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Mean-variance portfolio optimization by using time series approaches based on logarithmic utility function

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Abstract. Investments in stocks investors are also faced with the issue of risk, due to daily price of stock also fluctuate. For minimize the level of risk, investors usually forming an investment portfolio. Establishment of a portfolio consisting of several stocks are intended to get the optimal composition of the investment portfolio. This paper discussed about optimizing investment portfolio of Mean-Variance to stocks by using mean and volatility is not constant based on logarithmic utility function. Non constant mean analysed using models Autoregressive Moving Average (ARMA), while non constant volatility models are analysed using the Generalized Autoregressive Conditional heteroscedastic (GARCH). Optimization process is performed by using the Lagrangian multiplier technique. As a numerical illustration, the method is used to analyse some Islamic stocks in Indonesia. The expected result is to get the proportion of investment in each Islamic stock analysed.

1. Introduction

Investment is basically invest some capital into some form of instrument (asset), can be either fixed assets or financial assets. Investing in financial assets can generally be done by buying shares in the stock market. Investing in stocks, investors will be exposed to the risk that the magnitude of the problem along with the magnitude of the expected return [4]. The greater the expected return, generally the greater the risk to be faced. Investment risk is describing rise and fall stock price changes at any time can be measured by the value of variance [9].

The strategy is often used by investors in the face of the risks of investing is to form an investment portfolio. Establishment of an investment portfolio is essentially allocates capital in a few selected stocks, or often referred to diversify investments [5]. The purpose of the establishment of the investment portfolio is to get a certain return with minimum risk levels, or to get maximum returns with limited risk. To achieve these objectives, the investor is deemed necessary to conduct analysis of optimal portfolio selection. Analysis of portfolio selection can be done with optimum investment portfolio optimization techniques [8].

Therefore, this paper studied the paper on portfolio optimization model of Mean-Variance, where the average (mean) and volatility (variance) assumed the value is not constant, which is analyzed using time series model approach (time series). Non constant mean analyzed using models Autoregressive Moving Average (ARMA), whereas non constant volatility analyzed using models of the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) [8]. Methods such analysis is then used to analyze a stock in Indonesia. The purpose of this analysis is to obtain the proportion of investment capital allocation in some stocks are analyzed, which can provide a maximum return with a certain level of risk.

2. Methodology

In this section will discuss the stages of analysis includes the calculation of stock returns, mean modeling, volatility modeling and portfolio optimization.

2.1. Stocks Return

Suppose P_{it} stock price i at time t, and r_{it} stock return i at time t. The value of r_{it} can be calculated using the following equation.

$$r_{it} = \ln\left(\frac{P_{it}}{P_{it-1}}\right) \tag{1}$$

where $i = 1,...,N$ with N number of stocks that were analyzed, and $t = 1,...,T$ with T the number of stock price data observed ([9] , [10]).

2.2. Mean Models

Suppose $\{r_{it}\}$ is stock return i at time t are stationary, will follow the model of ARMA(p,q) if for every t have the following equation:

$$r_{it} - \phi_1 r_{it-1} - ... - \phi_p r_{it-p} = a_{it} + \theta_1 a_{it-1} + ... + \theta_q a_{it-q},$$

or can be written as

$$r_{it} = \phi_1 r_{it-1} + ... + \phi_p r_{it-p} + a_{it} + \theta_1 a_{it-1} + ... + \theta_q a_{it-q}, \tag{2}$$

where $\{a_{it}\}$ ~WN$(0,\sigma_i^2)$, which means sequence of residual $\{a_{it}\}$ normally distributed white noise with mean 0 and variance σ_i^2. Sequence $\{r_{it}\}$ is a model ARMA(p,q) with mean μ_{it}, if $\{r_{ti} - \mu_{it}\}$ is a model ARMA(p,q) ([3], [7]).

Stages of the process modeling the mean include: (i) identification of the model, (ii) parameter estimation, (iii) diagnostic test, and (iv) prediction [10].

