

Solving portfolio selection problems with minimum transaction lots based on conditional-value-at-risk

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Abstract. Portfolio selection problems conventionally means ‘minimizing the risk, given the certain level of returns’ from some financial assets. This problem is frequently solved with quadratic or linear programming methods, depending on the risk measure that used in the objective function. However, the solutions obtained by these method are in real numbers, which may give some problem in real application because each asset usually has its minimum transaction lots. In the classical approach considering minimum transaction lots were developed based on linear Mean Absolute Deviation (MAD), variance (like Markowitz’s model), and semi-variance as risk measure. In this paper we investigated the portfolio selection methods with minimum transaction lots with conditional value at risk (CVaR) as risk measure. The mean-CVaR methodology only involves the part of the tail of the distribution that contributed to high losses. This approach looks better when we work with non-symmetric return probability distribution. Solution of this method can be found with Genetic Algorithm (GA) methods. We provide real examples using stocks from Indonesia stocks market.

1. Introduction

In the investment world, portfolio selection problem conventionally means ‘minimizing the risk, given the certain level of returns’ from some financial assets. Since Markowitz’s seminal paper on 1952, this problem has found a mathematical formulation, and became a popular and interesting topic in financial mathematics. Markowitz introduced mean-variance portfolio optimization model, a quadratic programming problem with variance as risk measurement [1]. This model became a milestone in finance theory, but has high computational complexity and too many parameters that must be estimated from the data [2]. This models also rely on assumption that investor are risk averse and the return of assets are normally distributed [3]. Some research suggest that these assumptions are rarely satisfied in practical use.

Many researchs had done to simplify the portfolio selection problems and to certain that these assumptions are realistic. By change the risk measurement, one can convert the portfolio selection problems from mean-variance quadratic programming to a linear programming, which were simpler and easier for being solved than a quadratic programming. Some risk measurement that can be used i.e mean absolute deviation, as introduced by Konno [4], and Conditional Value-at-Risk or CVaR that has been developed by Rockafellar and Uryasev [5]. Pflug [6] noted that CVaR is a coherent risk measure, having the following properties: translation-equivariant, positively homogeneous, convex, monotonic with respect to stochastic dominance of order 1, and monotonic with respect to monotonic dominance



of order 2. These properties make CVaR looks attractive as a risk measure. Another approach that using CVaR as constraint had been studied by Uryasev [7] and Palmquist *et al.* [8].

On the other hand, the solution of the portfolio selection problem that obtained by both quadratic and linear programming method are in real numbers, which may generate some problem in real application. Each asset usually has its minimum transaction lots, which are different in many stock markets around the world. A transaction lot, also simply called lot or round, represents a standardized quantity associated with a specific asset, set by the regulatory body in the stock exchange. A lot represents the basis for a transaction on the financial asset. The number of lots must be integer, so the solution from the portfolio selection problem cannot be used immediately.

Some classical portfolio selection considering minimum transaction lots were developed based on Mean Absolute Deviation (MAD), variance (like Markowitz's model), and semi-variance as risk measure [2,3]. Regarding the attractive properties of Conditional Value-at-Risk (CVaR), in this paper we investigated the portfolio selection method considering minimum transaction lots with CVaR as risk measure. This method only involves the part of the tail of the distribution of the investment return that contributed to high losses. The CVaR also looks better when the probability distribution of the assets return is not symmetric. Different from the method that used by Angelelli *et al.* [10], in this paper we use genetic algorithm method to solve the optimization problem.

This paper organized as follows. In the section 2, we discuss the Conditional Value-at-Risk as risk measure and its properties. The related portfolio selection method, considering minimum transaction lots, discussed in the section 3. Empirical studies with stocks from Indonesia Stock Exchange (IDX) are presented in the section 4. The last, section 5 devoted for some conclusion.

2. Conditional Value-at-Risk

The definition of Conditional Value-at-Risk cannot be separated from the Value-at-Risk or VaR. With respect to a specified probability level β , the β -VaR of a portfolio is the lowest amount α in order to with probability β , the loss will not exceed α . In mathematical expression, we can write

$$VaR_{\beta}(R_p) = \min \{ R \mid P(-R_p \geq R) \leq 1 - \beta \}$$

where P denotes the probability function. Typical values for β are 0.90, 0.95, and 0.99. Although VaR is a well-known risk measure that used by most financial institutions around the world, this measure has several undesirable properties: VaR is not subadditive, lack of convexity, and does not take the magnitude or amount of the losses beyond the VaR value into account. These undesirable features motivated the development of Conditional Value-at-Risk.

