchanged conditions) even when the emissivity of the warm end the cold end are comparably small. With the help of the correlation we can estimate the point of intersection at an aspect ratio of approximately $AR \approx 6$ (see Figure 4). However, we have to keep in mind that this effect of higher radiative heat intake might be compensated or even cancelled out by higher thermal conduction. A cross-checking has to be done individually by the pipe designer.

Unless the new calculation model gives a more exact value for q_{rad} in comparison to the approach with plane-parallel surfaces and includes the pipe wall influence, one has to keep in mind that no experimental validation has been done. Moreover, the authors believe that an experimental validation is most likely very demanding with respect to very small total heat transfer values. The model generation applied here is based on numerical calculations with underlying assumptions. This is why the authors cannot give a typical deviation.

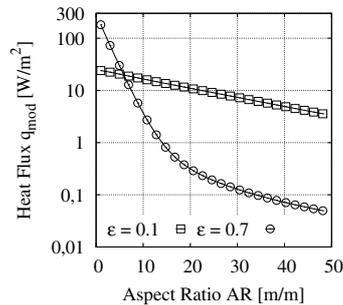


Figure 4. Comparison of $q_{rad,mod}$ for both copper with $\varepsilon = 0.7$ and stainless steel with $\varepsilon = 0.1$.

6. Conclusion

In previous investigations about interconnecting pipework in cryogenic applications it was shown that natural convection in combination with the heat conduction is the predominant effect of heat transfer along a typical pipe. Thermal radiation was either included in performed experiments or assumed to be negligible in numerical simulations. In this paper we tried to focus on methods to calculate the fraction of the radiative heat transfer – preferably with easy-to-use methods. In literature only the model of plane-parallel surfaces is available. But, this is only appropriate to calculate the lower or upper limit since the pipe wall influence is not considered at all. Hence, the calculable values do most probably not fit for the most technical configurations with pipes. This is why we extended the model of plane-parallel surfaces by additional terms which introduce the influence of the pipe wall by the emissivity. A correlation was fitted by a large number of numerical simulations. In the end we could gain an easy-to-use model equation for radiative heat transfer estimations with consideration of the pipe wall on the basis of the model of the plane-parallel plates. The new model equation can be used for more accurate estimations for the radiative heat transfer through a certain pipe in the overall heat balance. With respect to no experimental validation for the correlation, the authors recommend to handle the results as an estimation of the expectable order of magnitude. Further improvements of the model could be to extend the simulations to different emissivities for the warm and the cold end. This would give the opportunity to take into account the temperature dependency of the emissivities.

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