2.3. Volatility Models

Volatility models in time series data in general can be analyzed using GARCH models. Suppose $\{r_{it}\}$ is Islamic stock returns i at time t is stationary, the residuals of the mean model for Islamic stock i at time t is $a_{it} = r_{tt} - \mu_{it}$. Residual sequence $\{a_{it}\}$ follows the model GARCH(g,s) when for each has the following equation:

$$a_{it} = \sigma_{it}\varepsilon_{it},\ \sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^{g}\alpha_{ik} a_{it-k}^2 + \sum_{j=1}^{s}\beta_{ij}\sigma_{it-j}^2 + \varepsilon_{it}, \tag{3}$$

with $\{\varepsilon_{it}\}$ is a sequence of residual volatility models, namely the sequence of random variables are independent and identically distributed (IID) with mean 0 and variance 1. Parameter coefficients satisfy the property that $\alpha_{i0} > 0$, $\alpha_{ik} \geq 0$, $\beta_{ij} \geq 0$, and $\sum_{k=1}^{\max(g,s)} \alpha_{ik} + \beta_{ij} < 1$ ([8], [10]).

Volatility modeling process steps include: (i) the estimated mean model, (ii) test of ARCH effects, (iii) identification of the model, (iv) the estimated volatility models, (v) test of diagnosis, and (vi) prediction [10].

2.4. Prediction of l –Step Ahead

Using the mean and volatility models, aiming to calculate the prediction of mean $\hat{\mu}_{it} = \hat{r}_{ih}(l)$ and volatility $\hat{\sigma}_{it}^2 = \hat{\sigma}_{ih}^2(l)$, for l-period ahead of the starting point prediction h ([1], [10]). The prediction results of mean $\hat{\mu}_t = \hat{r}_{ih}(l)$ and volatility $\hat{\sigma}_{it}^2 = \hat{\sigma}_{ih}^2(l)$, will then be used for portfolio optimization calculations below.

2.5. *Risk Tolerance*

Each individual investor generally has a curve equation or utility functions that vary according to the preferences of its attitude towards an investment risk. Based on the utility function which is owned by an investor, we can determine the risk aversion function and risk tolerance function. Here is discussed a form of utility function in order to determine the function risk aversion and risk tolerance function. Suppose W stated the assets (funds) held in order to make an investment. Suppose that one investor has a logarithmic utility function form, the equation is as follows (Norstad, 2011):

$U(W) = \log_e^{(k+W)}$; with the provision $W > k$ and constant $k > 0$.

The first and second derivatives of the function of this utility are as follows:

$U'(W) = \frac{1}{k+W} = (k+W)^{-1} > 0$ and $U''(W) = -\frac{1}{(k+W)^2} < 0$.

Thus the function of risk aversion R_R can be determined as follows:

$$R_R(W) = -\frac{U''(W)}{U'(W)} = -\frac{(k+W)^{-1}}{-\frac{1}{(k+W)^2}} = \frac{1}{k+W}$$

and the function of risk tolerance τ is a constant function shaped as:

$$\tau(W) = \frac{1}{R_R(W)} = \frac{1}{\frac{1}{k+W}} = k + W. \quad (4)$$

2.6. *Portfolio Optimization Model*

Suppose r_{it} Islamic stock return i at time t, where $i = 1,...,N$ with N the number of stocks that were analyzed, and $t = 1,...,T$ with T the number of Islamic stock price data observed. Suppose also $\mathbf{w}' = (w_1,...,w_N)$ weight vector, $\mathbf{r}' = (r_{1t},...,r_{Nt})$ vector stock returns, and $\mathbf{e}' = (1,...,1)$ unit vector. Portfolio return can be expressed as $r_p = \mathbf{w}'\mathbf{r}$ with $\mathbf{w}'\mathbf{e} = 1$ ([5], [12]). Suppose $\boldsymbol{\mu}' = (\mu_{1t},...,\mu_{Nt})$, expectations of portfolio μ_p can be expressed as:

$$\mu_p = E[r_p] = \mathbf{w}'\boldsymbol{\mu}. \quad (5)$$

Suppose given covariance matrix $\boldsymbol{\Sigma} = (\sigma_{ij})_{i,j=1,...,N}$, where $\sigma_{ij} = Cov(r_{it}, r_{jt})$. Variance of the portfolio return can be expressed as follows:

$$\sigma_p^2 = \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}. \quad (6)$$

Definition 1. *A portfolio p^* called* (mean-variance) *efficient if there is no portfolio p with* $\mu_p \geq \mu_{p^*}$ and $\sigma_p^2 < \sigma_{p^*}^2$ [5].