Conditional Value-at-Risk (CVaR), also known as mean excess loss, mean shortfall, tail VaR, and expected tail loss (ETL), measures the expected amount of losses in the tail of the distribution of possible portfolio losses, beyond the VaR. Taking the portfolio return R_p as a random variable, we can define the CVaR as

$$CVaR_{\beta}(R_p) = E[-R_p \mid -R_p \geq VaR_{\beta}(R_p)] \quad (1)$$

To describe some mathematical properties of the CVaR measure, we introduced some notations below. Let us denote by \mathbf{w} , the n -dimensional portfolio vector such that each component w_i represent the amount of shares held in financial asset- i . Then, we denote by \mathbf{y} in \mathbb{R}^m a random vector describing the uncertain outcomes of the economy, e.g. market parameters. We define the function $f(\mathbf{w}, \mathbf{y})$ as *loss function*, represent the loss associated with the portfolio vector \mathbf{w} . Note that for each \mathbf{w} , the loss

function $f(\mathbf{w}, \mathbf{y})$ is a one-dimensional random variable in \mathbb{R} . Also we can assumed that the underlying probability distribution of \mathbf{y} in \mathbb{R}^m have density, denote by $p(\mathbf{y})$.

Following Rockafellar and Uryasev [5], since CVaR of the losses of portfolio \mathbf{w} is the expected value of the losses conditioned on the losses being in excess of VaR, assuming that all random values are discrete, we have

$$\begin{aligned} CVaR_{\beta}(\mathbf{w}) &= E[f(\mathbf{w}, \mathbf{y}) | f(\mathbf{w}, \mathbf{y}) \geq VaR_{\beta}(\mathbf{w})] \\ &= \frac{\sum_{\{\mathbf{y} | f(\mathbf{w}, \mathbf{y}) \geq VaR_{\beta}(\mathbf{w})\}} p(\mathbf{y}) f(\mathbf{w}, \mathbf{y})}{\sum_{\{\mathbf{y} | f(\mathbf{w}, \mathbf{y}) \geq VaR_{\beta}(\mathbf{w})\}} p(\mathbf{y})} \end{aligned} \quad (2)$$

In the continuous case, we get the equivalent form

$$\begin{aligned} CVaR_{\beta}(\mathbf{w}) &= E[f(\mathbf{w}, \mathbf{y}) | f(\mathbf{w}, \mathbf{y}) \geq VaR_{\beta}(\mathbf{w})] \\ &= (1 - \beta)^{-1} \int_{f(\mathbf{w}, \mathbf{y}) \geq VaR_{\beta}(\mathbf{w})} p(\mathbf{y}) f(\mathbf{w}, \mathbf{y}) d\mathbf{y} \end{aligned} \quad (3)$$

It can be proved that the CVaR is always at least as large as VaR. But, the most important property of CVaR is that the CVaR is a coherent risk measure, a property which was defined by Artzner *et al.* [9]. It means that (1) if there only positive returns, then the CVaR should be non-positive; (2) the risk of a portfolio of two assets be less than or equal to the sum of the risks of the individual assets; (3) if the portfolio is increased c times, the risk becomes c times larger, and (4) cash or another risk-free asset does not contribute to the portfolio risk. Another important property of CVaR, proved by Pflug [6] is that CVaR is convex in the following sense: For arbitrary (possibly dependent) random variables \mathbf{y}_1 and \mathbf{y}_2 , and $0 < \lambda < 1$,

$$CVaR_{\beta}(\lambda \mathbf{y}_1 + (1 - \lambda) \mathbf{y}_2) \leq \lambda CVaR_{\beta}(\mathbf{y}_1) + (1 - \lambda) CVaR_{\beta}(\mathbf{y}_2) \quad (4)$$

3. Minimum Lots Portfolio Selection with Conditional Value-at-Risk

The minimum lots portfolio selection models with conditional Value-at-Risk can be established from the Conditional Value-at-Risk portfolio optimization model.

3.1 CVaR Optimization

From the above formula, we need to determine the VaR before we calculate the CVaR. This problem cause CVaR optimization turn out to be somewhat tricky in practice. Fortunately, Rockafellar and Uryasev [5] discovering simpler approach. The key in this approach is a characterization of $VaR(\mathbf{w})$ and $CVaR(\mathbf{w})$ in terms of the function F_{β} on $X \times \mathbb{R}$, defined by

$$F_{\beta}(\mathbf{w}, \alpha) = \alpha + (1 - \beta)^{-1} \int_{\mathbf{y} \in \mathbb{R}^m} [f(\mathbf{w}, \mathbf{y}) - \alpha]^+ p(\mathbf{y}) d\mathbf{y} \quad (5)$$

where $[t]^+ = \max\{t, 0\}$, that can be used instead of CVaR. There are three important properties of that function: $F_\beta(\mathbf{w}, \alpha)$ is a convex and continuously differentiable function with respect to α , the $\text{VaR}_\beta(\mathbf{w})$ is a minimizer of $F_\beta(\mathbf{w}, \alpha)$, and the minimum value of $F_\beta(\mathbf{w}, \alpha)$ is $\text{CVaR}_\beta(\mathbf{w})$.

