

Theoretical analysis of start-up power in helium pulsating heat pipe

Monan Li^{1,2}, Rongjin Huang¹, Dong Xu¹, Laifeng Li¹

¹Key Laboratory of Cryogenics, Technical Institute of Physics and Chemistry, Chinese Academy of Sciences, Beijing, China

²University of Chinese Academy of Sciences, Beijing, China

LaifengLi@mail.ipc.ac.cn

Abstract. An analytical model for one-turn helium pulsating heat pipes (PHPs) with single liquid slug and vapor plug is established in present study. When an additional heat power takes place in the evaporating section, temperature and pressure will increase. The pressure wave travels through vapor and liquid phases at different speed, producing a pressure difference in the system, which acts as an exciting force to start up the oscillating motion. Results show that the start-up power of helium PHP is related to the filling ratio. The start-up power increases with the filling ration. However, there exist an upper limit. Furthermore, the start-up power also depends on the inclination angle of PHP. When the inclination angle increases, the heat input needed to start up the oscillating motion decreases. But for one-turn helium PHP, it can not be started up when the inclination angle is up to 90°, equalling to horizontal position,. While the inclination angle ranges between 0° (vertical position) and 75°, it can operate successfully.

1. Introduction

The conduction cooling method is widely used in cryogenic applications: the cryocooler which offers a reduced size cold finger connecting to the objects instead of any refrigerants such as liquid helium or liquid nitrogen, etc. That simplifies the cooling system, reducing the operating cost and releasing users from the bothersome handling.[1] Therefore, efficient thermal links are increasing essential for cold power transfer. Copper or sliver bar is usually used as thermal links, but the thermal conductivity is not as high as we need, which easily leads to significant temperature gradients, in fact, for a large distance between cooled object and cryocooler. What's more, the volume is large. To alleviate this constraint on the heat



transport, a novel heat transport means that can transport much more heat for the same temperature difference is needed. Pulsating heat pipe is proposed to be a highly effective two phase heat transfer device can transport several orders of magnitude larger heat loads than heat conduction of solids. It usually consists in a capillary tube bended into meandering tube, partially filled with a working fluid which disperses itself into vapor and liquid volumes within the internal structure via capillary action. The PHP essentially utilizes changes of the pressure and the temperature in volume expansion and contraction during phase changes to excite the pulsation motion of liquid plugs and vapor bubbles in capillary tube between the evaporator and the condenser.

Since pulsating heat pipe was introduced by Akachi [2] in the middle of 1990s, numerous experimental and theoretical work have been conducted by many researchers over decades, shedding light on the complex characteristics of heat transfer and dynamic pulsating behavior inside PHP at a capillary level[3-7]. However, most of researches are under room temperature, experiments and theoretical analysis for cryogenic PHPs are still lacking. PHP using liquid nitrogen as the working fluid has been studied for ultra-fast cooling of cells[8, 9]. Mito and Natsume investigated cryogenic PHPs with a G-M cryocooler using hydrogen, neon and nitrogen for cooling of high temperature superconducting (HTS) magnets. The effective thermal conductivities reached to 500-3000W/m·K for H₂ at 17-27K, 1000-8000W/m·K for Ne at 16-95K and 10000-18000W/m·K for N₂ at 17-70K. The cryogenic PHP based on helium at 4.2K was developed by Gully and Bonnet, and the maximum power of 145mW is transferred by the PHP made of Cu/Ni pipe with 5 turns at 40°. The measured effective thermal conductivities reached to 11700W/m·K at tilt angle 10° with 75mW, and the maximum capability increased with the enlarging of tilt angle. Luis and John studied helium based cryogenic PHP made of stainless steel pipe with 32 turns, and found that the measured effective thermal conductivities was up to 2200-2500W/m·K. Wang's group in Institute of Electrical Engineering, CAS built a nitrogen PHP system with the filling ratio of about 50%. The best thermal performance was obtained in bottom heat mode with thermal conductivity of about 16,000 W/m·K[10]. The hydrogen PHP built in Zhejiang University provides its highest effective thermal conductivity of 18.7 kW/m·K when the filling ratio is 35% and the heating power is 5 W. Li's group in Technical Institute of Physics and Chemistry, CAS built a helium PHP system. The effective thermal conductivity was 3600-15000W/m·K with the evaporator located at the bottom and at a liquid filling ratio of 54%[11].

