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Determination of stress concentration coefficients upon cyclic loading by photometric analysis of structure images

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Abstract. The development of an experimental method for the determination of stress concentration coefficient by the photometric analysis of surface reflectivity of material near stress concentrator and at a distance from it is considered in the paper. The experiments were performed with the AD-19 aluminum alloy samples under fatigue test conditions realized at a vibrating table with cantilevered sample. Notches 5 mm long and 0.4 mm wide spaced at constant intervals were applied by electrical discharge along one edge of the samples. The reflections of light from the fragments in the vicinity of the stress concentrator and from the opposite edge of the sample were compared with the help of photometric image analyzer. It is shown that such analysis can be used to determine the stress concentration coefficients.

1. Introduction

In metal structures, the presence of local cuts, notches, and grooves as well as the abrupt thickness differentials in the joints of metal sheets differing in thickness results in a significant reduction in structural strength. All they cause local overstresses in the metal and act as stress concentrators, which can lead to premature failure upon operation. The extent of their danger to operational capability of structures is characterized by stress concentration coefficient (K), which is the ratio of stress in the vicinity of the stress concentrator (σ_s) to the stress in a close position, in which its effect is negligible (σ_n):

$$K = \frac{\sigma_c}{\sigma_n} \tag{1}$$

However, Eq. (1) can be used if the experimental data obtained by strain gauge measurements are available. The theoretical estimates of the stress concentration coefficients were fulfilled within the frameworks of the theory of elasticity and allow for the geometry and mechanical properties of the material [1-3]. As a rule, the real material in such estimates is represented by solid structure less isotropic medium whose properties do not change upon plastic deformation. For this reason, the development of an experimental method for determining the stress concentration coefficient (K) from the data obtained by optical measurements of structure elements before and after loading is actual and important.

2. Experimental
The experimental part of the work was performed by fatigue tests of D19 aluminum alloy samples. Its chemical composition is given in Table 1.

Table 1. Chemical composition of the D19 alloy, mass. %.

Al	Cu	Mg	Mn	Be
base	3.6-4.3	1.7-2.3	0.5-1	0.0002-0.005

The samples were cut from a sheet 1.80 mm thick. The samples were 20 mm in width and 223 mm in total length. A thin photometric coating was applied on the sample surface, which before and after the test was examined by photometric analysis of structure images (PHASI). The sample is shown in Figure 1.

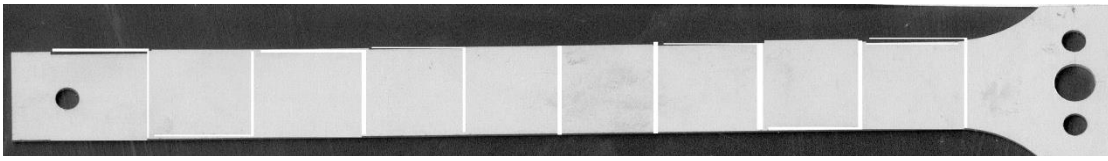

Figure 1. General view of the sample for fatigue tests.

Four notches 5 mm long at intervals of 41 mm were applied by electroerosion method at one end of the sample. The first cut was applied at a distance of 53 mm from the sample edge at the head side. The samples were tested on a vibrating table with electrodynamic excitation. They were attached to the vibrating table rod in a cantilever mode. The sample was excited by polyharmonic vibrations at the frequencies of the first three self-oscillation harmonics. The data on the vibration mode excited in the samples are given in Table 2. Letter A denoted the oscillation amplitude measured at the free end of the sample, and v means oscillation frequency.

Table 2. Loading modes of the samples.

Sample No.	Mode of load fluctuations					
	Mode I		Mode II		Mode III	
	$2A_I$, mm	ν_I, Hz	$2A_{II}$, mm	ν_{II}, Hz	$2A_{III}$, mm	ν_{III}, Hz
1-к	40	32-31	10	198-197	3.5	550
2-к	40	32-31	10	198-197	3.5	550
3-к	57	31.5-29	-	-	-	-

The effective stresses were calculated by the formulas of strength of materials [4] as a superposition of three bending stresses from three loading modes. Loading of the sample by alternating stresses according to the cantilever bending scheme leads to the formation of distributed loads with the intensity q determined by Eq. (2). Equation (3) was obtained from Eq. (2) of a flexible line describing the dependence of deflections, which are on the horizontal axis of the sample, on the coordinate z:

$$y(z) = \frac{q}{EJ_x}\left(\frac{lz^3}{6} - \frac{z^4}{24}\right) \tag{2}$$

where: q is the intensity of the distributed load, E is the normal module of the sample material, l is the sample length, J_x is the moment of inertia of the sample relative to the horizontal axis of symmetry of