Let y_j be the return on j -th instrument, and $\mathbf{y} = (y_1, y_2, \dots, y_n)$ is a random vector. Also, let $p(\mathbf{y})$ denote the distribution of \mathbf{y} , a joint distribution of the return of each instrument. Assume $p(\mathbf{y})$ is independent of \mathbf{w} . Using notations above, the return on the portfolio is the sum of the returns on each individual instruments in the portfolio, weighted by the proportions of each asset in the portfolio. The loss of this portfolio is the negative of the return, that is

$$f(\mathbf{w}, \mathbf{y}) = -[w_1 y_1 + \dots + w_n y_n] = -\mathbf{w}^T \mathbf{y} \quad (6)$$

Substituting equation (6) to equation (5), we get

$$F_\beta(\mathbf{w}, \alpha) = \alpha + (1 - \beta)^{-1} \int_{\mathbf{y} \in \mathbb{R}^m} [-\mathbf{w}^T \mathbf{y} - \alpha]^+ p(\mathbf{y}) d\mathbf{y} \quad (7)$$

The integral in equation (7) above needs to be approximated, since this method is proposed to use when \mathbf{y} doesn't follow a special distribution, such as normal distribution, student- t distribution, etc. One of the approximation can be done by sampling the probability distribution of \mathbf{y} according to its density $p(\mathbf{y})$ by simulation. Suppose that we have t data as sample set, $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_t$, Uryasev [7] suggest to use this approximate function

$$\tilde{F}_\beta(\mathbf{w}, \alpha) = \alpha + \frac{1}{(1 - \beta)q} \sum_{k=1}^t [-\mathbf{w}^T \mathbf{y}_k - \alpha] \quad (8)$$

The minimization problem above, minimizing \tilde{F}_β to get the approximate solution for the minimization of F_β , is equivalent to

$$\text{minimize } \alpha + \frac{1}{q(1 - \beta)} \sum_{k=1}^t u_k \quad (9)$$

subject to linear constraint

$$u_k \geq 0 \quad (10)$$

$$\mathbf{w}^T \mathbf{y}_k \geq -\alpha - u_k \quad (11)$$

$$\mathbf{w}^T \mathbf{1} = \sum_{j=1}^n w_j = 1 \quad (12)$$

Both the objective function and the constraint above are linear, so this optimization problem can be solved by standard linear programming techniques.

3.2 Additional Minimum Transaction Lots

A transaction lot, also simply called lot or round, represents a standardized quantity associated with a specific asset, set by the regulatory body in the stock exchange. A lot represents the basis for a

transaction on the financial asset, or minimum quantity of an asset that can be traded. For example, in Indonesian Stock Exchange regular transaction, the minimum transaction lot for stocks and for option contract is 100 units and 1 unit, respectively. It means that an investor should buy or sell stocks in multiplication of 100 unit, but can buy only one or two option contracts.

The number of lot assets that can be bought or sold by an investor must be integer. It means that there are minimum amount of capital that can be invested in that assets. The total capital invested in that assets has to be expressed as a multiple of the minimum amount. For example, if the Stock Exchange Board establishes that security A has to be bought in multiples of 100 unit, and the price is IDR 11,500, this implies that the security A has minimum round of IDR 1,150,000.

Let N_j be the number of units of security j required as minimum quantity on transaction (lot), and p_j be the market price for security j at the date of the purchase of the portfolio. We denote by c_j the purchasing price for the minimum lot of each security j , expressed in terms of money and is equivalent to $c_j = N_j p_j$. Trivially, when asset j traded without minimum lot, we take $N_j = 1$, so $c_j = p_j$. Next, we define the integer variable x_j , $j = 1, 2, \dots, m$, represents the number of minimum lots for each security j which will make part of the optimal portfolio. So, the quantity $c_j x_j$ represents the part of the total available budget that the investor decides to put to the security j . With these notations, we can define the portfolio selection problem with minimum transaction lot based on CVaR as minimizing

$$\min \text{CVaR}_\beta(R_p) \quad (13)$$

subject to

$$\sum_{j=1}^m x_j c_j \leq b \quad (14)$$

$$x_j \in \mathbb{Z} \quad (15)$$

As it can be observed above, equation (13) is the objective function which determines the CVaR as a risk measurement to be minimized. Equation (14) considered budget (b) that available for the investment, so that the total amount of investment is not greater than the available budget. Last, the (15) ensure that the number of lots should be integer.

