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The Mathematical Model of Image, Generated by Scanning Digital Radiography System

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Abstract. The mathematical model of image, generated by scanning digital radiography system is present. This model takes into account the X-ray energy spectrum transformation of the test object and a noise due to the quantum nature of radiation. The calculation results confirm the importance of fluctuations of the absorbed energy of the registered photon for the small size of the scintillation detectors.

1. Introduction
Digital radiography in its various implementations continues to be one of the most popular and developing NDT methods [1−3]. The scanning systems of digital radiography (SSDR) are widely used in industry, medicine and customs control. Quality of primary digital images (PDI) generated by these systems is determined by a significant number of parameters [4−6], characterizing the X-ray source and multi-channel X-ray detector, the test object (TO), the scanning geometry, etc. Designing SSDR is impossible without the development of an adequate PDI mathematical model describing the main characteristics of the image with the system's parameters. The PDI mathematical models available in the articles [7−9] does not fully take into account all the features of SSDR, therefore, the model need to be further improved.

2. Assumptions and constraints
To construct a mathematical model of PDI formed by a multi-scanning system of digital radiography, in analogy with [7] we assume:

− carried out a uniform continuous movement along the Ox axis of TO, and the system of source – line of detectors is fixed;

− measuring channels (combinations: a radiation detector − temporal integrator − analog-to-digital converter) are identical;

− line of detectors is formed in the direction of the Oy axis (perpendicular to the direction of the TO movement) and, moreover, symmetrically about the beam axis;

− the radiation source and the line of detectors are collimated by slit collimators, so that scattered by the TO radiation contribution to the registration results is negligible;

− the beam axis coincides with the Oz axis;

− the radiation source has azimuthally symmetrical angular distribution;

− the transverse dimensions of the apertures and detectors of the source focal spot are much less than the focal distance (distance from the source to the detectors along the line of O*z* axis);
− the radiation registration is carried out in an analog (mean current) mode;
− the quantization step of analog-digital converters is small compared to fluctuations in the radiation flux.

3. The Mathematical Model of Image, Generated by Scanning System of Digital Radiography

The primary digital image, generated by SSDR, is a matrix **B**, consists of M lines and K columns. The number of columns K is equal to the number of detectors (measurement channels) in the line of detectors. In view of the assumptions and limitations the matrix **B** is equal to the sum of two matrices $\overline{\mathbf{B}}$ and **N**

$$\mathbf{B} = \overline{\mathbf{B}} + \mathbf{N}. \tag{1}$$

The elements of matrix $\overline{\mathbf{B}}$ are equal to the corresponding mean values of the radiometric signals, and the elements of matrix **N** are characterized an image noise.

The expression for elements of matrix $\overline{\mathbf{B}}$ has a form

$$\overline{B}_{mk} = A_{mk}\lambda_{mk}\int_{-\infty}^{\infty}\varphi(t)dt, \tag{2}$$

where A_{mk} is a mean value of the electrical pulse amplitudes (electrical charges, coulomb) from output of k-th detector from the line of detectors for m-th image line; λ_{mk} is density of electrical pulses from output k-th detector at time t_m, 1/sec; $\varphi(t)$ is a temporal integrator impulse response.

For the system of ideal integrators with reset the impulse response $\varphi(t)$ is described by a step function

$$\varphi(t) = \begin{cases} 1, & 0 \le t \le T; \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where T is the time constant of the integrator (time radiation measurement).

The values A_{mk} are connected with mean value of the absorbed energy by k-th detector of the registered quantum of radiation $\overline{E^{ab}_{mk}}$, MeV by expression

$$A_{mk} = \gamma_c \overline{E^{ab}_{mk}}, \tag{4}$$

here γ_c is the conversion factor of absorbed X-ray energy into an electric charge, coulomb/MeV.

