

## Mathematical modelling of convective processes in a weld pool under electric arc surfacing

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**Abstract.** The authors develop the mathematical model of convective processes in a molten pool under electric arc surfacing with flux-cored wire. The model is based on the ideas of how convective flows appear due to temperature gradient and action of electromagnetic forces. Influence of alloying elements in the molten metal was modeled as a non-linear dependence of surface tension upon temperature. Surface tension and its temperature coefficient were calculated according to the electron density functional method with consideration to asymmetric electron distribution at the interface “molten metal / shielding gas”. Simultaneous solution of Navier-Stokes and Maxwell equations according to finite elements method with consideration to the moving heat source at the interface showed that there is a multi-vortex structure in the molten metal. This structure gives rise to a downward heat flux which, at the stage of heating, moves from the centre of the pool and stirs it full width. At the cooling stage this flux moves towards the centre of the pool and a single vortex is formed near the symmetry centre. This flux penetration is ~ 10 mm. Formation of the downward heat flux is determined by sign reversal of the temperature coefficient of surface tension due to the presence of alloying elements.

### 1. Introduction

At present time the method of arc surfacing with flux-cored wire is gaining popularity for producing wear-resistance coating [1]. It allows producing bimetallic products characterized by high strength and low production cost combined with long service life. The main part of the product can be manufactured from low-alloy steels. Application of this method for repairing worn-out parts allows reducing the amount of spare parts produced for the operated equipment, reducing the breakdown time during the repairs, reducing maintenance costs. On the other hand, in the process of arc surfacing, in spite of the simple construction of the applied units and their high productivity, coarsely dispersed non-uniform structures with low impact strength are formed in the surface layer, quality of the surface requires its further machining [2]. This determines the necessity of searching for such surfacing conditions that ensure forming a structure with high physical-mechanic properties [3]. To achieve this goal we have to understand the hydrodynamic processes taking place in the molten pool. The specific features of these processes will determine the geometry of the surfaced layer, its structure, phase composition and mechanic properties [3]. The hydrodynamic processes in the molten pool under arc surfacing are studied in many works, [4 – 9] to be mentioned among them. In the given works the

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authors complete a computational investigation of heat exchange in the liquid and liquid flowing under pulse-arc welding and direct current welding. In [4] the authors come to the conclusion that surface-tension gradient has the most significant impact upon the weld pool geometry while other factors, such as magnetic conductivity, thermal expansion coefficient of the material are less important. In [5] the authors obtain the results which demonstrate that application of pulse current results in a more qualitative weld joint. Thermocapillary convection in the weld pool was studied in [6 – 8]. In [6] the authors conducted the computational investigation of Marangoni effect influence upon the parameters of various metals under closed arc welding. As a result the current intensity ranges under which the thermocapillary convection plays the dominating role were established. Further increase of current intensity does not result in increase of penetration depth, thus, the linear dependence of surface tension coefficient of pure molten metals upon the temperature deteriorates the penetrating ability of the electric arc. In [7] it is shown that, for pure metals, thermocapillary convection plays an important role only at the initial stages of metal melting and then gravity-induced deformation of liquid becomes the source of convective flows. On the other hand study of the concentrated energy flows action upon metals shows that heat-gravitational convection is insignificant under the great values of power density [8]. It is also necessary to take the action of alloying elements into consideration [8]. In [9] the researchers study influence of these elements upon surface tension of liquid iron. It is shown that addition of sulfur results in surface tension increase as temperature grows and its further decrease, i.e. there is a non-linear thermal dependence. If sulfur is introduced directly into the molten pool it leads to tension growth, so the pattern of vortex flows will change significantly. The substances involved into vortex flow will move to the centre, then downward, to the bottom of the pool. The downward flow of hot metal in the centre of the pool works like a thermal drill bit. This results in penetration depth increase and pool geometry change. In [10] the scientists suggest the mathematical model of pulse radiation impact with consideration to the dependence of surface tension upon the superficially active substance in the molten metal. The numerical modeling results showed that in the presence of superficially active substance multi-vortex structure of flows is formed ensuring more uniform distribution of alloying component.

Thus, while developing the model of convective flows in the molten pool it is important to consider action of alloying elements upon the hydrodynamic flow. It is relevant for studying the convective flows in the weld pool when surfacing with flux-cored wires. The action of the alloying elements is modeled by the dependence of surface tension coefficient upon the temperature. The alloying elements in the composition of flux change the character of this dependence which results in alteration of the convective flow pattern and this, in its turn, effects the structure-phase composition of the surfaced metal. That is why the aim of our work is developing the mathematical model of convective flows which take these elements into consideration.