3.3 Solution

The CVaR (13) can be linearized by substituting the linear function (7) and its approximation (8) or by using the equivalent form (9) and the additional constraint (11), become

$$\min \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^t u_k$$

subject to

$$\sum_{j=1}^m x_j c_j \leq b$$

$$\mathbf{x}^T \mathbf{y}_k \geq -\alpha - u_k$$

$$x_j \in \mathbb{Z}$$

It can be understood that the portfolio optimization model based on CVaR considering transaction lots is a mixed integer linear program, because only the x_j is restricted to integer values. There are some methods to obtain solution from a mixed integer linear program, for example by branch-and-bound method and cutting plane method [11]. Unfortunately, neither of the two methods can be claimed for being uniformly effective in solving the integer linear program. Mansini, Ogryczak, dan Speranza [12] also noted that integer linear programming models with CVaR measure are harder to solve than the corresponding models with mean absolute deviation (MAD) measure. An increasing number of assets also makes the model harder to solve.

Another approach that can be used to solving complex optimization problem is heuristic method, for example genetic algorithm. Proposed by Holland [13] in 1975, this method were developed from the Darwin's principal of natural selection and genetic theory. In genetic algorithms, an initial population consist of several solution, represented as chromosomes, is selected randomly as the first generation. Two chromosomes as "parents" were selected among them, and produce "child" by cross over and mutation operators. A fitness function, which is originated from the objective function of the optimization problem, used to evaluate the desirability of each solution represented by each chromosome. Chang *et al.* [3] explained basic steps of genetic algorithm as

Generate an initial population

Evaluate fitness of individuals in the population

Repeat

Select parents from the population

Recombine (mate) parents to produce children using cross over and mutation operators

Evaluate fitness of the children

Replace some or all of the population by the children

Until a satisfactory solution has been found

Since the study by Arnone *et al.*[14], genetic algorithm had used to solving portfolio selection problems. Many studies have showed that genetic algorithm can efficiently find near optimal or even the optimal solutions for optimization problems. Specially, the use of genetic algorithm in optimizing portfolio based on the Conditional Value at Risk (CVaR) has been described by Lin and Ohnishi [15]. This study can be seen as a development from their study, as in this study there is additional constraint i.e. minimum transaction lots.

4. Empirical Study

In this empirical study, we choose ten stocks from LQ-45 index, an blue-chips stock list published by Indonesian Stock Exchange (IDX) for period July 2015 to July 2016. We compute the continuously compounded return or log return, based on definition by Tsay [16]. The descriptive statistics of log return for each stock are presented in Table 1.

In Table 1 we can see that some stocks have large excess kurtosis, for example ADHI and SRIL. There are also relatively large difference between skewness of each stock return. It means that standard deviation (or variance) is not enough to describe the risk, so we use the conditional value at risk (CVaR). Here, the CVaR were calculated by historical method, so it is not influenced by the probability distribution of the loss.

Assume that our transaction are happened in Indonesian Stock Exchange (IDX), with regularly minimum lots equivalent to 100 unit stock. The opening price of each stock as given in Table 2.

Table 1. Descriptive statistics and 5% Conditional VaR of log return for some stocks

Stock Code	Mean	Standard deviation	Skewness	Excess Kurtosis	CVaR 5 %
PTPP	0.00056020	0.0193677	0.3554030	2.554045	0.03888579
ADHI	0.00166336	0.0300079	0.0929876	4.492607	0.06302641
PTBA	0.00027220	0.0337169	0.7406433	1.504704	0.06038276
LPPF	0.00111219	0.0282435	0.5336275	1.162816	0.05238701
SRIL	0.00024344	0.0435830	1.0833290	6.817095	0.08953712
BBNI	0.00014429	0.0229853	0.5762866	1.397740	0.04318361
BMTR	-0.00004335	0.0346544	0.3274879	0.746279	0.06991387
ICBP	0.00142647	0.0199368	0.8724702	2.460618	0.03476379
JSMR	-0.00002437	0.0221391	0.4233325	2.990921	0.04902375
TLKM	0.00136858	0.0167517	-0.2146708	0.523264	0.03657948

Table 2. Price of each stocks (in Indonesian Rupiah, IDR)

Stock Code	Price/unit	Price/lot	Stock Code	Price/unit	Price/lots
PTPP	3,870.00	387,000.00	BBNI	5,400.00	540,000.00
ADHI	2,830.00	283,000.00	BMTR	1,045.00	104,500.00
PTBA	9,875.00	987,500.00	ICBP	8,700.00	870,000.00
LPPF	20,000.00	2,000,000.00	JSMR	5,400.00	540,000.00
SRIL	268.00	26,800.00	TLKM	4,350.00	435,000.00

4.1 CVaR Portfolio Optimization without Transaction Lots

Suppose that an investor wants to invest IDR 1,000,000,000 into a portfolio consist of ten stocks above. To obtain the weights of each assets in the portfolio, we use genetic algorithm method. Parameters of the proposed genetic algorithm are set according to Hopgood [17], Lin and Liu [18], and Chang et al. [3] as follows. All computation were conducted on a notebook with an Intel i5 2.8 GHz processor for about 10 minutes.

