A collaborative vendor-buyer production-inventory systems with imperfect quality items, inspection errors, and stochastic demand under budget capacity constraint: a Karush-Kuhn-Tucker conditions approach

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Abstract. In this paper, we develop an integrated inventory model considering the imperfect quality items, inspection error, controllable lead time, and budget capacity constraint. The imperfect items were uniformly distributed and detected on the screening process. However there are two types of possibilities. The first is type I of inspection error (when a non-defective item classified as defective) and the second is type II of inspection error (when a defective item classified as non-defective). The demand during the lead time is unknown, and it follows the normal distribution. The lead time can be controlled by adding the crashing cost. Furthermore, the existence of the budget capacity constraint is caused by the limited purchasing cost. The purposes of this research are: to modify the integrated vendor and buyer inventory model, to establish the optimal solution using Kuhn-Tucker's conditions, and to apply the models. Based on the result of application and the sensitivity analysis, it can be obtained minimum integrated inventory total cost rather than separated inventory.

1. Introduction

Nowadays, integrated inventory management has enjoyed a great deal of attention. The integration model exists because the inventory replenishment decisions are treated separately from the viewpoints of buyer and vendor based on economic order quantity (EOQ) and economic production quantity (EPQ). The results shows that the optimal EPQ solution for the vendor is not acceptable to the buyer, and vice versa. To identify the efficient solutions, both buyer and vendor should coordinate and collaborate to achieve good inventory management. The supply chain collaborative advantages from a firm's perspective have been showed by Chao and Zhang [5], such as process efficiency, offering flexibility, business synergy, quality and innovation. Goyal [7] was a pioneer in the study of the integrated inventory model. He developed a joint economic lot-sized model for single buyer and single vendor with infinite production rate. Banerjee [3] developed the model by incorporating a finite production rate and following a lot-for-lot policy for the vendor. After that, more researchers considered different cases in integrated inventory model about permissible delay in payments [6], inflation [4], service level constraint [18], price-dependent demand [22], and fuzzy random framework [23]. However, a common unrealistic assumption of the above integrated inventory models is that all items received by a buyer are of good quality.

In a real system, due to the imperfect production process of the vendor, damage in transit, or other unforeseeable circumstance, goods that were received by the buyer often contain some defective items. Further, Goyal et al. [8] and Huang [12] developed integrated vendor-buyer inventory models for items with imperfect quality. They assumed that items of poor quality detected in the screening process of a lot are sold at a discounted price. Several researchers, including Hsu and Hsu [10], Lin [20], Bag and Chakraborty [2], Kurdhi et al. [16], Jindal and Solanki [13] proposed integrated vendor-buyer production-inventory model under imperfect quality items in various conditions. However, the above mentioned papers assumed that there is no error in the inspection process. Khan [14] stated that there are still wastes generated by errors in screening, although there have been advancements in technology. This means that the screening errors may occur with imperfect quality in practice. Moreover, Khan [14] asserted that the inspection accuracy is influenced by some factors. More specifically, these factors can be categorized into three groups: inspector related factors, task related factors and environmental factors. Lin [19] proposed an integrated vendor-buyer model for items with imperfect quality and inspection errors. He suggested two types errors usually could be found in inspection process. These are Type I error, in which the inspector may incorrectly classify a non-defective item as defective; and Type II error, in which a defective item is classified as non-defective. The Lin's [19] model assumed that both the Type I and Type II inspection errors are known constants. Hsu and Hsu [11] then developed a production-inventory model with defective items and inspection errors, where the both type of errors are viewed as random variable. There are more papers related to inspection errors on integrated model such as Widianto et al. [25], Khan et al. [15], Priyan and Uthayakumar [21].

In the integrated inventory management system with both imperfect quality items and inspection errors, the integrated inventory model under deterministic demand to be widely discussed. However, it is noticed that the inventory literature on the integrated inventory model under stochastic demand discussing screening errors is quite sparse. Nearly all integrated models that consider the inspection errors assume no uncertainty in demand and shortages are not allowed. On the other hand, Al-Salamah [1] proposed two EOQ models with stochastic demand, imperfect quality, and inspections errors. In the models, shortages are backordered and demand during lead time has a certain probability density function. However, the Al-Salamah's [1] models focused on determining optimal policy for the buyer only (one-echelon model). These models neglect the opportunity that buyer and vendor can cooperate and negotiate with each other to obtain a better integrated policy. We also notice that in the inventory models under inspection errors, shortages, reorder point, lead time reduction, and budget constraint were not considered. Separately, Priyan and Uthayakumar [5] considered budget capacity constraint on an integrated inventory model. On the model, the buyer has limited capacity to purchase products. On the other hand, there is an upper bound on the purchase of products. According to Kurdhi et al. [16, 17], in many practical situations, lead time can be reduced, by an additional crashing cost, customer service level improved, inventory in safety stocks reduced, and the competitive edge in business increased; in other words, it is controllable.

Summarizing the above description, we seek to investigate a two-echelon production-inventory model by considering shortage backlogging, imperfect quality items, inspection errors, lead time, budget capacity constraint, and stochastic demand. In this study, the percentage of defective item and lead time demand are assumed to follow a uniform distribution and a normal distribution, respectively. It is also assumed that the partial backorder situation is considered. This means that the shortages are partial backordered and partial lost sale with a certain backorder rate. The aim is to minimize the integrated inventory system total cost by optimizing the order quantity, reorder point, lead time, and the number of deliveries per batch production run. This paper continuous with notations and assumptions in Section 2. Section 3 and 4 present model formulation and solution methodology, respectively. A numerical example and sensitivity analysis are discussed in Section 5. Finally, conclusions are given in Section 6.

2. Notations and Assumptions

To develop the proposed model, the following notations and assumptions are introduced.