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the sample cross section. Therefore, the expression for determining the intensity of a distributed inertial load q takes the form:

$$q = \frac{yE\,J_x}{\left(\frac{lz^3}{6} - \frac{z^4}{24}\right)} \tag{3}$$

In general, the effective stresses in each cross section of the sample can be expressed by the formula:

$$\sigma(z,t) = \sum_{i=1}^{i=3} \sigma_i(z) \cdot \sin\,(2\pi\nu_i t) \tag{4}$$

After the determination of $q(z)$ by Eq. (3) and using the data from Table 2, we have found the distribution of bending moments over the sample length for the loading scheme used in the experiment:

$$M(z) = \frac{q}{2} \cdot (l^2 - z^2) \tag{5}$$

Then the amplitude distribution along the sample length is expressed by the formula:

$$\sigma(z) = \frac{M(z)}{W} = \frac{q \cdot (l^2 - z^2)}{2W} \tag{6}$$

where: W is the moment resistance to be determined by the formula:

$$W = \frac{bh^2}{6} \tag{7}$$

According to the results of the calculations by the above method, the resultant stresses operating in a sample were determined as a function of time and location of cross section in the samples. Figure 2 shows the time dependence of the resulting stresses at the time interval corresponding to 5 oscillation periods of the i-th harmonic for self oscillations of the samples of selected shapes and sizes.

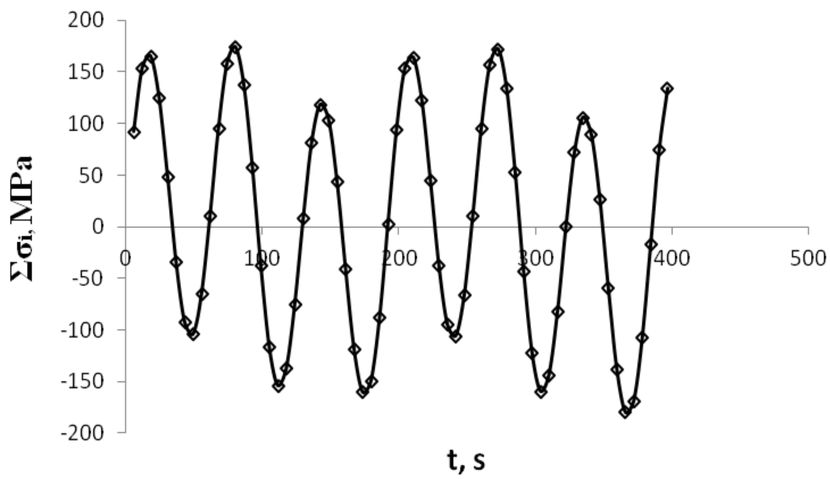

Figure 2. Time dependence of the resulting stresses in the sample upon fatigue test.

Polyharmonic loading at the selected oscillation parameters insignificantly changed the sinusoidal character of the resulting load. It is seen that the amplitude of the resulting load is variable. Its maximum value of 149.76 MPa was used for the calculations. Figure 3 shows the effective stress distribution of the over the sample length. Hereafter, we are interested in the stress in the cross sections weakened by the notch reducing its moment resistance by a factor of 1.33. The sample fragments containing notches were selected for the photometric analysis of structural images (PHASI) taken before and after the fatigue tests up to failure of the samples.

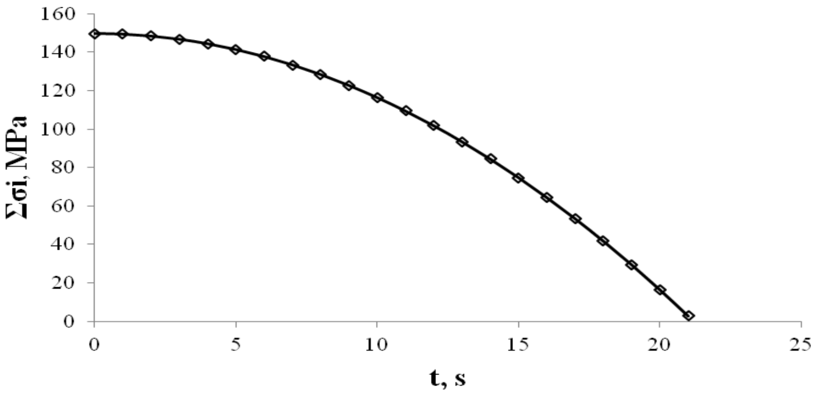

Figure 3. Distribution of the maximum effective stress over the sample length upon fatigue test.