For helium pulsating heat pipe, there is not any theoretical analysis now. In the current investigation, a simplified model of one-turn helium pulsating heat pipe with single liquid slug and vapor plug is used to determine the filling ratio and inclination angle effect on the startup power of helium pulsating heat pipe[12].

2. Theoretical modeling

Generally, looped pulsating heat pipes are consist of several loops. The experiments on He-PHP were all about closed loop PHP. So closed loop PHP is chosen to do theoretical analysis here, so that the results can be compared with the results in experiments. Considering the pulsating motion in each loop is similar, we establish an analytical model for one-turn

helium pulsating heat pipe with single liquid slug and vapor plug here, which is shown in Fig.1.

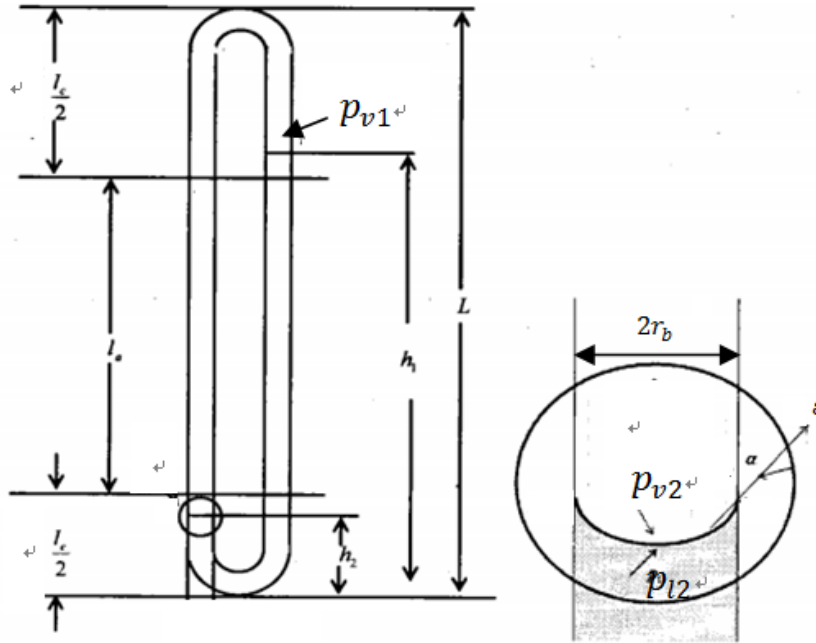


Fig.1. Schematic of one-turn PHP[12]

It is assumed that the behavior of vapor plugs in the evaporator can be modeled by ideal gas law. Considering Laplace-Young equation, the pressure difference across the liquid-vapor interface is given as

$$\Delta p_1 = p_{v1} - p_{l1} = \frac{2\varepsilon}{r_b \cos \alpha_1} = \frac{2\varepsilon \cos \alpha_1}{r_b} \quad (1)$$

$$\Delta p_2 = p_{v2} - p_{l2} = \frac{2\varepsilon}{r_b \cos \alpha_2} = \frac{2\varepsilon \cos \alpha_2}{r_b} \quad (2)$$

$$\text{Considering the effect of gravitational force, } p_{l2} - p_{l1} = (\rho_l - \rho_v)g(h_2 - h_1)\cos\beta \quad (3)$$

The pressure difference between two sides of the vapor bubble, $p_{v2} - p_{v1}$, can be described as

$$p_{v2} - p_{v1} = \Delta p_2 - \Delta p_1 + p_{l2} - p_{l1} = \frac{2\varepsilon (\cos \alpha_2 - \cos \alpha_1)}{r_b} + (\rho_l - \rho_v)g(h_2 - h_1)\cos\beta \quad (4)$$

where α_1 and α_2 are contact angles of liquid-vapor, β is the inclination angle of PHP. The pressure defined by Eq(4) is the total pressure need to be overcome to start up the oscillating motion in pulsating heat pipe.