The dependence $\overline{E^{ab}_{mk}}$ from the TO parameters, X-ray source with the maximal energy E_0 and k-th detector has form

$$\overline{E^{ab}_{mk}} = \frac{\int_0^{E_0} P(E)\mathrm{e}^{-\mu(E)H_{mk}} E_{ab\,k}(E)\varepsilon(E)dE}{\int_0^{E_0} P(E)\mathrm{e}^{-\mu(E)H_{mk}}\varepsilon(E)dE}, \tag{5}$$

where $P(E)$ is the energy spectrum of the radiation source by the number of photons, $1/(\text{MeV}\cdot\text{sec})$; $\int_0^{E_0} P(E)dE$ is the total output of photons from all sources per unit time in the solid angle 4π; $\mu(E)$ is the linear attenuation coefficient of photons with energy E for the TO material; H_{mk} is the TO beam size along the line connecting the transmitter center and the center of the k-th detector at time t_m (using

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ideal integrators with reset $t_m = mT$); $E_{ab\,k}$ is the mean value of the absorbed energy by k-th detector for one registered quantum with the energy E, MeV; $\varepsilon(E)$ is the detection efficiency of photons with energy E by a separate detector from the line within its aperture.

The intensity flow λ_{mk} is calculated by using the formula

$$\lambda_{\text{mk}} = \frac{\psi(\theta_k)}{|\mathbf{r}_k|^2} S \int_0^{E_0} P(E)\,\mathrm{e}^{-\mu(E)H_{mk}}\,\varepsilon(E)dE, \tag{6}$$

where θ_k is an angle between Oz axis (axis of the radiation flow symmetry) and the vector $\mathbf{\Omega}_k$ is the direction of beam from X-ray source center to the center of k-th detector of line; θ is the angle between Oz axis and the direction of photon's flight; $\Psi(\theta)$ is the angular radiation distribution, which considered normalized on 1 quantum for solid angle 4π; $\mathbf{r}_k$ is the radius-vector of the aperture center of k-th detector; S is the square of aperture of a single detector from the line of detectors.

The elements N_{mk} of matrix $\mathbf{N}$ are characterized a noise due to the quantum nature of radiation. Its mean value, covariance and variance are equal correspondingly:

$$\overline{N_{mk}} = 0;\ \ \overline{N_{mk}N_{ij}} = \begin{cases} 0, & j \neq k, \\ \overline{A^2_{mk}}\lambda_{mk} \int_{-\infty}^{\infty} \varphi(t)\varphi(t+|m-i|\Delta t)dt, & j = k \end{cases};\ \ \sigma^2[N_{mk}] = \overline{A^2_{mk}}\lambda_{mk} \int_{-\infty}^{\infty} \varphi^2(t)dt. \tag{7}$$

The mean value of square of the electrical pulse amplitudes from output of k-th detector $\overline{A^2_{mk}}$ is connected with the mean value of square absorbed energy by k-th detector for the single registered quantum of radiation $\overline{E^{ab\,2}_{mk}}$, MeV^2 by expression

$$\overline{A^2_{mk}} = \gamma_c^2 \overline{E^{ab\,2}_{mk}}. \tag{8}$$

The value $\overline{E^{ab\,2}_{mk}}$ is calculated similar to the expression (5)

$$\overline{E^{ab\,2}_{mk}} = \frac{\int_0^{E_0} P(E)\,\mathrm{e}^{-\mu(E)H_{mk}}\,\overline{E^2_{ab}}(E)\varepsilon(E)dE}{\int_0^{E_0} P(E)\,\mathrm{e}^{-\mu(E)H_{mk}}\,\varepsilon(E)dE}, \tag{9}$$

where $\overline{E^2_{ab}}(E)$ is the mean square of the absorbed energy by separate detector for one registered quantum with energy E, MeV^2.