2.1. Notations

Q_p : size of production batch of items at the vendor
Q : size of the deliveries from the vendor to the buyer (decision variable)
r : reorder point
k : safety factor (decision variable)
n : number of deliveries per batch production run (decision variable)
D : annual demand of the buyer
P : annual production rate at the vendor
y : inspection rate
S_v : setup cost per production run for the vendor
S_B : ordering cost per order for the buyer
γ : probability that an item produced is defective
$f(\gamma)$: probability density function of γ
e_1 : probability of a Type I error (classifying a non-defective item as defective)
e_2 : probability of a Type II error (classifying a defective item as non-defective)
c_s : buyer's inspection cost per unit
c_w : vendor's unit cost for producing a defective item (warranty)
c_{aB} : buyer's cost of a post-sale defective item
c_{av} : vendor's cost of a post-sale defective item
c_a : cost of accepting a defective item ($c_a = c_{aB} + c_{av}$)
c_r : cost of rejecting a non-defective item
h_v : holding cost per unit per year for the vendor
h_B : holding cost per unit per year for the buyer
F : transportation cost per delivery
B_1 : number of items that are classified as defective in each delivery of Q units
B_2 : number of items that are returned from the market in each delivery of Q units
T : time interval between successive deliveries of Q units
T_1 : period during which the vendor produces
T_2 : period during which the vendor supplies from inventory
T_c : cycle time
L : length of lead time for the buyer (decision variable)
$C(L)$: lead time crashing cost
α : buyer's shortage cost (penalty cost)
α_0 : buyer's marginal profit
β : fraction of the demand during the shortage period that will be backordered, $\beta \epsilon [0,1]$
p : buyer's purchasing price per unit item
B : buyer's maximum available budget to purchase products
X : lead time demand with finite mean DL and standart deviation $\sigma\sqrt{L} \geq 0$
$E(\cdot)$: mathematical expectation
x^+ : maximum value of x and 0, i.e., $x^+ = \max\{x, 0\}$
$*$: superscript representing optimal value.

2.2. Assumptions

1. There are a single vendor and a single buyer with a single product.
2. The buyer's inventory is continuously reviewed and replenishments are made whenever the inventory level falls to the reorder point r. The reorder point r = expected demand during lead

time + safety stock (SS), and $SS = k \times$ (standard deviation of lead time demand), i.e., $r = DL + k\sigma\sqrt{L}$, where k is the safety factor.

3. The production processes are imperfect and may produces defective items. The defective percentage (γ) is variable random uniformly distributed with probability density function $f(\gamma)$. The inspection process is also imperfect. There are two types of possibilities. The first is type I of inspection error (when a non-defective item classified as defective) and the second is type II of inspection error (when a defective item classified as non-defective).
4. The lead time L has m mutually independent components. The ith component has a normal duration b_i, a minimum duration a_i, and a crashing cost per unit time c_i. Furthermore, for convenience, we rearrange c_i such that $c_1 \le c_2 \le \cdots \le c_m$. Then, it is clear that the reduction of lead time should first occur on component 1 (because it has the minimum unit crashing cost), and then component 2, etc. If we let $L_0 = \sum_{j=1}^{m} b_j$ and L_i be the length of components $1, 2, \ldots, i$ crashed to their minimum duration, then L_i can be expressed as
$$L_i = \sum_{j=1}^{m} b_j - \sum_{j=1}^{i}\left(b_j - a_j\right), i = 1,2,\ldots,n;$$
and the lead time crashing cost $C(L)$ per cycle for a given $L \in [L_i, L_{i-1}]$, is given by
$$C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j\left(b_j - a_j\right).$$
5. Shortage is allowed for the buyer and partially backordered.
6. The purchasing cost for all products is limited, mathematically,
$$pQ \le B.$$

3. Model Formulation

In this paper, an integrated single-vendor and single-buyer inventory model involving defective items, inspection errors, stochastic demand, controllable lead time, and partial backorder under budget capacity constraint. The coordination mechanism is such that the vendor receives the buyer's demand and produces the single product at a finite production rate P. The vendor replenishes the order in a number of equal-sized shipments. It is assumed that the vendor's production processes are imperfect and devective items may be produced. Thus, once the buyer receives the lot-size Q, a 100% screening process is conducted. The length of the screening process is $t_1 = Q/y$ year. The screening process is also imperfect in that an inspector may incorrectly classify a non-defective product as defective (Type I error), or a defective product as non-defective (Type II error). It is assumed that the customers who buy the defective items will detect the quality problem and return them to the buyer and receive a good item from the buyer. Later, the buyer returns all items classified as defective and those returned from the customers to the vendor, and receives a full price refund from the vendor. Both the vendor and the buyer incur a post-sale failure cost (e.g., loss of good will) for the items returned from the market. The goal of this study is to simultaneously optimize the order quantity, safety factor, lead time and the number of lots delivered from vendor to buyer with the objective of minimizing the total supply chain integrated cost and the constraint is satisfied.

3.1. The buyer's cost formulation

The buyer's total inventory cost consists of ordering cost, transportation cost, screening cost, post-sale failure cost for the buyer, crashing lead time cost, holding cost, and shortage cost. The behaviour of the inventory level over time for the buyer is described in Fig.1. The buyer's inventory system with random lead time demands, the inventory level is continuous reviewed and new order is triggered until the inventory level declines to r units. If the lead time demand exceeds the reorder point, the inventory system will experience shortages. The shortages are partial lost sale and partial backordered with backorder rate β. Let $(X - r)^+ = \max\{X - r, 0\}$. Then the number of demand backordered is $\beta E[(X - r)^+]$, and the number of demand lost is $(1 - \beta)E[(X - r)^+]$, where $E[(X - r)^+]$ is the expected inventory shortage at the end of a cycle. We note that the lead time demand, X, follows a normal distribution with *p.d.f.* $f(\cdot)$, mean DL and standard deviation $\sigma\sqrt{L}$; and the reorder point $r = DL + k\sigma\sqrt{L}$ (assumption 2). Then we have

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$$E[(X-r)^{+}] = \int_{r}^{\infty}(x-r)f(x)dx = \sigma\sqrt{L}\Psi(k),$$

where $\Psi(k) = \phi(k) - k[1-\Phi(k)]$, and ϕ and Φ are standard normal distribution and cumulative distribution function, respectively. Hence, the shortage cost for the buyer per production cycle will be

$$n[\alpha + \alpha_0(1-\beta)]\sigma\sqrt{L}\Psi(k),$$

where α and α_0 represent the buyer's penalty cost and marginal profit.