## 2. Problem formulation

It is relevant to start modeling of convective flows from determining the volume forces effecting the liquid as well as from calculating the electromagnetic fields. The volume force effecting the liquid comprises the gravity and the electromagnetic forces.

$$\vec{F} = \vec{F}_g + \vec{j} \times \vec{B} = \rho_0 \vec{g} - \rho_0 \vec{g} \beta (T - T_{ref}) + \vec{j} \times \vec{B} \quad (1)$$

Electromagnetic fields can be calculated according to Maxwell-Lorentz equation

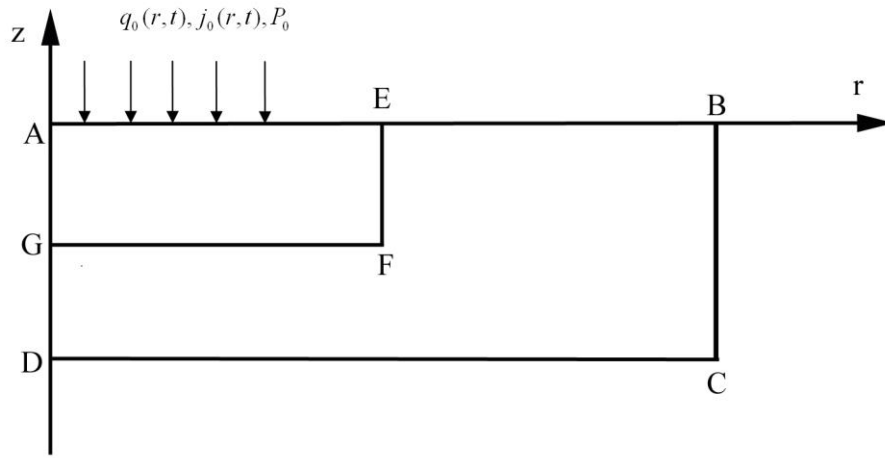
$$\begin{aligned} \nabla \cdot \left( \sigma \nabla V + \sigma \frac{\partial \vec{A}}{\partial t} \right) &= 0, \quad \sigma \frac{\partial \vec{A}}{\partial t} + \frac{1}{\mu_0} \nabla \times (\nabla \times \vec{A}) + \sigma \nabla V \\ \vec{E} &= -\nabla V - \frac{\partial \vec{A}}{\partial t}, \quad \vec{j} = \sigma \vec{E}, \quad \vec{B} = \nabla \times \vec{A} \end{aligned} \quad (2)$$

With consideration to (1), (2) let us put down Navier-Stokes and thermal conductivity equations

$$\begin{aligned}\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \nabla \vec{u} \right) &= -\nabla p + \mu \Delta \vec{u} + \vec{F}_V, \nabla \cdot \vec{u} = 0, \\ \rho C_p \left( \frac{\partial T}{\partial t} + \vec{u} \nabla T \right) &= \nabla \cdot (k \nabla T) + Q_V\end{aligned}\quad (3)$$

where  $\vec{u}$  - velocity vector,  $p$  - pressure,  $\rho$  - density,  $\mu$  - dynamic viscosity,  $\vec{F}_V$  - volume forces which are given by the total of Lorentz force and gravity force,  $T$  - temperature,  $C_p$  - specific heat capacity,  $k$  - thermal conductivity coefficient,  $Q_V$  - volume heat sources. For cathode and anode only Joule effect is the volume heat source  $Q_V = \vec{j} \cdot \vec{E}$ .

Let us consider the action of electric arc plasma upon a steel plate (Figure 1).



**Figure 1.** Computational region

On the top surface AB we set density of the current, pressure and heat flow which result from plasma action upon the metal surface like in [3, 5, 11].

$$\begin{aligned}-\vec{n} \cdot \vec{q} &= q_0(r) = \frac{dU\eta}{\pi r_0^2} G(r, t), \quad -\vec{n} \cdot \vec{j} = j_0(r) = \frac{dI}{\pi r_0^2} G(r, t) \\ p &= p_0(r) = \frac{\mu I}{4\pi} j_0(r)\end{aligned}\quad (3)$$

where  $q_0, j_0, p_0$  - heat flow, density of current, pressure accordingly;  $U, I$  - напряжение voltage and intensity of current;  $\eta$  - process efficiency coefficient;  $G(r, t)$  - function of surface and time distribution of the action:

$$G(r, t) = \exp \left( -\frac{d(r^2 + (r_0 - V_e t)^2)}{r_0^2} \right) \quad (4)$$

$V_e$  - electrode motion speed,  $d, r_0$  - parameters of distribution. On the bottom surface CD we set the condition of convective heat transfer and electromagnetic continuity:

$$-\vec{q} \cdot \vec{n} = h_0(T - T_0), \quad \vec{j} \cdot \vec{n} = 0 \quad (5)$$

On the side surface BC we set the condition of convective heat transfer and scalar electric potential equaling zero:

$$-\vec{q} \cdot \vec{n} = h_0(T - T_0), V = 0 \quad (6)$$

For the magnetic field we set the condition of continuity at all boundaries:  $\vec{n} \times \vec{A} = 0$ .  
At the boundaries EF and FG the slip condition was fulfilled for the speed.