To get efficient portfolio, typically using an objective function to maximize

$2\tau\mu_p - \sigma_p^2$, $\tau \geq 0$

where the parameters of the investor's risk tolerance. Means, for investors with risk tolerance τ $(\tau \geq 0)$ need to resolve the problem of portfolio

$$\text{Maximize}\{2\tau\mathbf{w}'\boldsymbol{\mu} - \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\} \tag{7}$$

The condition $\mathbf{w}'\mathbf{e} = 1$

Please note that the completion of (6), for all $\tau \in [0, \infty)$ form a complete set of efficient portfolios. Set of all points in the diagram-(μ_p, σ_p^2) related to efficient portfolio so-called efficient surface (efficient frontier) ([2], [11]).

Equation (6) is the optimization problem of quadratic convex [5]. Lagrangian multiplier function of the portfolio optimization problem is given by

$$L(\mathbf{w}, \lambda) = 2\tau\mathbf{w}'\boldsymbol{\mu} - \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} + \lambda(\mathbf{w}'\mathbf{e} - 1) \tag{8}$$

Based on the Kuhn-Tucker theorem, the optimality condition of equation (7) is $\frac{\partial L}{\partial \mathbf{w}} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$. Completed two conditions of optimality mentioned equation, the equation would be the optimal portfolio weights as follows:

$$\mathbf{w}^* = \frac{1}{\mathbf{e}'\boldsymbol{\Sigma}^{-1}\mathbf{e}}\boldsymbol{\Sigma}^{-1}\mathbf{e} + \tau\{\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \frac{\mathbf{e}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}'\boldsymbol{\Sigma}^{-1}\mathbf{e}}\boldsymbol{\Sigma}^{-1}\mathbf{e}\} \tag{9}$$

Furthermore, with substituting $\mathbf{w}^*$ into equation (4) and (5), respectively obtained the values of the expectation and variance of the portfolio [9]. As a numerical illustration, will be analyzed some stocks as the following.

3. Illustrations

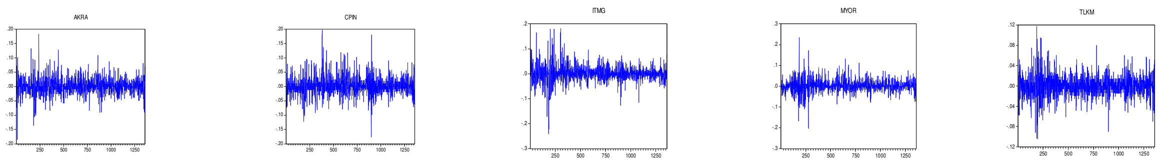
In this section will discuss the application of the method and the results of the analysis stage of the observation that includes stocks data, the calculation of Islamic stock returns, modelling the mean of stocks, volatility modelling, prediction of the mean and variance values, and the process of optimization. In this section we explore numerical illustration of estimation of CAPM regression, estimation of distributed lagged CAPM, and estimation of Koyck transformation CAPM.

3.1. Stocks Data

The data used in this study is secondary data, in the form of time series data (time series) of some of the daily stock price, which includes the names of stocks: AKRA, CPIN, ITMG, MYOR, and TLKM. Stock price data, the data used is the closing stock price, for over 1360 days starting from January 1, 2010 till June 30, 2015 were downloaded from www.finance.yahoo.com [6]. Stock prices data will then be processed by using statistical software of Eviews-6 and Matlab.

3.2. Stocks Return and Stationarity

Stock returns of five firms in this study were calculated using equation (1). Figure-1 below indicates the five stock chart returns are analyzed.