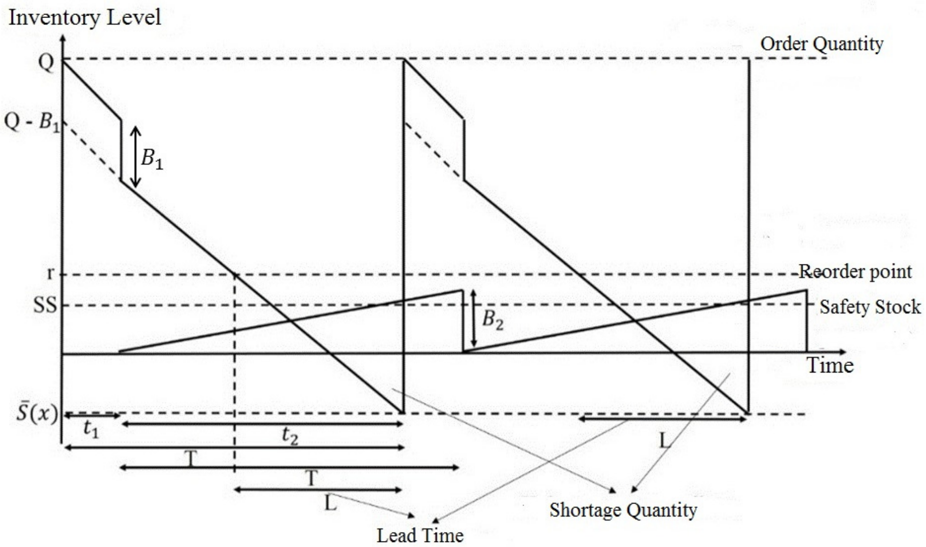

Figure 1. Behavior of the buyer's inventory level over time

By definition, the number of items that are classified as defective include those that are non-defective, $Q(1-\gamma)$, and incorrectly classified as defective (with probability e_1), and those that are defective, $Q\gamma$, and classified as defective (with probability $1-e_2$); thus, we get

$$B_1 = Q(1-\gamma)e_1 + Q\gamma(1-e_2).$$

The number of defective items returned from the market are those that are defective, $Q\gamma$, and incorrectly classified as non-defective (with probability e_2); thus, we have

$$B_2 = Q\gamma e_2.$$

Since we assume that the defective items returned from the market are replaced with good items, the inventory will be depleted at a rate of $D' = D + \frac{B_2}{T}$. By definition, the cycle length

$$T = \frac{Q-B_1}{D'} = \frac{Q(1-\gamma)(1-e_1)}{D}.$$

Hence, the holding cost per production cycle is

$$HC_B = nh_B\left(B_1 t_1 + \frac{B_2 T}{2}\right) + nh_B T\left(\frac{Q-B_1}{2} + r - DL + (1-\beta)\sigma\sqrt{L}\Psi(k)\right)$$

$$= nh_B\left[\frac{Q^2[(1-\gamma)e_1 + \gamma(1-e_2)]}{y} + \frac{Q(1-\gamma)(1-e_1)}{D}\left(\frac{Q}{2}[1-(e_1+\gamma)+\gamma(e_1+2e_1)] + k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\varphi(k)\right)\right].$$

The ordering cost, transportation cost, screening cost, and crashing lead time cost per delivery cycle are S_B, F, $c_i Q$, and $C(L)$, respectively. There are n deliveries in one production cycle, then the ordering cost, transportation cost, screening cost, and crashing cost per production cycle are nS_B, nF, $nc_s Q$, and $nC(L)$, respectively. Further, the buyer incur a post-sale failure cost c_{aB} for each item being returned from the market. Then, the post-sale failure cost for the buyer per production cycle is $nc_{aB}Q\gamma e_2$.

Then, the buyer's total cost in a production cycle is the sum of ordering, transportation, screening, post-sale failure, crashing, holding, and the shortage costs

$$ETC_B(Q,k,L) = nS_B + nF + nC(L) + nc_sQ + nc_{aB}Q\gamma e_2 + n[\alpha+\alpha_0(1-\beta)]\sigma\sqrt{L}\Psi(k)$$
$$+\, nh_B\left[\frac{Q^2[(1-\gamma)e_1+\gamma(1-e_2)]}{y} + \frac{Q(1-\gamma)(1-e_1)}{D}\left(\frac{Q}{2}[1-(e_1+\gamma)+\gamma(e_1+2e_1)] + k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\varphi(k)\right)\right].$$

3.2. *The vendor's cost formulation*

The vendor prepares for the repeating flow of orders size $Q_p = nQ$ from the buyer by producing items in in batches of size Q_p and by planning to have each batch delivered to the buyers in n deliveries, each with a lot of Q units. Fig. 2 shows the vendor's holding cost per cycle. After adding setup, warranty, type I and type II errors, and holding costs, the expected cost per year for the vendor is

$$ETC_B(n,Q) = \frac{DS_v}{nQ(1-E[\gamma])(1-e_1)} + \frac{D[c_we[\gamma]+c_r(1-E[\gamma])e_1+c_{av}E[\gamma]e_2]}{(1-E[\gamma])(1-e_1)}$$
$$+h_v\left[\frac{QD}{P(1-E[\gamma])(1-e_1)} - \frac{nQD}{2P(1-E[\gamma])(1-e_1)} + \frac{(n-1)Q}{2}\right].$$