For completeness of the model description, we present a expressions to calcule the values $\overline{E_{ab}}(E)$ and $\overline{E^2_{ab}}(E)$. We consider the case use SSDR scintillation detectors containing scintillators (crystals) of cylindrical shape of radius r and thickness h (size in the direction of the incident X-rays) only. Then according to [10] we get:

$$\begin{aligned} \overline{E_{ab}}(E) &= \overline{E_{\min}(E)} + (\overline{E_{\max}(E,h)} - \overline{E_{\min}(E)})(1-\mathrm{e}^{-g(E,h)r}), \\ \overline{E^2_{ab}}(E) &= \overline{E^2_{\min}(E)} + (\overline{E^2_{\max}(E,h)} - \overline{E^2_{\min}(E)})(1-\mathrm{e}^{-g(E,h)r}). \end{aligned} \tag{10}$$

Here E is the quantum of energy, experienced the interaction with the scintillator material, MeV; $\overline{E_{\min}}$, $\overline{E^2_{\min}}$ and $\overline{E_{\max}}$, $\overline{E^2_{\max}}$ are minimal and maximal (corresponding to minimal and maximal radius of scintillator) mean values of absorbed and square of absorbed energy of registered quantum;

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the coefficient g is characterized the rate of increase of the relevant function of the radius of the sensitive volume of the detector (scintillator).

The values $\overline{E_{\min}}$ и $\overline{E^2_{\min}}$ are depend from energy of registered quantum and the material of scintillator. The calculation expressions are [10]:

$$\begin{aligned}
&\overline{E_{min}} = E\,\mu_{foto}/\mu_{sc} + (E-1{,}02)\,\mu_{par}/\mu_{sc} + \pi r_0^2 N_e 0{,}511/\mu_{sc} \times \\
&\times\left[\frac{-20\alpha^4+102\alpha^3+186\alpha^2+102\alpha+18}{3\alpha(1+2\alpha)^3} - \frac{2\alpha+3-\alpha^2}{\alpha^2}\ln(1+2\alpha)\right]; \\
&\overline{E^2_{min}} = E^2\,\mu_{foto}/\mu_{sc} + (E-1{,}02)^2\,\mu_{par}/\mu_{sc} + \pi r_0^2 N_e 0{,}511^2/\mu_{sc} \times \\
&\times\left[\frac{-68\alpha^5+184\alpha^4+566\alpha^3+494\alpha^2+180\alpha+24}{3(1+2\alpha)^4} - \frac{2\alpha+4-\alpha^2}{\alpha}\ln(1+2\alpha)\right].
\end{aligned} \tag{11}$$

Here $r_0^2 = 7.94\cdot 10^{-26}\,\mathrm{cm}^2$; N_e is number of electrons in 1 cm^3 of scintillator's material; μ_{sc}, μ_{foto}, μ_{par} are the full linear attenuation factor of radiation in the scintillator's material and the linear attenuation factors due to the photo-absorption effect and the effect of pair creation; $\alpha=E/0.511$, where energy E is presented in MeV.

The dependencies $\overline{E_{max}}$ and $\overline{E^2_{max}}$ from thickness h of scintillator are described with sufficient accuracy for practical purposes (up to 3%) by expressions [10]:

$$\begin{aligned}
\overline{E_{max}} &= \overline{E_{min}} + (1-\overline{E_{min}})\left(1-e^{-t\mu_{sc}(E)h}\right); \\
\overline{E^2_{max}} &= \overline{E^2_{min}} + (1-\overline{E^2_{min}})\left(1-e^{-t\mu_{sc}(E)h}\right).
\end{aligned} \tag{12}$$

Here t is a coefficient, depending from detector's material (see Table 1).

The formula to calculate the coefficient $g(E,h)$ has form [10]:

$$g(E,h) = b_1\mu_{sc}(E) + \frac{b_2}{\mu_{sc}(E)h}, \tag{13}$$

where b_1, b_2 are the approximation coefficients, depending from the material of the sensitive volume of detector (scintillator). The values of coefficients for some scintillators are given in Table 1.

Table 1. The approximation coefficients t, b_1, b_2 for different scintillators [10].