The liquid helium and helium gas in the tube is under saturation state condition, and the gas could be taken as an ideal gas. Differentiating ideal gas law with respect to time gives

$$\frac{dp_v}{dt} = RT \frac{d\rho_v}{dt} + R\rho_v \frac{dT}{dt} \quad (5)$$

If we set reference values of vapor density and temperature, the transient density and temperature can be expressed as $\rho_v = \rho_{v0} + \Delta\rho$ and $T = T_0 + \Delta T$, respectively. Substituting them into Eq(5), yields

$$\frac{dp_v}{dt} = R(T_0 + \Delta T) \frac{d\rho_v}{dt} + R(\rho_{v0} + \Delta\rho) \frac{dT}{dt} \quad (6)$$

Considering that $T_0 \gg \Delta T$ and $\rho_{v0} \gg \Delta\rho$, Eq (6) can be simplified as

$$\frac{dp_v}{dt} = RT_0 \frac{d\rho_v}{dt} + R\rho_{v0} \frac{dT}{dt} \quad (7)$$

To obtain the minimum heat input needed to start the oscillating motion, we assume that all the heat added to the heating section is used to generate vapor exactly before the pulsating heat pipes start up, i.e., $q_v = \dot{m}_v h_{lv}$ (8), where $\dot{m}_v$ is the vapor mass generation per unit time, i.e., $\dot{m}_v = \frac{dm_v}{dt}$. The total vapor mass can be obtained by $m_v = \rho_v V_v$ (9), where $V_v = 2L(1 -$

$x)\pi r_b^2$. Combining two equations, the heat addition can be written as $q_v = \frac{d\rho_v}{dt} V_v h_{lv}$ (10)

Rearranging Eq.(10) and substituting it into Eq. (7) yields

$$\frac{dp_v}{dt} = RT_0 \frac{q_v}{V_v h_{lv}} + R\rho_{v0} \frac{dT}{dt} \quad (11)$$

Integrating Eq. (11) with time t

$$\Delta p_v = RT_0 \frac{q_v}{V_v h_{lv}} t + R\rho_{v0} \Delta T \quad (12)$$

Where $\Delta p_v = p_{v1} - p_{v2}$, $\Delta T = T_t - T_0$. The pressure difference Δp_v is caused by the heat addition in the evaporator and heat rejection in the condenser. When Δp_v shown in Eq(12), generated after time t in the system is larger than pressure defined by Eq (4), the oscillating motion in pulsating heat pipe can start up.

Combining Eq. (12) and (4)

$$\frac{2\varepsilon (\cos\alpha_2 - \cos\alpha_1)}{r_b} + (\rho_l - \rho_v)g(h_2 - h_1)\cos\beta = RT_0 \frac{q_v}{V_v h_{lv}} t + R\rho_{v0} \Delta T \quad (13)$$

Rearranging Eq.(13), the expression of heat input to start up the oscillating motion can be obtained

$$q_v = \frac{(2\varepsilon(\cos\alpha_2 - \cos\alpha_1) + r_b((\rho_l - \rho_v)g(h_2 - h_1)\cos\beta - R\rho_{v0}\Delta T))V_v h_{lv}}{RT_0 r_b t} \quad (14)$$

When heat is added to the evaporating section, vapor pressure increases. Then pressure wave travels through both vapor and liquid phases. The speed of pressure wave in the vapor phase is different from that in the liquid phase, which results in an excited pressure difference. The time difference for these two pressure waves reach another interface can be found as

$$t = \frac{2L(1-x)}{u_v} - \frac{2Lx}{u_l} \quad (15)$$

Where L is the length of OHP, x is the liquid filling ratio, u_v is the pressure wave speed in the vapor phase and u_l is the pressure wave speed in the liquid phase. For the wave speed in the liquid phase, it is well known that, $u_l = \sqrt{\frac{K}{\rho_l}}$, where K is bulk modulus. The vapor phase can be considered as the ideal gas. The speed of pressure wave in the vapor phase is determined by $u_v = \sqrt{kRT}$, where k is adiabatic coefficient. Substituting (15) into Eq.(14) and rearranging Eq.(14), the heat input to generate the oscillating motion can be found as

$$q_v = \frac{(2\sigma(\cos\alpha_2 - \cos\alpha_1) + r_b((\rho_l - \rho_v)g(h_2 - h_1)\cos\beta - R\rho_{v0}\Delta T))(1-x)\pi r_b u_v u_l h_{lv}}{RT_0(u_l - x(u_v + u_l))} \quad (16)$$