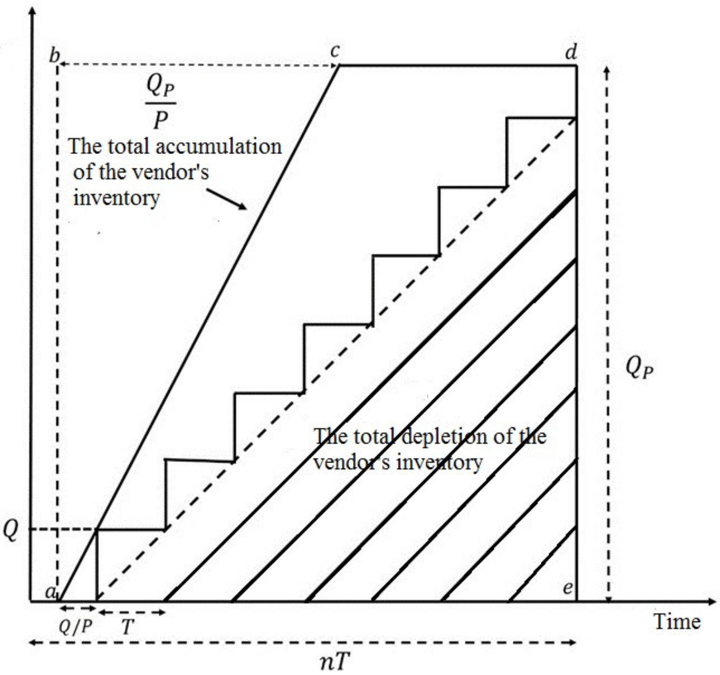

Figure 2. Time-weighted inventory for the vendor

3.3. *An integrated vendor-buyer production-inventory model*

The expected total annual cost of the vendor and buyer is

$$ETC(n,Q,k,L) = \frac{D\left(S_B+F+C(L)+[\alpha+\alpha_0(1-\beta)]\sigma\sqrt{L}\varphi(k)\right)}{Q(1-E[\gamma])(1-e_1)} + \frac{D[c_s+c_{aB}E[\gamma]e_2]}{(1-E[\gamma])(1-e_1)}$$
$$+h_B\left[\frac{QD\left((1-E[\gamma])e_1+E[\gamma](1-e_2)\right)}{y(1-E[\gamma])(1-e_1)} + \frac{QE[A]}{2(1-E[\gamma])(1-e_1)} + k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}\varphi(k)\right]$$
$$+\frac{DS_v}{nQ(1-E[\gamma])(1-e_1)} + \frac{D[c_we[\gamma]+c_r(1-E[\gamma])e_1+c_{av}E[\gamma]e_2]}{(1-E[\gamma])(1-e_1)}$$
$$+h_v\left[\frac{QD}{P(1-E[\gamma])(1-e_1)} - \frac{nQD}{2P(1-E[\gamma])(1-e_1)} + \frac{(n-1)Q}{2}\right].$$

The purchasing cost for all products is limited, mathematically, $pQ \leq B$. The integrated vendor- buyer inventory model with budget capacity constraint is

Minimize $ETC(n,Q,k,L) =$

$$\frac{D(S_B+F+C(L)+[\alpha+\alpha_0(1-\beta)]\sigma\sqrt{L}\varphi(k))}{Q(1-E[\gamma])(1-e_1)}+\frac{D[c_s+c_{aB}E[\gamma]e_2]}{(1-E[\gamma])(1-e_1)}+h_B\left[\frac{QD((1-E[\gamma])e_1+E[\gamma](1-e_2))}{y(1-E[\gamma])(1-e_1)}+\right.$$
$$\left.\frac{QE[A]}{2(1-E[\gamma])(1-e_1)}+k\sigma\sqrt{L}+(1-\beta)\sigma\sqrt{L}\varphi(k)\right]+\frac{DS_v}{nQ(1-E[\gamma])(1-e_1)}+$$
$$\frac{D[c_w e[\gamma]+c_r(1-E[\gamma])e_1+c_{av}E[\gamma]e_2]}{(1-E[\gamma])(1-e_1)}+h_v\left[\frac{QD}{P(1-E[\gamma])(1-e_1)}-\frac{nQD}{2P(1-E[\gamma])(1-e_1)}+\frac{(n-1)Q}{2}\right], \quad (1)$$

subject to $pQ \le B$.

4. Solution Technique

Taha [24] discussed how to solve the optimum solution of nonlinear programming problem subject to inequality constraints by using the Kuhn-Tucker conditions. The development of the Kuhn-Tucker conditions is based on the Lagrangian method. Suppose that the problem (1) can be written as follow.

Minimize $f(z) = ETC(n, Q, k, L)$, (2)

subject to $g(z) = pQ - B \le 0$.

A new function, i.e the Lagrangian function $ETC(n, Q, k, L, \lambda)$ is formed by introducing Lagrangian multiplier λ, then we have

$$ETC(z,\lambda) = f(z) - \lambda g(z)$$
$$= \frac{D\left(S_B+F+C(L)+[\alpha+\alpha_0(1-\beta)]\sigma\sqrt{L}\varphi(k)\right)}{Q(1-E[\gamma])(1-e_1)}+\frac{D[c_s+c_{aB}E[\gamma]e_2]}{(1-E[\gamma])(1-e_1)}+\frac{DS_v}{nQ(1-E[\gamma])(1-e_1)}+$$
$$\frac{D[c_w e[\gamma]+c_r(1-E[\gamma])e_1+c_{av}E[\gamma]e_2]}{(1-E[\gamma])(1-e_1)}+h_B\left[\frac{QD\left((1-E[\gamma])e_1+E[\gamma](1-e_2)\right)}{y(1-E[\gamma])(1-e_1)}+\frac{QE[A]}{2(1-E[\gamma])(1-e_1)}+k\sigma\sqrt{L}+\right.$$
$$\left.(1-\beta)\sigma\sqrt{L}\varphi(k)\right]+h_v\left[\frac{QD}{P(1-E[\gamma])(1-e_1)}-\frac{nQD}{2P(1-E[\gamma])(1-e_1)}+\frac{(n-1)Q}{2}\right]-\lambda(pQ-B). \quad (3)$$

The Kuhn-Tucker conditions need z and λ to be a stationary point of minimization problem which can be summarized as following:

$$\begin{cases}\nabla f(z) - \lambda \nabla g(z) = 0,\\ \lambda g(z) = 0,\\ g(z) \le 0,\\ \lambda \ge 0.\end{cases} \quad (4)$$