In the selected fragments, the zones adjacent to the notch were cut off, and the remainder of the fragment was divided into two equal parts as shown in Figure 4. The PHASI is a software-analytical complex that analyzes the brightness spectra of the visible light reflection from the surface fragments examined before and after the actions of different physical nature. The areas under the spectral curves in physical sense represent the light energy reflected from the test fragment in arbitrary units. The difference in the areas under the spectral curves obtained from each fragment is determined by the intensity of external action (in our case, by the effective stress [4, 5]). According to [6], this energy (*U*) is related to the internal energy of the object *(E*) by the formula:

$$U = A \cdot E \tag{8}$$

where: *A* is the coefficient characterizing the probability of spontaneous emission of radiant energy by the solid body. The difference in the areas under the spectral curves recorded from each notched fragment before and after fatigue test in arbitrary units is related to the elastic energy of the corresponding micro fragments by Eq. (9):

$$A_i = \frac{U_i}{E_i} \tag{9}$$

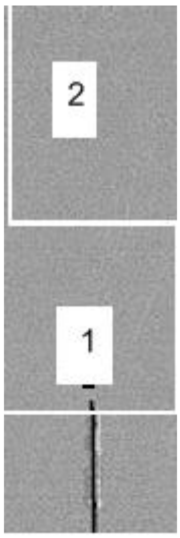

Figure 4. Scheme of the division of the sample fragment with a notch for PHASI: (1) micro fragment closest to the notch and (2) notch-free fragment near the sample edge.

3. Results and Discussion

The areas under the spectral curves have been identified by PHASI for all micro fragments of types 1 and 2. For each micro fragment, the specific surface energy E_i was calculated by Eq. (10):

$$E_i = \frac{\sigma_i^2}{2E} \tag{10}$$

Then the A_i values were calculated by Eq. (9) and the $A_i = f(\sigma)$ dependence was constructed (Figure 5).

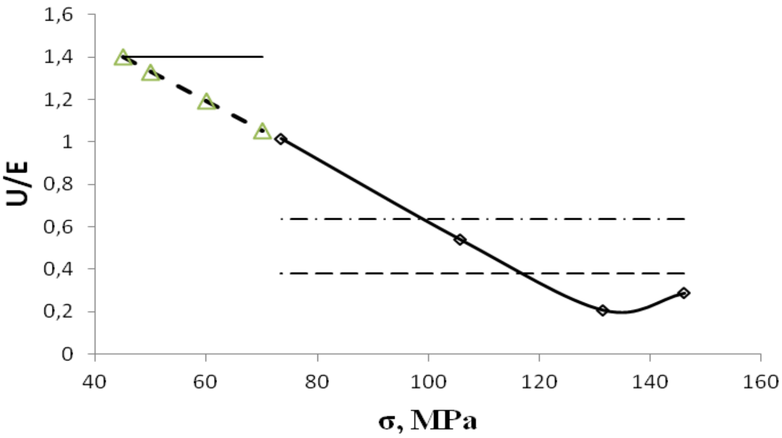

Figure 5. Dependence of the size coefficient A_i on stress σ_i for the fragments with notches.

The horizontal lines in Figure 5 are drawn through the A_i values corresponding to micro fragments of type 2. It is clear that the nominal stresses in the fragments of both types (1 and 2) are the same and they are calculated with allowance for the weakening of cross sections by the notches. For this reason, the difference between the A_i values of these micro fragments can be explained by the effect of stress concentrators in the immediate vicinity of the micro fragments of type 1. Then K can be defined as the relationship between the stresses corresponding to the A_i values for micro fragments 1 and 2 of the same cross section containing the notch, i.e.,

$$K = \frac{\sigma_{1i}}{\sigma_{2i}} \tag{11}$$

The obtained K values are given in Table 3.

Table 3. Stress concentration coefficient determined from the PHASI data on the sample tested for fatigue.

No. of cross section with a notch	σ, MPa	K without allowance for weakening the cross section	K with allowance for weakening the cross section
1	190.77	1.28	1.70
2	169.23	1.31	1.74
4	132.88	2.84	3.79

The last column of Table 3 shows the *K* values determined with allowance for weakening the cross section by the notch. The data show that, with increasing stress, *K* tends to decrease. This can be caused by the development of local plastic deformations at the top of the notch.

4. Conclusions

1. The developed method for evaluating stress concentration coefficient *K* is based on the measurements of the brightness of the visible light reflection from the objects under study.
2. The method is suitable for the study of *K* upon the development of plastic deformation at the notch tip.
3. The development of plastic deformation at the stress concentrator tip leads to the reduction of stress concentration coefficient *K*.

Acknowledgments
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