The model of phase transition from the solid into the liquid state was based on the idea of dynamic viscosity coefficient growth in the solid region. It looked as follows:

$$\mu = \mu_L(T)(1 - f_s) + \mu_s f_s \quad (7)$$

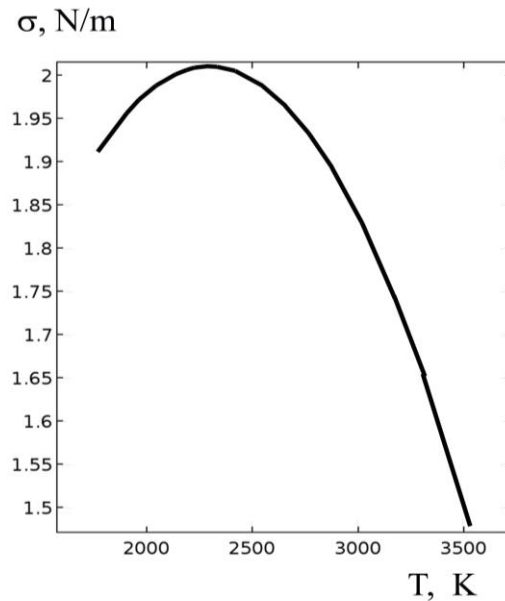
where  $f_s$  - the coefficient determining the solid phase,  $\mu_L$  - viscosity of the liquid phase,  $\mu_s$  - viscosity of the solid phase large enough to stop motion in the solid phase.

The calculations were completed according to finite elements method in the pack Comsol Multiphysics. In Table 1 we provide the characteristics of the material and treatment conditions.

**Table 1.** The input parameters of the problem

Symbol	Description	Value
$U$	Voltage	30V
$I$	Intensity of current	250 A
$r_0$	Effective action radius of the plasma jet	10mm
$T_l$	Temperature of the liquid phase	1723K
$\rho_s$	Density of the solid phase	7500kg m <sup>-3</sup>
$\rho_l$	density of the liquid phase	6350kg m <sup>-3</sup>
$C_{pl}$	Heat capacity ratio of the liquid phase	720 J kg <sup>-1</sup> K <sup>-1</sup>
$C_{ps}$	Heat capacity ratio of the solid phase	602 J kg <sup>-1</sup> K <sup>-1</sup>
$k_l$	Thermal conductivity coefficient	20 W m <sup>-1</sup> K <sup>-1</sup>
$k_s$	Thermal conductivity coefficient	26 W m <sup>-1</sup> K <sup>-1</sup>
$\sigma$	Electric conductivity of metal	$7.7 \times 10^5 \Omega^{-1} \text{m}^{-1}$
$\mu$	Dynamic viscosity coefficient	0.05 kg m <sup>-1</sup> s <sup>-1</sup>
$L_f$	Melting heat	$247 \times 10^3 \text{ J kg}^{-1}$