By the method of Kuhn-Tucker conditions, consider the two cases $\lambda = 0$ and $\lambda \neq 0$.

i. For $\lambda = 0$, from (3) and (4), we have

$$Q^* = \sqrt{\frac{D\left(nS_B+S_v+nF+nC(L)\right)+nD(\alpha+\alpha_0(1-\beta))\sigma\sqrt{L}\varphi(k)}{M}}, \quad (5)$$

and

$$\Phi(k^*) = 1 - \frac{h_B Q(1-E[\gamma])(1-e_1)}{h_B Q(1-E[\gamma])(1-\beta)+D(\alpha+\alpha_0(1-\beta))}, \quad (6)$$

with

$$M = nh_B\left\{\frac{D}{y}[(1-E[\gamma])e_1+E[\gamma](1-e_2)]+\frac{E[A]}{2}\right\}nh_v\left\{\frac{D}{P}-\frac{nD}{2P}+\frac{(n-1)}{2}(1-E[\gamma])(1-e_1)\right\}.$$

The explicit general solution for Q and k cannot be obtained by solving equations (5) and (6) because the evaluation each of the expressions requires knowledge of the value of the other. The value of (Q, k) can be obtained by adopting a similar graphical technique used in Hadley and Within [9]. The same numerical search technique also has been used in Lin [19, 20] and Kurdhi et al. [16, 17]. Therefore, the following iterative algorithm is established to determine the solution of (n, Q, k, L) for $\lambda = 0$.

Algorithm 1

a. Set $n = 1$ and $\lambda = 0$.
b. For each L_i, $i = 0,1,2, \dots, m$, perform (i) to (vii).
   (i) Start with $k_{i1} = 0$, $\phi(k_{i1}) = 0.39894$, and $\Phi(k_{i1}) = 0.5$.
   (ii) Compute $\varphi(k_{i1})$.
   (iii) Substituting $\varphi(k_{i1})$ into Equation (5) to evaluate Q_{i1}.

(iv) Utilizing Q_{i1} to determine $\Phi(k_{i2})$.
(v) Check $\Phi(k_{i2})$ on standard normal table to determine k_{i2} and then $\phi(k_{i2})$.
(vi) Repeat (i) to (v) until no change occurs in the values of Q_i and k_i. These can be notated by Q_i^* and k_i^*.
(vii) Compute $g(z)$ using Q_i^*.

c. Compute $ETC(n, Q_i^*, k_i^*, L_i)$, $i = 0,1,2,\dots,m$.
d. Set $ETC(n, Q_n^*, k_n^*, L_n^*) = min_{i=0,1,2,\dots,m}\{ETC(n, Q_i^*, k_i^*, L_i)\}$, then (n, Q_n^*, k_n^*, L_n^*) is the solution for fixed n.
e. Set $n = n + 1$ and repeat step (b)-(d) to get $ETC(n, Q_n^*, k_n^*, L_n^*)$.
f. If $ETC(n, Q_n^*, k_n^*, L_n^*) \le ETC(n-1, Q_{n-1}^*, k_{n-1}^*, L_{n-1}^*)$, then go to step (e), otherwise go to step (g).
g. Set $ETC(n^*, Q^*, k^*, L^*)= ETC(n-1, Q_{n-1}^*, k_{n-1}^*, L_{n-1}^*)$, then $ETC(n^*, Q^*, k^*, L^*)$ is the solution.

ii. For $\lambda \neq 0$, from (3) and (4), we have

$$\begin{aligned}\lambda^* = &-\frac{D(nS_B+nF+C(L)+S_V)}{pnQ^2(1-E[\gamma])(1-e_1)} - \frac{D[\alpha+\alpha_0(1-\beta)]\sigma\sqrt{L}\varphi(k)}{pQ^2(1-E[\gamma])(1-e_1)} \\ &+\frac{h_B}{p}\left\{\frac{D\big((1-E[\gamma])e_1+E[\gamma](1-e_2)\big)}{y(1-E[\gamma])(1-e_1)} + \frac{E[A]}{2(1-E[\gamma])(1-e_1)}\right\} \\ &+\frac{h_V}{p}\left\{\frac{D}{P(1-E[\gamma])(1-e_1)} - \frac{nD}{2P(1-E[\gamma])(1-e_1)} + \frac{(n-1)}{2}\right\},\end{aligned} \tag{7}$$

$$Q^* = \sqrt{\frac{D\big(nS_B+S_v+nF+nC(L)\big)+nD\big(\alpha+\alpha_0(1-\beta)\big)\sigma\sqrt{L}\varphi(k)}{W}}, \tag{8}$$

and

$$\Phi(k^*) = 1 - \frac{h_B Q(1-E[\gamma])(1-e_1)}{h_B Q(1-E[\gamma])(1-\beta)+D\big(\alpha+\alpha_0(1-\beta)\big)}, \tag{9}$$

with

$$\begin{aligned}W = &\, nh_B\left\{\frac{D}{y}[(1-E[\gamma])e_1 + E[\gamma](1-e_2)] + \frac{E[A]}{2}\right\} \\ &+nh_v\left\{\frac{D}{P} - \frac{nD}{2P} + \frac{(n-1)}{2}(1-E[\gamma])(1-e_1)\right\} - n\lambda p(1-E[\gamma])(1-e_1).\end{aligned}$$

With the same argument as case $\lambda = 0$, the following iterative algorithm is established to determine the solution of (n, Q, k, L) for $\lambda \neq 0$.