Table 3. Some method and parameters of the Genetic Algorithm

Method/Parameters	Value
Chromosome length	10 (equal to the number of assets in this portfolio)
Representation of genes	real number between 0 and 1
Number of population	50
Crossover method	Uniform crossover
Crossover probability	0.7
Mutation method	Random mutation
Mutation probability	0.1
Selection method	Tournament (roulette-wheel) selection
Elitism	3

Without regarding the minimum transaction lots, the objective function is just minimizing the CVaR of the portfolio. On other side, genetic algorithm's goal is to get the chromosome with largest fitness, so we state the 'minimizing CVaR' as 'maximizing the negative of CVaR'. The result of this algorithm is stated in Table 4 below.

Table 4. Number of lots of each assets on the optimal portfolio *without* regarding the transaction lots, $\beta = 0.95$

Stock Code	Iteration 100x	Iteration 250x	Iteration 500x	Iteration 1000x
PTPP	120	233	227	233
ADHI	113	53	6	8
PTBA	31	25	21	22
LPPF	30	5	5	0
SRIL	918	577	1023	1069
BBNI	463	299	259	237
BMTR	170	88	63	14
ICBP	344	500	540	553
JSMR	52	70	92	120
TLKM	476	457	421	410
Total Investment	995,823,400	997,559,600	996,748,900	996,501,200

From result on table 4 above, we can conclude that different number of generations in genetic algorithm can yield slightly different number of lots on each assets. The limitation to only buy assets in regular lot transaction leads to some difference between the total money spent to bought the asset and the total available money for investment purpose.

4.2 CVaR Portfolio Optimization with Transaction Lots

In this sub section, we look for portfolio weight with CVaR as a risk measure and minimum transaction lots as additional constraint. First, we calculate the weight of each assets in the portfolio. The number of lots were calculated by multiply the weight of each assets by the total investment and divide it by each price per lot asset. Last, we measure the 'true' weight, as the number of lots, multiply by the price, and divide by the total assets. The portfolio's CVaR which will be minimized were calculated from this 'true' weight. This more complex fitness function needs more time to execute when implemented in the genetic algorithm.

For the convenience, the parameters of Genetic Algorithm is same as that used in sub section 4.1 above, i.e. that mentioned in table 3. The result of portfolio optimization with minimum transaction lots are shown in table 5, 6, and 7 below for different β of CVaR.

Table 5. Number of lots of each assets on the optimal portfolio regarding the transaction lots, with CVaR $\beta = 0.90$

Stock Code	Iteration 100x	Iteration 250x	Iteration 500x	Iteration 1000x
PTPP	355	409	405	360
ADHI	113	41	54	63
PTBA	50	48	55	52
LPPF	18	8	3	3
SRIL	623	483	522	739
BBNI	156	160	164	136
BMTR	234	143	87	35
ICBP	313	364	388	414
JSMR	159	148	126	153
TLKM	592	576	563	552
Total Investment	995,793,400	994,709,900	994,448,100	994,295,700

Table 6. Number of lots of each assets on the optimal portfolio regarding the transaction lots, with CVaR $\beta = 0.95$

Stock Code	Iteration 100x	Iteration 250x	Iteration 500x	Iteration 1000x
PTPP	245	196	216	247
ADHI	121	37	35	18
PTBA	40	30	27	21
LPPF	27	10	3	3
SRIL	675	629	962	959
BBNI	325	300	308	250
BMTR	169	65	48	73
ICBP	376	480	483	511
JSMR	118	88	90	105
TLKM	394	480	467	454
Total Investment	996,018,500	995,502,700	995,218,600	994,499,700

Table 7. Number of lots of each assets on the optimal portfolio regarding the transaction lots, with CVaR $\beta = 0.99$

Stock Code	Iteration 100x	Iteration 250x	Iteration 500x	Iteration 1000x
PTPP	120	270	339	302
ADHI	79	88	63	78
PTBA	74	67	73	60
LPPF	14	9	1	5
SRIL	757	499	312	547
BBNI	442	266	292	257
BMTR	156	40	27	17
ICBP	442	501	500	530
JSMR	119	122	78	84
TLKM	236	274	295	287
Total Investment	996,564,600	995,656,200	997,381,100	994,689,100

From Tables 5, 6, and 7 above, it can be seen that both the β value of CVaR and the number of generation influence the number of lots of each assets on the portfolio. However, the number of generation seems not related with the total investment.