AKRA CPIN ITMG MYOR TLKM

Figure-1 FiveStockCharts

It can be seen in Figure-1 that the five stock return data were analyzed have stationary. For stationary testing is done using the ADF test. Statistic results respectively values are -34.24848; -33.79008; -30.20451; -40.04979; and -28.36974. Further, if the specified level of significance $\alpha =$ 5%, can be obtained by a standard normal distribution critical value is -2.863461. It is clear that the value of the test statistic for all ADF of stocks are analyzed located in the rejection region, so that everything is stationary.

3.3. Mean Modelling of Stocks Return

Stock return data will be used to estimate mean model using the software of Eviews-6. First, the identification and estimation of mean model carried out through the sample autocorrelation function (ACF) and partial autocorrelation function (PACF). Based on the pattern of ACF and PACF, tentative models can be specified from each stock returns. The estimation results indicate that the best models are respectively ARMA(1,0), ARMA(1,0), ARMA(1,0), ARMA(7,0), and ARMA(2,0).Second, the significance test for parameters and significance tests for models indicate that the mean model for all stocks analyzed have significance. Third, diagnostic tests for these models are done by using the data residual correlogram and Ljung-Box test hypotheses. The test results show the residuals of the models are white noise. Results residual normality test showed normal distribution. Therefore residuals for all stock has been analyzed are white noise.

Equations of the results mean modeling for the five stocks will be written simultaneously with the volatility models each of stocks, which will be estimated following.

3.4. Volatility Modelling of Stocks Return

Modeling of volatility in this section is done by using statistical software of Eviews-6. First, carried out the detection elements of autoregressive conditional heteroscedasticity (ARCH) to the residual a_t, using the ARCH-LM test statistic. Statistical value of the results obtained χ^2 (obs*R-Square) each of stock returns AKRA, CPIN, ITMG, MYOR, and TLKM respectively are: 31.76757; 76.75431; 40.55526; 48.58576; 9.270781, and 125.2410 by probability each of 12:050.0000 or smaller, which means that there are elements of ARCH.

Second, the identification and estimation of volatility models. This study used models of generalized autoregressive conditional heteroscedasticity (GARCH) which refers to equation (3). Based on squared residuals correlogram a_t^2, the ACF and PACF graphs of each selected model of volatility that might be tentative. Volatility model estimation each of stock return performed simultaneously (synchronously) with mean models. After going through tests of significance for parameters and significance tests for models, all equations are written below have been significant. The best model are respectively:

- Stock of AKRA follow the model ARMA(1,0)-GARCH(1,1) with equation:
  $r_t = 0.073891\ r_{t-1} + a_t$
  and
  $\sigma_t^2 = 0.000014 + 0.040015\alpha_{t-1}^2 + 0.9404318\sigma_{t-1}^2 + \varepsilon_t$
- Stock of CPIN follow the model ARMA(1,0)-GARCH(1,1) with equation:
  $r_t = 0.089639\ r_{t-1} + a_t$
  and
  $\sigma_t^2 = 0.000052 + 0.134049\alpha_{t-1}^2 + 0.820716\sigma_{t-1}^2 + \varepsilon_t$
- Stock of ITMG follow the model ARMA(1,0)-GARCH(1,1) with equation:
  $r_t = 0.193825\ r_{t-1} + a_t$

and

$\sigma_t^2 = 0.000012 + 0.066024\,\alpha_{t-1}^2 + 0.923108\,\sigma_{t-1}^2 + \varepsilon_t$

- Stock of MYOR follow the model ARMA(7,0)-GARCH(1,1) with equation:

  $r_t = 0.102007\, r_{t-7} + a_t$

  and

  $\sigma_t^2 = 0.000009 + 0.044332\,\alpha_{t-1}^2 + 0.945801\,\sigma_{t-1}^2 + \varepsilon_t$

- Stock of TLKM follow the model ARMA(2,0)-GARCH(1,1) with equation:

  $r_t = -0.084289 r_{t-2} + a_t$

  and

  $\sigma_t^2 = 0.000019 + 0.139166\alpha_{t-1}^2 + 0.824540\sigma_{t-1}^2 + \varepsilon_t$

Based on the ARCH-LM test statistics, the residuals of the models for stock AKRA, CPIN, IMTG, MYOR, and TLKM there is no element of ARCH, and also has white noise. Mean and volatility models are then used to calculate the values $\hat{\mu}_t = \hat{r}_t(l)$ and $\hat{\sigma}_t^2 = \sigma_t^2(l)$ recursively.