3. Results and discussion

Substitute the physical parameters of helium and design parameters of $r_b = 0.5 \times 10^{-3} \text{m}$, $h_2 - h_1 = 0.1 \text{m}$, $T_0 = 4.2 \text{K}$, $\Delta T = 0.8 \text{K}$ into Eq. (14). Then q_v is just the function of the inclination angle β and the filling ratio x . Fig. 2 illustrates the filling ratio effect on the required startup power taking $\beta = \frac{10}{21}\pi$.

The negative value of heat load q_v is meaningless, which means the helium PHP can not start up when the filling ratio is higher than a maximum filling limit. At the experimental condition shown in Fig. (2), the calculated maximum filling limit is near 0.7.

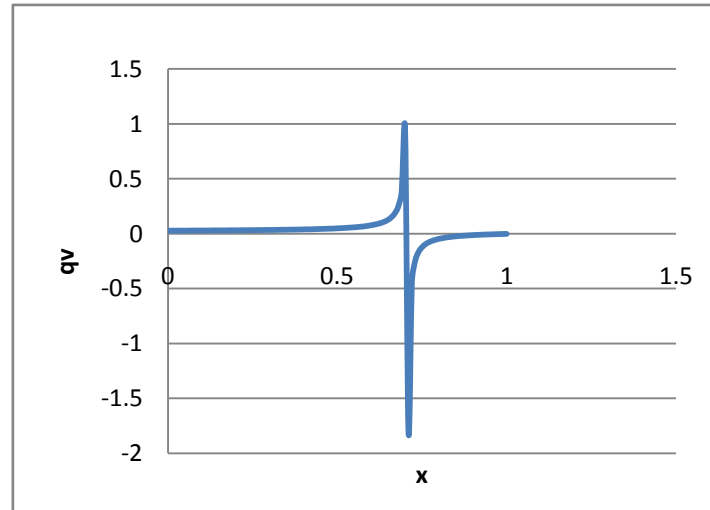


Fig. 2. Heat input vs filling ratio ($\beta = \frac{10}{21}\pi$).

When $\beta = 0, \frac{1}{8}\pi, \frac{1}{4}\pi, \frac{3}{8}\pi, \frac{1}{2}\pi$, the inclination angle effect on q_v is shown in Fig.3. As shown in Fig.3, the required startup power diminishes with the pulsating heat pipe

tends to level. While, when the PHP is in horizontal, the startup power is negative, which means single loop PHP can not start in horizontal. This result is consistent with facts.