4.3 Portfolio performance

From practical point of view, after an investment portfolio had constructed, it is need to be monitored and evaluated periodically. The purpose of monitoring and evaluating are to maintain the portfolio return and prevent the larger risk. In this case, performance of each portfolio above measured daily for 30 market days after. Figure 1 stated the LQ45 index movement for this period.

From figure 1 above, we could say that the market of stock in LQ45 index is bullish. As we could compute the daily continuously compounded return or log return, we could compute the continuously compounded multiperiod return as the sum of daily log return [17]. The portfolio total return then calculated as the sum of 100 times number of lots for each assets times return of each assets.

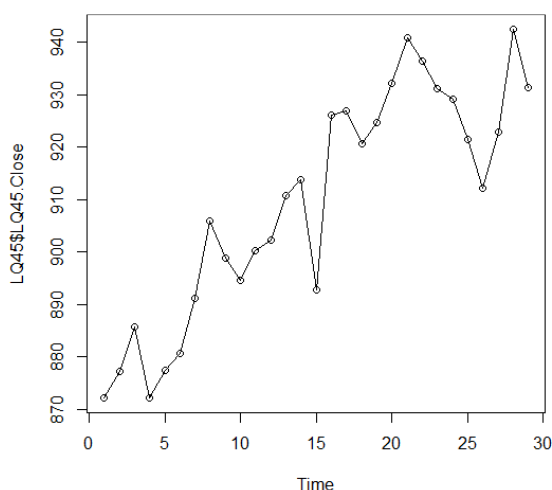


Figure 1. Movement of LQ45 index in 30 market days after portfolio selection

It is look necessary to know, how the additional constraint about minimum lot transaction affect the portfolio performance, i.e. the return and CVaR of the portfolio. Comparison between two portfolios with same β but different additional constraint can be examined from figure 2 below.

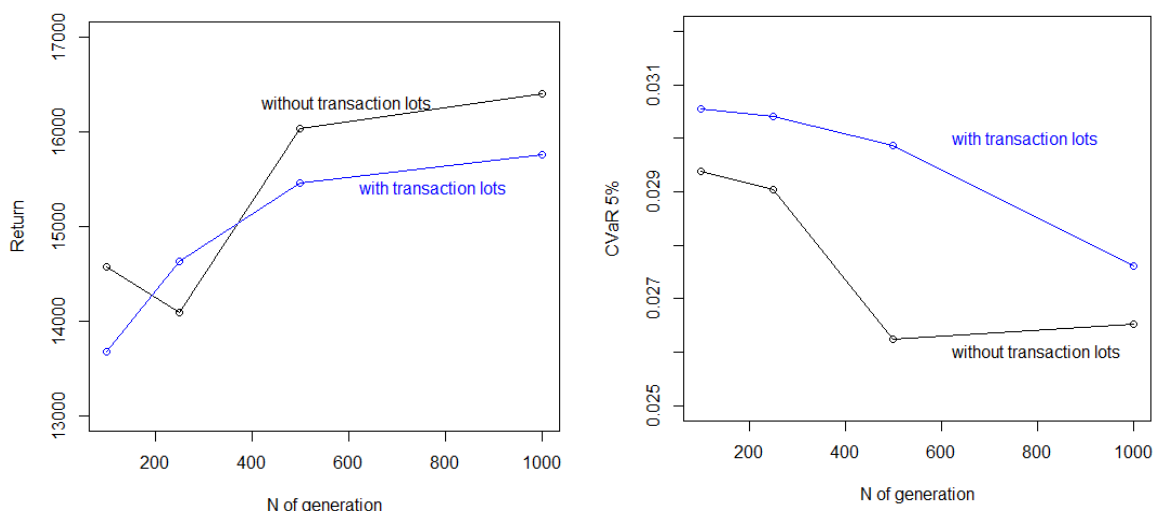


Figure 2. Portfolio return and CVaR for 30 market days for CVaR optimization with and without minimum transaction lots

Figure 2 above shows that the additional minimum transaction lots in the portfolio optimization give higher CVaR than portfolio without considering minimum transaction lots. However, for larger number of generations, it seems that both portfolio will give approximately same value of CVaR. For the return, its can looks that when the number of generation equal to 250, both of the portfolio give same return.

From figure 3 below, we could see the relationship between confidence level (β) of CVaR to the return and CVaR of the portfolio. For small number of generation, different confidence level yield different portfolio return. However, it will converge to approximately same return when the number of generation become larger, more than 1000. For larger number of generation, different beta CVaR also yielding portfolio with different risk, as measured by CVaR on the right side.