3.5. Prediction of Mean and Variance Values

Models of mean and volatility estimation results of five stocks in the previous stage, is then used to perform one step ahead prediction for the mean and variance values. Results predicted mean and volatility values are given in Table 1 below.

Table 1. Predictive Values of Mean and Variance One Period Ahead for Each Stocks

Islamic Stocks	Model of Mean-Volatility	Mean ($\hat{\mu}_t$)	Variance ($\hat{\sigma}_t^2$)
AKRA	ARMA(1,0)-GARCH(1,1)	0.001407	0.001200
CPIN	ARMA(1,0)-GARCH(1,1)	0.004919	0.001840
IMTG	ARMA(1,0)-GARCH(1,1)	0.014656	0.001078
MYOR	ARMA(7,0)-GARCH(1,1)	0.004940	0.000956
TLKM	ARMA(2,0)-GARCH(1,1)	0.004406	0.001399

3.6. Mean-Variance Portfolio Optimization

In this part we did the process of portfolio optimization calculations. Portfolio optimization is done with referring to the equation (6). The data used for process optimization are the values of the mean and variance are given in Table 2. Using the values of mean in Table 2, column $\hat{\mu}_t$, used to form the mean vector as: $\boldsymbol{\mu}^T = (0.001407 \quad 0.004919 \quad 0.014656 \quad 0.004940 \quad 0.004406)$, amount of the stock that were analyzed.

Furthermore, by using the values of variance in Table-2, column $\hat{\sigma}_t^2$, and together with the values of the covariance between stocks, used to form the covariance matrix $\boldsymbol{\Sigma}$ and the inverse matrix $\boldsymbol{\Sigma}^{-1}$ as follows:

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.001200 & 0.000136 & 0.000251 & 0.000113 & 0.000401 \\ 0.000136 & 0.001840 & 0.000092 & 0.000315 & 0.000225 \\ 0.000251 & 0.000092 & 0.001078 & 0.000512 & 0.000133 \\ 0.000113 & 0.000315 & 0.000512 & 0.000956 & 0.000075 \\ 0.000401 & 0.000225 & 0.000133 & 0.000075 & 0.001399 \end{pmatrix}$$

and

$$\boldsymbol{\Sigma}^{-1} = 10^3 \times \begin{pmatrix} 0.9613 & -0.0345 & -0.2020 & 0.0257 & -0.2522 \\ -0.0345 & 0.5904 & 0.0738 & -0.2237 & -0.0801 \\ -0.2020 & 0.0738 & 1.3037 & -0.6955 & -0.0406 \\ 0.0257 & -0.2237 & -0.6955 & 1.4880 & 0.0150 \\ -0.2522 & -0.0801 & -0.0406 & 0.0150 & 0.8030 \end{pmatrix}$$

Optimization done in order to determine the composition of the portfolio weights, and thus the portfolio weight vector is determined by using equation (8). The weight vector calculation process, the values of risk tolerance τ determined by the simulation begins value $\tau = 0.000$ with an increase of 0.001. If it is assumed that short sales are not allowed, then the simulation is stopped when the value of τ=0.036, because it has resulted in a portfolio weight at least there is a negative value. The portfolio weights calculation results are given in Table 2.