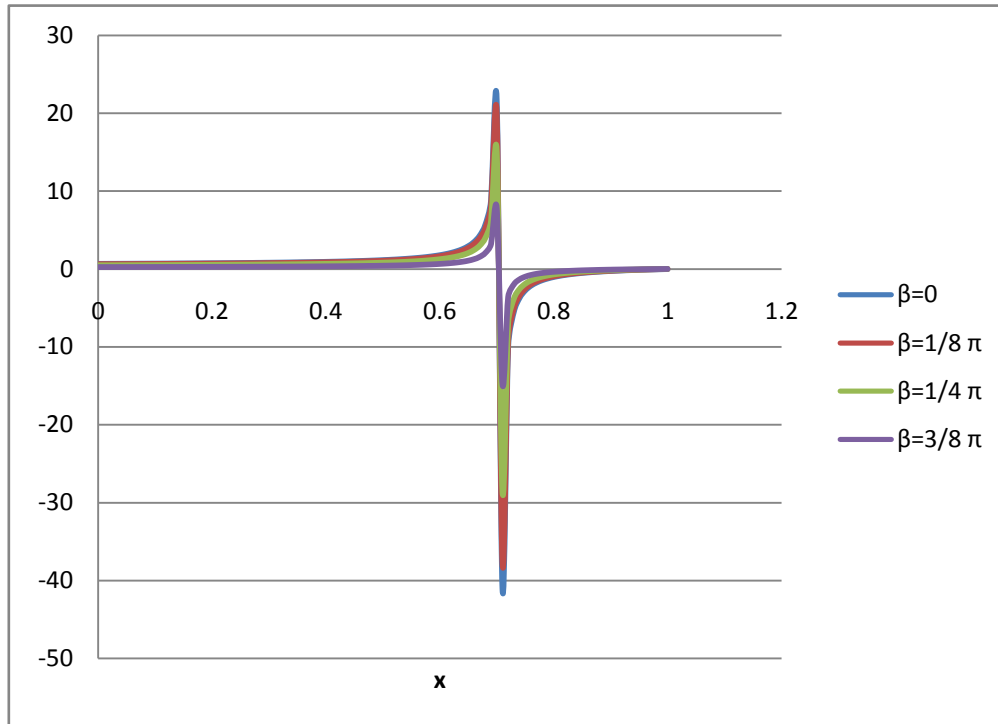


Fig.3. Heat input vs inclination angle

($\beta = 0, \frac{1}{8}\pi, \frac{1}{4}\pi, \frac{3}{8}\pi$ from top to bottom respectively)

4. Conclusions

The model is established based on the fact that pressure wave travels in different speed in vapor phase and liquid phase. The following conclusions were obtained:

- 1) As shown in Fig.2 and Fig.3, when the filling ratio is lower than 40%, required heat input changes little. While, it increases more and more rapidly with the filling ratio when the filling ratio is higher than 40% to the maximum filling limit. A conclusion is drawn that the effect of the filling ratio on the startup power is little when the filling ratio is low. With the increase of the filling ratio, the effect gets stronger.
- 2) As shown in Fig.2 and Fig.3, for each inclination angle, there is a upper limit for filling ratio. When the filling ratio is larger than it, the heat input becomes negative suddenly, which means the PHP can not start up. The maximum filling ratio is related to the structure parameters and operating parameters of the PHP.
- 3) As shown in Fig.3, when the inclination angle β increases, the heat input needed to start up the oscillating motion decreases.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No.51406217, No. 51577185, No.51522705), the fund of Key Laboratory of Cryogenics,

CRYOQN201505, the fund of the State Key Laboratory of Technologies in Space Cryogenic Propellants, SKLTSCP1504.

References

- [1] K. Natsume, T. Mito, N. Yanagi, H. Tamura, T. Tamada, K. Shikimachi, N. Hirano, S. Nagaya 2011 *Cryogenics* **51** 309-14.
- [2] H. Akachi 1990 *US Patent* No. 4,921041.
- [3] M. Arab, M. Soltanieh, M.B. Shafii 2012 *Exp. Therm. F.* **42** 6-15.
- [4] C.-Y. Tseng, K.-S. Yang, K.-H. Chien, M.-S. Jeng, C.-C. Wang 2014 *Exp. Therm. F.* **54** 85-92.
- [5] V.K. Karthikeyan, K. Ramachandran, B.C. Pillai, A. Brusly Solomon 2014 *Exp. Therm. F.* **54** 171-8.
- [6] M. Mameli, V. Manno, S. Filippeschi, M. Marengo 2014 *Exp. Therm. F.* **59** 222-9.
- [7] X.H. Wang, H.C. Zheng, M.Q. Si, X.H. Han, G.M. Chen 2015 *Int. J. Heat.* **83** 586-90.
- [8] A.J. Jiao, H.B. Ma, J.K. Critser 2009 *Int. J. Heat.* **52** 3504-9.
- [9] H.M. Xu Han, Anjun Jiao 2008 *Cryobiology* **56** 195-203.
- [10] Y. Li, Q. Wang, S. Chen, B. Zhao, Y. Dai 2014 *Int. J. Heat.* **79** 713-9.
- [11] D. Xu, L. Li, H. Liu 2016 *Exp. Therm. F.* **1** 61-8.
- [12] D. Yin, H. Rajab, H.B. Ma 2014 *Int. J. Heat.* **74** 353-7.