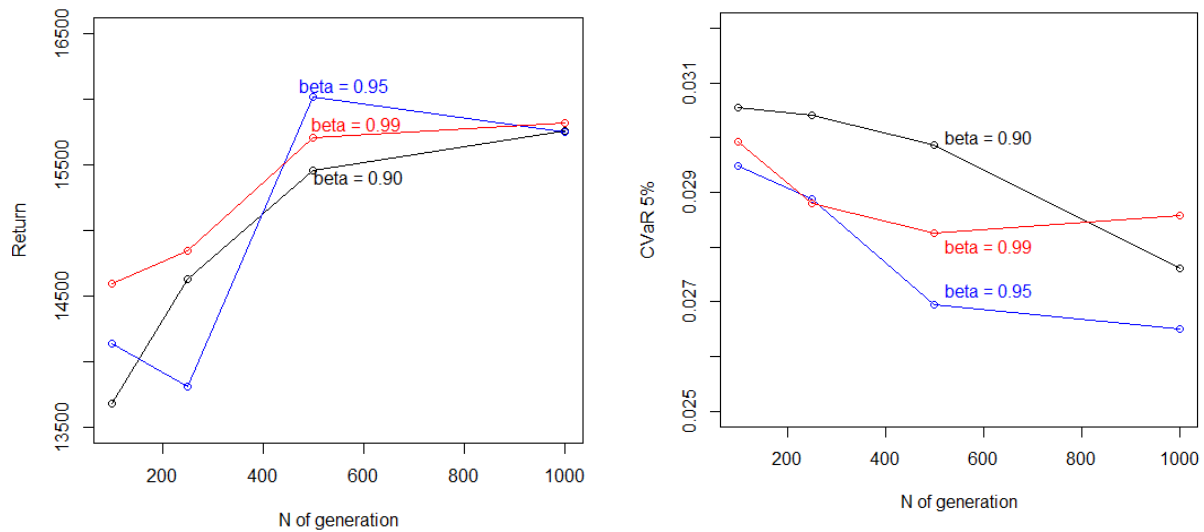


Figure 3. Portfolio return and CVaR for 30 market days for different β -CVaR optimization with minimum transaction lots

As explained in Table 3, there are some parameters in the genetic algorithm approach, i.e. cross over probability and mutation probability. Both of them also analyzed to find whether they affect the return and/or the risk of the optimal portfolio.

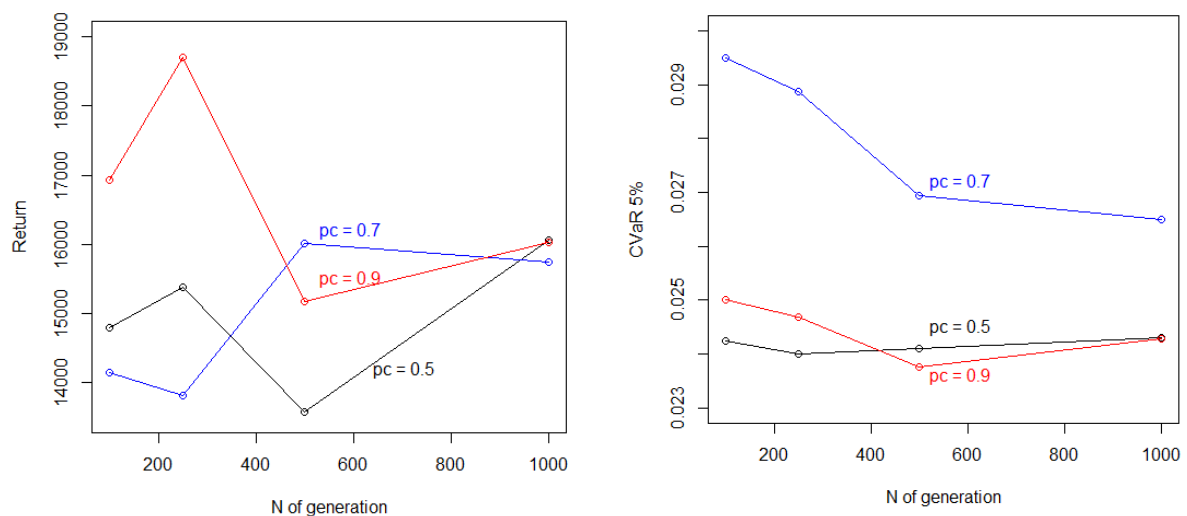


Figure 4. Portfolio return and CVaR for 30 market days for β -CVaR optimization with minimum transaction lots, with different crossover probability

From Figures 4 and 5, it looks that the probability of mutation and probability of crossing over of genetic algorithm affect the return and the CVaR of the portfolio. For the CVaR, which wanted to be minimized, too small mutation probability cause the CVaR down slowly, and may difficult to reach the global minima. By compared figure 4 to figure 2 above, we could carefully select the parameters of the genetic algorithm so the portfolio optimization problem constrained with minimum transaction lots will yield higher return than portfolio without minimum transaction lots.