Algorithm 2

a. Set $n = 1$.
b. For each L_i, $i = 0,1,2,\dots,m$, perform (i) to (ix).
(i) Start with $\lambda_{i1} = 0.1$, $k_{i1} = 0$, $\phi(k_{i1}) = 0.39894$, and $\Phi(k_{i1}) = 0.5$.
(ii) Compute $\varphi(k_{i1})$.
(iii) Substituting $\varphi(k_{i1})$ into (8) to evaluate Q_{i1}.
(iv) Utilizing Q_{i1} to determine $\Phi(k_{i2})$.
(v) Substituting Q_{i1} and $\varphi(k_{i1})$ into (7) to evaluate λ_{i2}.
(vi) Check $\Phi(k_{i2})$ on standard normal table to determine k_{i2}, then $\phi(k_{i2})$ and $\varphi(k_{i2})$.
(vii) Substituting λ_{i2} and $\varphi(k_{i2})$ into (8) to evaluate Q_{i2}.
(viii) Repeat (iv) to (vii) until no change occurs in the values of Q_i, k_i, and λ_i. These can be notated by $Q_i{}^*$, $k_i{}^*$, and $\lambda_i{}^*$.
(ix) Compute $g(z)$ using $Q_i{}^*$.

c. Compute $ETC(n, Q_i^*, k_i^*, L_i, \lambda_i^*)$, $i = 0,1,2,\dots,m$.
d. Set $ETC(n, Q_n^*, k_n^*, L_n^*, \lambda_n^*) = min_{i=0,1,2,\dots,m}\{ETC(n, Q_i^*, k_i^*, L_i, \lambda_i^*)\}$, then $(n, Q_n^*, k_n^*, L_n^*, \lambda_n^*)$ is the solution for fixed n.
e. Set $n = n + 1$ and repeat step (b)-(d) to get $(n, Q_n^*, k_n^*, L_n^*, \lambda_n^*)$.

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f. If $ETC(n,\ Q_n^*,\ k_n^*, L_n^*, \lambda_n^*) \leq ETC(n-1,\ Q_{n-1}^*,\ k_{n-1}^*, L_{n-1}^*, \lambda_{n-1}^*)$, then go to step (e), otherwise go to step (g).

g. Set $ETC(n^*, Q^*, k^*, L^*, \lambda^*)= ETC(n-1, Q_{n-1}^*, k_{n-1}^*, L_{n-1}^*, \lambda_{n-1}^*)$, then $ETC(n^*, Q^*, k^*, L^*, \lambda^*)$ is the solution.

5. A numerical example

In this section, an example is solved using the proposed solution in the previous section. The purpose is to illustrate the solution procedure and conduct some sensitivity analysis for important model parameters. Consider an integrated vendor and buyer inventory model with the following parameters:

Production rate (P)	= 160,000 units/year
Demand rate (D)	= 50,000 units/year
Inspection rate (y)	= 155,200 units/year
Setup cost for vendor (S_V)	= \$300/ production run
Ordering cost for buyer (S_B)	= \$100/order
Holding cost for vendor (h_v)	= \$2/unit/year
Holding cost for buyer (h_B)	= \$5/unit/year
Transportation cost (F)	= \$25/delivery
Inspection cost (c_s)	= \$0.5/unit
The cost of producing a defective item (c_w)	= \$50/unit
The cost of rejecting a non-defective item (c_r)	= \$100/unit
The buyer's post sale failure cost (c_{aB})	= \$200/unit
The vendor's post sale failure cost (c_{aV})	= \$300/unit
Buyer's shortage cost (α)	= \$25/unit
Buyer's marginal profit(α_0)	= \$75/unit
Buyer's purchasing price (p)	= \$10/unit
Buyer's maximum available budget to purchase products (B)	= \$30,000/year.

The lead time has three components with data shown in Table 1. Moreover, we assume that the defective percentage is uniformly distributed with *p.d.f.* as

$$f(\gamma) = \begin{cases} \dfrac{1}{\mu}, 0 \leq \gamma \leq \mu; \\ 0, \text{otherwise}. \end{cases}$$

Table 1. Lead time data.

Lead time component, i	Normal duration, b_i (days)	Minimum duration, a_i (days)	Unit crashing cost, c_i (days)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

By the method of Kuhn-Tucker conditions, consider the two cases 1) $\lambda = 0$ and 2) $\lambda \neq 0$. Table 2 shows the solutions of n, Q, k, L, r, and the minimum integrated inventory total cost ETC for $\lambda = 0$.

Table 2. The solutions for $\lambda = 0$.

λ	n	L (weeks)	Q (unit)	$k(r)$ (unit)	$pQ - B$	$ETC(.)$ (\$)
0	1	8	2860.37	2.47194(7702.00)	-1396.26	325244.74
0	2	8	2066.32	2.58600(7702.45)	-4336.79	323597.14
0	3	8	1710.55	2.65040(7702.70)	-12894.50	323432.15
0	3	8	1495.70	2.69540(7702.88)	-10043.00	323663.70

From Table 2, we obtain $n^* = 3$ times, $L^* = 8$ weeks, $Q^* = 1710.55$ units, $k^* = 2.65040$, $r = 7702.70$ units, and the minimum total expected annual cost is \$323432.15. We also have $pQ - B = -12894.50 < 0$, then the lot size satisfy the budget constraint. The Kuhn-Tucker conditions are satisfied, so the solution is optimal and feasible. Table 3 shows the solutions of n, Q, k, L, r, and the minimum integrated inventory total cost ETC for $\lambda \neq 0$. From the table, we get $\lambda^* = 0.190555$, $n^* = 1$ times, $L^* = 8$ weeks, $Q^* = 5119.92$ units, $k^* = 2.25670$, $r = 7701.16$ units, and the minimum total expected annual cost is \$323992.96. However, the lot size does not satisfy the budget constraint, that is $pQ - B = 21199.22 > 0$. Hence, the solution is not feasible.

Table 3. The solutions for $\lambda \neq 0$.

λ	n	L (weeks)	Q (unit)	$k(r)$ (unit)	$pQ - B$	$ETC(.)$ (\$)
0.190555	1	8	5119.92	2.25670(7701.16)	21199.22	323992.96
0.250710	2	8	3973.81	2.35230(7701.53)	9738.11	324329.39
0.306326	3	8	3398.39	2.40981(7701.76)	3983.90	325679.13

In order to study how the parameters affect the integrated optimal solution, the sensitivity analyses for transportation cost (F), vendor's holding cost (h_v), buyer's holding cost (h_B), upper limit of defective percentage (μ), probability of Type I error (e_1), and probability of Type II error (e_2) are performed.