Table 2. Process of Mean-Variance Portfolio Optimization

τ	w_1	w_2	w_3	w_4	w_5	$\mathbf{w}^T\mathbf{e}$	$\hat{\mu}_p$	$\hat{\sigma}_p^2$	$\hat{\mu}_p - \hat{\sigma}_p^2$	$\hat{\mu}_p / \hat{\sigma}_p^2$
	AKRA	CPIN	IMTG	MYOR	TLKM	Sum	Mean	Variance	Maximum	Ratio
0.000	0.2150	0.1406	0.1895	0.2629	0.1920	1	0.0059	0.00043136	0.00546864	13.7161
0.001	0.2093	0.1411	0.2025	0.2555	0.1916	1	0.0061	0.00043151	0.00566849	14.0507
0.002	0.2036	0.1417	0.2155	0.2480	0.1913	1	0.0062	0.00043195	0.00576805	14.6893
0.003	0.1978	0.1422	0.2285	0.2406	0.1909	1	0.0064	0.00043268	0.00596732	14.6893
0.004	0.1921	0.1428	0.2414	0.2331	0.1905	1	0.0065	0.00043370	0.00606630	14.9921
0.005	0.1864	0.1433	0.2544	0.2257	0.1902	1	0.0066	0.00043502	0.00616498	15.2832
0.006	0.1807	0.1439	0.2674	0.2182	0.1898	1	0.0068	0.00043663	0.00636337	15.5621
0.007	0.1750	0.1444	0.2803	0.2108	0.1894	1	0.0069	0.00043853	0.00646147	15.8284
0.008	0.1693	0.1450	0.2933	0.2033	0.1891	1	0.0071	0.00044073	0.00665927	16.0817
0.009	0.1636	0.1455	0.3063	0.1959	0.1887	1	0.0072	0.00044322	0.00675678	16.3217
0.010	0.1579	0.1461	0.3193	0.1884	0.1883	1	0.0074	0.00044600	0.00695400	16.5481
0.011	0.1522	0.1466	0.3322	0.1810	0.1880	1	0.0075	0.00044907	0.00705093	16.7608
0.012	0.1465	0.1472	0.3452	0.1736	0.1876	1	0.0077	0.00045344	0.00724656	16.9597
0.013	0.1407	0.1477	0.3582	0.1661	0.1872	1	0.0078	0.00045610	0.00734390	17.1445
0.014	0.1350	0.1483	0.3711	0.1587	0.1587	1	0.0080	0.00046005	0.00753995	17.3154
0.015	0.1293	0.1488	0.3841	0.1512	0.1865	1	0.0081	0.00046430	0.00763570	17.4724
0.016	0.1236	0.1494	0.3971	0.1438	0.1461	1	0.0083	0.00046884	0.00783116	17.6155
0.017	0.1179	0.1499	0.4101	0.1363	0.1858	1	0.0084	0.00047367	0.00792633	17.7449
0.018	0.1122	0.1505	0.4230	0.1289	0.1854	1	0.0086	0.00047879	0.00812121	17.8608
0.019	0.1065	0.1510	0.4360	0.1214	0.1850	1	0.0087	0.00048421	0.00821579	17.9633
0.020	0.1008	0.1516	0.4490	0.1140	0.1847	1	0.0088	0.00048992	0.00831008	18.0528
0.021	0.0951	0.1521	0.4619	0.1065	0.1843	1	0.0090	0.00049592	0.00850408	18.1295
0.022	0.0893	0.1527	0.4749	0.0991	0.1840	1	0.0091	0.00050221	0.00859779	18.1937
0.023	0.0836	0.1533	0.4879	0.0916	0.1836	1	0.0093	0.00050880	0.00879120	18.2459
0.024	0.0779	0.1538	0.5009	0.0842	0.1832	1	0.0094	0.00051568	0.00888432	18.2863
0.025	0.0722	0.1544	0.5138	0.0767	0.1829	1	0.0096	0.00052285	0.00907715	18.3154
0.026	0.0665	0.1549	0.5268	0.0693	0.1825	1	0.0097	0.00053032	0.00916968	18.3336
0.027	0.0608	0.1555	0.5398	0.0619	0.1821	1	0.0099	0.00053808	0.00936192	**18.3413**
0.028	0.0551	0.1560	0.5527	0.0544	0.1818	1	0.0100	0.00054613	0.00945387	18.3390
0.029	0.0494	0.1566	0.5657	0.0470	0.1814	1	0.0102	0.00055447	0.00964553	18.3270
0.030	0.0437	0.1571	0.5787	0.0395	0.1810	1	0.0103	0.00056311	0.00973689	18.3059
0.031	0.0380	0.1577	0.5917	0.0321	0.1807	1	0.0105	0.00057204	0.00992796	18.2760
0.032	0.0322	0.1582	0.6046	0.0246	0.1803	1	0.0106	0.00058126	0.01001874	18.2379
0.033	0.0265	0.1588	0.6176	0.0172	0.1799	1	0.0107	0.00059078	0.01010922	18.1919
0.034	0.0208	0.1593	0.6306	0.0097	0.1796	1	0.0109	0.00060059	0.01029941	18.1386
0.035	0.0151	0.1599	0.6435	0.0023	0.1792	1	0.0110	0.00061069	0.01038931	18.0783
0.036	0.0094	0.1604	0.6565	-0.0052	0.1788	1	0.0112	0.00062108	0.01057892	18.0115