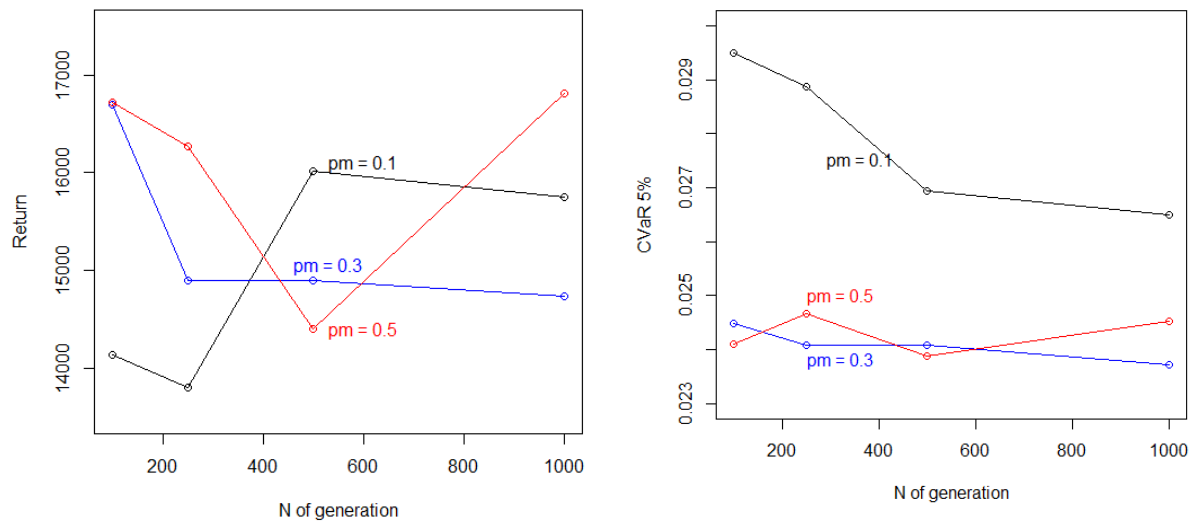


Figure 5. Portfolio return and CVaR for 30 market days for β -CVaR optimization with minimum transaction lots, with different mutation probability

Table 8. Reward-to-variability (Sharpe) Ratio of some optimum portfolio with minimum transaction lots and CVaR as risk measure. The stock description is refer to Table 3, unless mentioned.

Stock Description	Iteration 100x	Iteration 250x	Iteration 500x	Iteration 1000x
Varying significance level				
$\beta = 0.90$	3.269	3.853	4.156	3.998
$\beta = 0.95$	3.702	3.806	3.788	3.699
$\beta = 0.99$	3.933	4.771	5.362	4.977
Varying the mutation probability				
pm = 0.1	3.794	3.725	3.725	4.223
pm = 0.3	3.702	3.806	3.788	3.699
pm = 0.5	4.639	3.394	3.893	3.754
Varying the crossover probability				
pc = 0.5	3.946	4.116	3.989	3.943
pc = 0.7	3.702	3.806	3.788	3.699
pc = 0.9	3.031	3.797	4.004	3.620

Last, to compare all models above simultaneously, we compute the reward-to-variability ratio, which popular known as Sharpe ratio (See Elton *et al.* [19]). Here, reward is measured by difference between mean return of the portfolio and the risk-free-rate. The variability measured by variance of the portfolio. Note that the larger the ratio, the better the performance of the portfolio. As a rule of thumb, portfolio with Sharpe ratio > 1 considered good, Sharpe ratio > 2 considered very good, and Sharpe ratio > 3 considered excellent. The result is provided in the table 8.

From the reward-to-variability ratio above, we conclude that the number of generation or iteration should be more than 100. Using higher confidence level of CVaR, reward or excess return that will received by the investor also higher. Consistent to explanation above, the mutation and cross over probability also influence both the return and the variance.

5. Conclusion and Future Research

Portfolio optimization using Conditional Value at Risk (CVaR) as risk measurement has attractive property, i.e. the coherent property of CVaR. To build a portfolio optimization based on CVaR with minimal transaction lots as an additional constraint, genetic algorithm become an useful tool to solving the model and get the amount of asset that should be bought by the investor. However, the properties and method of genetic algorithm may influence the solution that obtained, and the portfolio can't well compared each other. In the future, more research is needed to get better solution, using other heuristic method and/or adding more constraint such as cardinality constraint and asset weight constraint.

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