Table 4 shows the optimal solutions for different transportation costs. When the transportation cost increases, the joint total cost also increase. The percentage increase in joint total cost is 0.05%. We may also observe that increasing in transportation cost, will increase the order quantity and decrease the reorder point. The smaller the transportation cost up to \$10, the larger the cost reduction of the integrated model in comparison to independent decision is.

Table 4. Optimal solution for different transportation cost.

F (\$)	L (weeks)	n	Q (unit)	$k\ (r)$ (unit)	ETC Independent (\$)	ETC Integrated (\$)	Cost reduction (\$)
5	8	3	1632.97	2.6660 (7702.76)	322826.05	322794.44	31.61
10	8	3	1652.71	2.6620 (7702.75)	322979.18	322936.44	42.74
15	8	3	1672.21	2.6580 (7702.73)	323132.31	323117.01	15.30
20	8	3	1691.49	2.6542 (7702.72)	323285.25	323275.47	9.78
25	8	3	1710.55	2.6504 (7702.70)	323437.83	323432.15	5.68

Table 5 shows the optimal solutions for different vendor's holding cost per unit. From the table, one can see that the higher the vendor's holding cost is, the greater the joint total cost is. The percentage increase in joint total cost is 0.40%. One can also see that when the vendor's holding cost decreases, it is more profitable to decrease the order quantity and to deliver the items more frequently from the vendor to the buyer. The greater the vendor's holding cost up to \$8, the larger the cost reduction of the integrated model in comparison to independent decision is.

Table 5. Optimal solution for different vendor's holding cost per unit.

h_v (\$)	L (weeks)	n	Q (unit)	$k\ (r)$ (unit)	ETC Independent (\$)	ETC Integrated (\$)	Cost reduction (\$)
2	8	3	1710.55	2.65040 (7702.70)	323437.83	323432.15	5.68
4	8	2	1818.33	2.62970 (7702.62)	325591.62	325526.24	65.38
6	8	1	2568.64	2.51013 (7702.15)	328187.55	327038.22	1149.33
8	8	1	2452.67	2.52640 (7702.22)	329288.64	327869.74	1418.90
10	8	1	2351.09	2.54120 (7702.74)	329839.19	328665.40	1173.79

Table 6 shows the optimal solutions for different buyer's holding cost per unit. One can see that the smaller the buyer's holding cost is, the greater the advantages to deliver the items more rarely from the vendor to the buyer. Moreover, when the buyer's holding cost increases, the joint total cost also increase. The percentage increase in joint total cost is 0.82%.

Table 6. Optimal solution for different buyer's holding cost per unit.

h_B ($)	L (weeks)	n	Q (unit)	$k\ (r)$ (unit)	ETC Independent ($)	ETC Integrated ($)	Cost reduction ($)
5	8	3	1710.55	2.65040 (7702.70)	323437.83	323432.15	5.68
10	8	4	1219.21	2.52840 (7702.22)	327036.45	327027.61	8.84
15	8	5	981.97	2.46144 (7701.96)	329769.97	329757.70	12.27
20	8	6	838.54	2.41455 (7701.84)	332056.19	332056.83	0.64
25	8	7	741.07	2.37813 (7701.69)	334094.30	334091.79	2.51

Table 7 shows the optimal solutions for different probability of Type I error. One can see that when the probability of Type I error increases, both the buyer and the vendor incur a higher joint total annual cost. The percentage increase in joint total cost is 82.16%. The smaller the probability of Type I error up to 0.08, the smaller the cost reduction of the integrated model in comparison to independent decision is.

Table 7. Optimal solution for different probability type I error.

e_1	L (weeks)	n	Q (unit)	$k\ (r)$ (unit)	ETC Independent ($)	$ETC(.)$ Integrated ($)	Cost reduction ($)
0.04	8	3	1710.55	2.65040 (7702.70)	323437.83	323432.15	5.68
0.08	8	3	1757.22	2.65560 (7702.72)	554497.57	554492.15	5.42
0.12	8	3	1805.93	2.66140 (7702.75)	806567.65	806562.20	5.45
0.16	8	3	1856.77	2.66772 (7702.77)	1082649.96	1082644.15	5.81
0.20	8	3	1909.78	2.67470 (7702.79)	1386346.88	1386340.27	6.62

Table 8 shows the optimal solutions for different probability of Type II error. From the table, we may see that the probability of Type II error has a similar impact as the probability of Type I error. When the probability of Type II error increases, the joint total cost also increases. The percentage increase in joint total cost is 6.57%. The smaller the probability of Type II error up to 0.12, the smaller the cost reduction of the integrated model in comparison to independent decision is.

Table 8. Optimal solution for different probability type II error.

e_2	L (weeks)	n	Q (unit)	$k(r)$ (unit)	ETC Independent ($)	ETC Integrated ($)	Cost reduction ($)
0.04	8	3	1710.55	2.6504(7702.7)	323437.83	323432.15	5.68
0.08	8	3	1709.97	2.6505(7702.7)	344701.18	344695.41	5.77
0.12	8	3	1709.39	2.6506(7702.7)	365964.52	365958.66	5.58
0.16	8	3	1708.81	2.6507(7702.7)	387227.87	387221.92	5.95
0.20	8	3	1708.23	2.6508(7702.7)	408491.22	408485.17	6.05

Table 9 shows the optimal solutions for different upper limit of defective percentage. From the table, we may see that when the upper limit of defective percentage increases, the joint total cost and the order quantity tend to increase. The percentage increase in joint total cost is 25.78%. The smaller the upper limit of defective percentage up to 0.08, the smaller the cost reduction of the integrated model in comparison to independent decision is.

Table 9. Optimal solution for different upper limit of defective percentage.