Coefficient	Plastic	CsI	$CdWO_4$
T	0.51	0.85	0.75
b_1	1.51	2.01	2.33
b_2	0.25	0.61	0.73

Note that the expressions (10) − (13) can be used to estimate $\overline{E^{ab}_{mk}}$, $\overline{E^{ab\,2}_{mk}}$ not only for the cylindrical scintillators, but also for scintillators with square cross-section. This conclusion is following from monotony increasing of $\overline{E^{ab}_{mk}}$, $\overline{E^{ab\,2}_{mk}}$ and η_{mk} from the radius. The values of analyzed variables take the intermediate position for the cylindrical scintillators with radiuses of inscribed and circumscribed in a square circles. The preliminary calculation results for scintillators with transversal sizes 1 mm or smaller, show the closeness of the top and bottom estimations of the analyzed variables. The deviations are less then 0.001 %.

The set of expressions (1) − (13) and Table 1 allow to simulate the primary digital radiographic image of TO with arbitrary cross-section.

4. Calculation Example

On base of formulas (10) − (13) and (5), (9) and with abovementioned note, as example, we calculated values $\overline{E_{mk}^{ab}}$, $\overline{E_{mk}^{ab\,2}}$ and coefficient η_{mk} of electrical pulse amplitude variation from output k-th detector of line, which defined by expression [10]

$$\eta_{mk} = \frac{\sqrt{\overline{A_{mk}^2}}}{\overline{A_{mk}}} = \frac{\sqrt{\overline{E_{mk}^{ab\,2}}}}{\overline{E_{mk}^{ab}}}.$$

With respect to the central detector of detector's line (k=K/2, θ_k=0) under the following assumptions:
− TO has shape of plate from carbon filled plastic with the density 1 g/cm^2 and with thickness 5 mm;
− scintillator from cadmium tungstate ($CdWO_4$) in shape of cuboid with cross-section 0.6 mm×0.6 mm and with thickness (size in direction of incident radiation) 3 mm;
− the energy spectrum of the radiation source by number of quanta is described by expression

$$P(E) = \frac{1}{E}\varphi(E) = \begin{cases} \dfrac{2}{EE_0}\left(1 - \dfrac{E}{E_0}\right), \text{ if } 0 \le E \le E_0; \\ 0, \qquad \text{otherwise,} \end{cases},$$

where $\varphi(E)$ is the energy spectrum of radiation intensity, described by Kramers formula (in normalized form)

$$\varphi(E) = \begin{cases} \dfrac{2}{E_0}\left(1 - \dfrac{E}{E_0}\right), \text{ if } 0 \le E \le E_0; \\ 0, \qquad \text{otherwise.} \end{cases}$$

The energy spectrum of radiation by number of quanta after TO is a continuous function in point E= 0, for any thickness of the attenuating barrier H≠0

$$\lim_{E \to 0+} P(E) e^{-\mu(E)H} = 0.$$

Under those assumptions and with using the data base [11] about the photon radiation attenuation, we get:

for E_0 = 200 KeV − $\overline{E_{ab}} = 0.333E_0 = 66.6\,\text{KeV}$, $\overline{E_{ab}^2} = 0.171E_0^2 = 6840\,\text{KeV}^2$, $\eta_0 = 1.241$;

for E_0 = 250 KeV − $\overline{E_{ab}} = 0.292E_0 = 73\,\text{KeV}$; $\overline{E_{ab}^2} = 0.138E_0^2 = 8625\,\text{KeV}^2$; $\eta_0 = 1.273$;

for E_0 = 300 KeV − $\overline{E_{ab}} = 0.258E_0 = 77.4\,\text{KeV}$; $\overline{E_{ab}^2} = 0.112E_0^2 = 10080\,\text{KeV}^2$; $\eta_0 = 1.298$.

From result analysis we can conclude about significance of the absorbed energy fluctuations of the registered photon for the small size of the scintillation detectors.

5. Conclusion

The paper shows the mathematical model of the image generated by the scanning system of the digital radiography. The model allows for the transformation of the X-ray energy spectrum by the test object, a noise due to the quantum nature of radiation, and the radiation registration with considering the leakage of secondary photons. The calculation results confirm the importance of fluctuations of the absorbed energy of the registered photon for the small size of the scintillation detectors. The model can be easily adapted to the panel X-ray detectors.

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