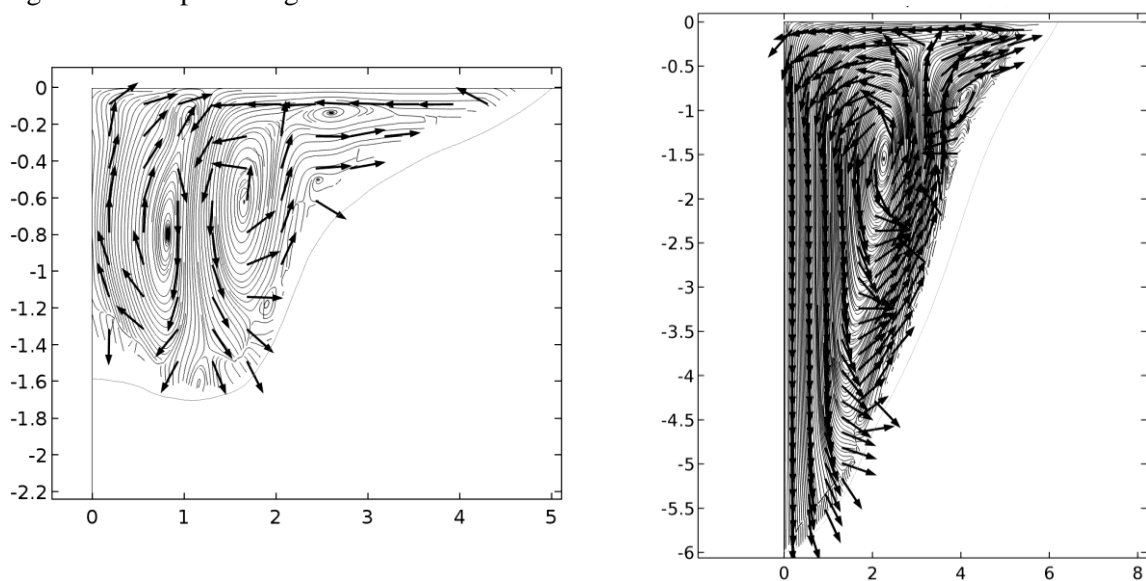
Surface tension and its temperature coefficient were calculated according to the methods described in [12, 13]. The method of electron density functional was chosen as the basic method. The functions which take into consideration the asymmetrical distribution of electrons near the surface were taken as testing functions [13]. The possible combinations of iron and second element in the surfaced material were considered. They were determined according to the constitution diagrams. In Figure 2 we show the dependence of surface tension which looks like a parabola.



**Figure 2.** Dependence of surface tension upon temperature

### 3. Results and discussion

It is known that the result in the molten pool is determined by marangoni convection which is characterized by the temperature gradient of surface tension  $d\sigma/dT$ . In Figure 2 we can see that  $d\sigma/dT > 0$  under the temperatures below critical  $T < T_c = 2300\text{K}$  and  $d\sigma/dT < 0$  for  $T > T_c$ . In Figure 3 the evolution of molten metal flow is shown at various times. We can see that before the heat flow reaches its peak values the downward flux formed by two symmetrical vortices moves from the centre of the pool stirring the molten metal full-width. When the heat action of the plasma jet decreases the cooling stage starts characterized by movement of the downward flux towards the centre of the pool and further formation of the single vortex near the centre of symmetry (Figure 3b). This flux penetrates the depth of 6mm. Formation of the given downward flux can be explained by the sign change of the temperature gradient of surface tension  $\partial\sigma/\partial T$ .



**Figure 3.** The lines of current of the molten metal and the field of velocities at various times:  
 a)  $t=2.7\text{ s}$ ; b)  $t=5.3\text{ s}$

#### 4. Conclusion

We completed numerical modeling of the hydrodynamic flow in the molten pool under the action of electric arc discharge with consideration to Marangoni effect. Presence of alloying elements in the metal has a significant impact upon the dependence of surface tension on temperature which, in its turn, determines the pattern of convective flow of the molten metal.

#### 5. Acknowledgements

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#### References

- [1] Zahiri R, Sundaramoorthy R, Lysz P, Subramanian C, 2014 *Surface & Coatings Technology* **260** 220
- [2] Berger L-M 2015 *Int. Journal of Refractory Metals and Hard Materials* **49** 350
- [3] Traidia A, Roger F, Guyot E, Schroeder J, Lubineau G, 2012 *International Journal of Heat and Mass Transfer* **55** 3946
- [4] Tong L G, Gu J C, Wang L, Yin S W 2015 *International Journal of Heat Mass Transfer* **90** 968
- [5] Traidia A, Roger F 2011 *International Journal of Heat Mass Transfer* **54** 2163
- [6] Sultangazieva R, Amankulova N 2015 *Vestnik KGUSTA* **2** 78
- [7] Kim Y, Hossain A and Nakamura Y 2013 *Journal of Thermal Science and Technology* **8** 136
- [8] Volkov N B, Leivi A Ya, Talala K A and Yalovets A P 2010 *Technical Physics* **55** 484
- [9] DebRoy T 2015 *Welding Journal* **94** 58
- [10] Popov V N, Kovalev O B, Smirnova E M and Tsivinskaya Yu S 2012 *Vestnik NSU* **7** 114
- [11] Wu C S, Zhao P C and Zhang Y M 2004 *Welding Journal* **83** 330
- [12] Toshihiro T 2014 Surface tension models Treatise on process metallurgy (Process Phenomena vol 2) ed S Seetharaman (Amsterdam: Elsevier) chapter **1.4** 35–59
- [13] Kanchukoev V Z, Kashezhev A Z, Mambetov A Kh and Sozaev V A 2002 *Technical Physics Letters* **28** 515