μ	L (weeks)	n	Q (unit)	$k\ (r)$ (unit)	ETC Independent ($)	ETC Integrated ($)	Cost reduction ($)
0.04	8	3	1710.55	2.65040 (7702.70)	323437.83	323432.15	5.68
0.08	8	3	1732.57	2.65300 (7702.71)	401606.19	401600.57	5.62
0.12	8	3	1755.06	2.65580 (7702.72)	483101.76	483096.14	5.62
0.16	8	3	1778.04	2.65860 (7702.74)	568141.59	568135.91	5.68
0.20	8	3	1801.50	2.66162 (7702.75)	656961.98	656956.16	6.82

6. Conclusions

In this paper, we propose the two-echelon supply chain inventory model by considering shortage backlogging, imperfect quality items, inspection errors, lead time, budget capacity constraint, and stochastic demand. An analytic solution procedure is developed to determine the optimal number of shipments, the size of each shipment, the reorder point, and the lead time. An investigation of the effects of six important parameters (transportation cost, vendor's holding cost, buyer's holding cost, upper limit of defective percentage, Type I error, and Type II error) on the optimal solution is also made. Numerical results show that (1) the joint total annual cost is more sensitive to the variation of the upper limit of percentage defective, Type I error, and Type II error; (2) the cost reduction of the integrated model in comparison to independent decision can be increased by decreasing the transportation cost; or by increasing the upper limit of defective percentage, the probability of Type I and II errors, and the vendor's holding cost.

Possible extensions for future research could be by considering learning in production. In this case, the vendor experiences learning in the production process while some of the units are defective. The effect of learning in buyer's inspection errors on supply chain cost also can be investigated. Furthermore, the other possibility is considering the alternative inventory models such as periodic review.

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References

[1] Al-Salamah M 2012 Economic order quantity with stochastic demand, imperfect quality, and inspection errors *Journal of Engineering and Computer Sciences* **5** 2 pp 131-145.

[2] Bag S and Chakraborty D 2014 An integrated inventory model of imperfect quality products with fuzzy chance constraints *Journal of Uncertainty in Mathematics Science* **2014** jums-00013 19 Pages.

[3] Banerjee A 1986 A joint economic-lot-size model for purchaser and vendor *Decision Sciences* **17** 3 pp 292–311.

[4] Bansal K K and Ahalawat N 2012 Integrated inventory models for decaying items with exponential demand under inflation *International Journal of Soft Computing and Engineering* **2** 3 pp 578-587.

[5] Cao M and Zhang Q 2010 Supply chain collaborative advantage: A firm's perspective *International Journal of Production Economics* **128** 1 pp 358-367.

[6] Chen L-H and Kang F-S 2007 Integrated vendor–buyer cooperative inventory models with variant permissible delay in payments *European Journal of Operational Research* **182** 2 pp 658–673.

[7] Goyal S K 1976 An integrated inventory model for a single supplier-single customer problem *International Journal of Production Research* **15** 1 pp 107–111.

[8] Goyal S K, Huang C K, and Chen K C 2003 A simple integrated production policy of an imperfect item for vendor and buyer *Production Planning & Control* **14** pp 596- 602.

[9] Hadley G and Whitin T M 1963 *Analysis of Inventory Systems* Prentice-Hall, Englewood Cliffs, New Jersey.

[10] Hsu J-T and Hsu L F 2012 An integrated vendor-buyer cooperative inventory model for items with imperfect quality and shortage backordering *Advances in Decision Sciences* **2012** 679083 19 pages.
[11] Hsu J-T and Hsu L F 2012 An integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors *International Journal of Industrial Engineering Computations* **3** pp 703-720.
[12] Huang C K 2004 An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration *International Journal of Production Economics* **91** pp 91-98.
[13] Jindal P and Solanki A 2016 Integrated vendor-buyer inventory models with inflation and time value of money in controllable lead time *Decision Science Letters* **5**, pp 81–94.
[14] Khan M 2010 Inventory control in a two-level supply chain with learning, quality and inspection errors *Theses and dissertations* Paper 740.
[15] Khan M, Ahmad A R, and Jaber M Y 2014 An integrated supply chain model with errors in quality inspection and learning in production *Omega* **42** 1 pp 16-24.
[16] Kurdhi N A, Lestari S M P, and Susanti Y 2015 A fuzzy collaborative supply chain inventory model with controllable setup cost and service level constraint for imperfect items *International Journal of Applied Management Science* **7** 2 pp 93-122.
[17] Kurdhi N A, Prasetyo J, and Handajani S S 2015 An inventory model involving back-order price discount when the amount received is uncertain *International Journal of Systems Science* **47** 3 pp 662-671.
[18] Li G, Guan X, Liu M-Q, and Shan M-Y. 2013 Two-echelon inventory model with service level constraint and controllable lead time sensitive to order quantity *Journal of Central South University* **20** 11 pp 3324-3333.
[19] Lin T-Y 2009 Optimal policy for a simple supply chain system with defective items and returned cost under screening errors *Journal of the Operations Research Society of Japan* **52** 3 pp 307-320.
[20] Lin H-J 2013 An integrated supply chain inventory model with imperfect-quality items, controllable lead time and distribution-free demand *Yugoslav Journal of Operations Research* **23** 1 pp 87-109.
[21] Priyan S and Uthayakumar R 2015 An integrated production–distribution inventory system involving probabilistic defective and errors in quality inspection under variable setup cost *International Transactions in Operational Research* doi:10.1111/itor.12202 pp 1-38.
[22] Rad M A, Khoshalhan F and Setak M 2014 Supply chain single vendor – single buyer inventory model with price - dependent demand *Journal of Industrial Engineering and Management* **7** 4 pp 733-748.
[23] Soni H N and Patel K A 2015 Optimal policies for integrated inventory system under fuzzy random framework *The International Journal of Advanced Manufacturing Technology* **78** 5 pp 947-959.
[24] Taha H A 1997 *Operations Research* Prentice Hall New Jersey USA.
[25] Widianto I P, Jauhari W A, and Rosyidi C N 2014 Cooperative Vendor-Buyer Inventory Model with Imperfect Quality and Inspection Errors *Proceedings of the International MultiConference of Engineers and Computer Scientists* **II** pp 990-994.