Based on the results of the optimization process are given in Table 2, the pair of points ($\hat{\mu}_p$, $\hat{\sigma}_p^2$) efficient portfolio can be formed or the so-called efficient frontier as given in Figure 2a. This graph shows the efficient frontier decent area for investors with different levels of risk tolerance to make an investment. Also by using the optimization process results in Table 2, it can be calculated ratio value $\hat{\mu}_p$ towards $\hat{\sigma}_p^2$ for each level of risk tolerance. The ratio calculation results can be shown as inFigure2.b. This ratio shows the relationship between the optimum portfolio return expected with variance as a measure of risk.

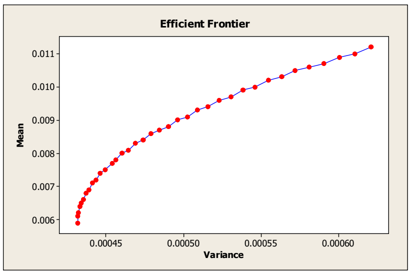


Figure 2a. Efficient Frontier

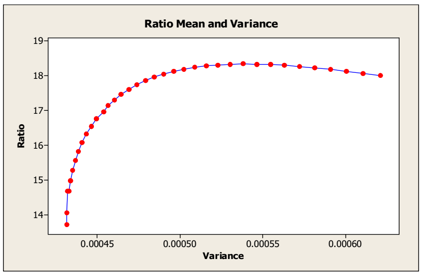


Figure 2b. Ratio of Mean-Variance

Based on the results of the calculation of portfolio optimization, the optimum value is achieved when the value of the portfolio's risk tolerance $\tau = 0.027$. The portfolio produces mean value of $\hat{\mu}_p$ = 0.0099 with the value of risk as the variance $\hat{\sigma}_p^2$ = 0.00053808.

Composition weight of the maximum portfolio respectively are: 0.0608, 0.1555, 0.5398, 0.0619, and 0.1821. This provides reference to investors that invest in stocks of AKRA, CPIN, ITMG, MYOR, and TLKM, in order to achieve the maximum value of the portfolio, the composition of the portfolio weights are as mentioned above.

4. Conclusions

In this paper we analyzed the Mean-Variance portfolio optimization on some stocks by using non-constant mean and volatility models approaches in some stocks traded in the Islamic capital market in Indonesia. The analysis showed that some of stocks which analyzed follow the ARMA(p,q)-GARCH(g,s) models. Whereas, based on the results of the calculation of portfolio optimization, it is produced that the optimum is achieved when the composition of the portfolio investment weights in stocks of AKRA, CPIN, ITMG, MYOR, and TLKM, respectively are 0.0608, 0.1555, 0.5398, 0.0619, and 0.1821. The composition of the portfolio weights thereby will produce a portfolio with mean value of 0.0099 and the value of risk, measured as the variance of 0.00053808.

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