

Mode I stress intensity factors of slanted cracks in plates

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Abstract. This paper presents the roles of slanted cracks on the stress intensity factors (SIF) under mode I tension and bending loading. Based on the literature survey, lack of solution of SIFs of slanted cracks in plain strain plates are available. In this work, the cracks are modelled numerically using ANSYS finite element program. There are two important parameters such as slanted angles and relative crack length. SIFs at the crack tips are calculated according to domain integral method. Before the model is further used, it is validated with the existing model. It is found that the present model is well agreed with the previous model. According to finite element analysis, there are not only mode I SIFs produced but also mode II. As expected the SIFs increased as the relative crack length increased. However, when slanted angles are introduced (slightly higher than normal crack), the SIFs increased. Once the angles are further increased, the SIFs decreased gradually however they are still higher than the SIFs of normal cracks. For mode II SIFs, higher the slanted angles higher the SIFs. This is due to the fact that when the cracks are slanted, the cracked plates are not only failed due to mode I but a combination between both modes I and II.

1. Introduction

The concept of fracture mechanics is successfully used to characterize the elastic behavior of cracks in mechanical components. There are two important tools generally used for such purposes such as stress intensity factor (SIF) and J-integral approaches. The first approach is the traditional method where it is relatively difficult to use as compare with the second tool. Therefore, this work focuses more on the implementation of J-integral and then convert it to SIFs. The advantages of J-integral over traditional SIF can be found in [1]. On the other hand, some basic applications of SIF method can also be found in [2].

Another important fracture criterion can be found in [3]. They have reviewed the developments and application of crack tip opening angle or displacement (CTOA/CTOD). This paper also discussed on the current issues regarding to CTOA/CTOD. Based on the literature, the works on the edge cracks are tremendously found especially on the normal cracks. However, the solution of SIFs for the slanted cracks are considerably difficult to find. For example Albinmousa et al. [4] studied slanted cracks under mode I tension. They demonstrated the applicability, simplicity and flexibility of the new approach where it is then validated through comparison with the existing analytical and numerical solution. It is found that their finding are well agreed with experimental results. However, they are assumed that the cracks are in the plane stress but not in plain strain.



Dempsey and Mu [5] used an approach of weight function for calculating the stress intensity factors of in-situ edge-cracked rectangular plated under arbitrary crack face loading. Various crack geometries are investigated and stress intensity factor (SIF), crack opening displacement (COD) and crack opening angle (COA) are determined and then compared with the existing results. It is found that a good accuracy is obtained for all crack geometries considered.

Matsumoto et al. [6] computed the SIFs of interfacial cracks based on the interaction energy release rates and boundary element method (BEM) sensitivity analysis. Several crack configurations are considered. In their analyses, the present method can give more accurate results with a coarse mesh than the method based on the displacement extrapolation. Sensitivity analysis is conducted and found that the computational cost are comparatively small. Hammond and Fawaz [7] reviewed the solutions of SIFs of various size single edge-cracked tension specimens. Comparison are made between the literature reported data and the present effort's results with generally satisfactory correlation. Significant differences are also found for the short crack with the plate aspect ratio of less than 3.

Kuang and Chen [8] used a displacement extrapolation method for analyzing the problem of mixed mode loading of single edge cracks. According to their reports, the proposed modified extrapolation method associated with collapsed triangular point element is the most reliable method for determining the SIFs for both normal and inclined cracks.

Ismail et al. [9-12] investigated the surface cracks in round bars under bending, tension and their combination of stresses. They found that the stress intensity factors significantly affected by the crack geometries and for combined stresses conditions, the combined stress intensity factors can be predicted using a superposition technique. While the stress intensity factors from different loading modes can also be predicted using an equivalent von-Mises stress however there some discrepancies when compared with the values predicted using finite element method. He is also studied the stress intensity factors of three-dimensional slanted cracks in round bars. The detail of the solution of stress intensity factors can be found in [13-15].

According to the literature above, it is found that lack of the solutions of stress intensity factors are available especially for slanted cracks under mode I loading. Therefore, this paper presents the solutions of SIFs for various single edge crack geometries and angles embedded in the plain strain plates.

2. Numerical Modelling

Single edge crack is placed at one edge of the plain strain plate subjected to mode I loading (tension and bending forces). Two types of crack configurations are considered such as normal and slanted cracks. The schematic diagram of the cracked plate is shown in Figure 1. The width of the plate is $W = 50\text{mm}$ while the total height is $2H = 100\text{mm}$. There are seven relative crack length, a/L are used such as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7 where a is a crack length and L is a plate width. The normal crack condition, $W = L$. Another important parameter is slanted angle, θ then they are normalized against 90° . Therefore, the normalized angles used are 0.00, 0.056, 0.111, 0.167, 0.222, 0.278, 0.333, 0.389, 0.444 and 0.500. The crack geometries are then modelled using ANSYS finite element program through the use of ANSYS Parametric Design Language (APDL). Since the problem is elastic then modulus of elasticity, $E = 200\text{GPa}$ and Poisson's ratio, $\nu = 0.3$ is used. The cracked plated is also assumed to fulfil the plain strain condition and iso-parametric 8-node quadrilateral element called PLANE183 is used to model the plate. A special attention is paid to the crack tip where the nodes around it is shifted to the location of $1/4$ close to the tip. This is to ensure the singularity around the crack tip.

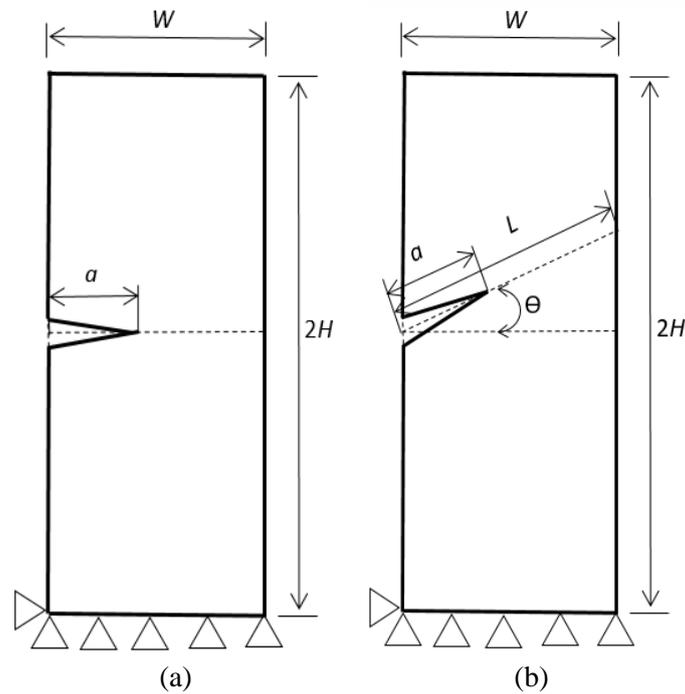


Figure 1 Schematic diagram of (a) normal and (b) slanted cracks.

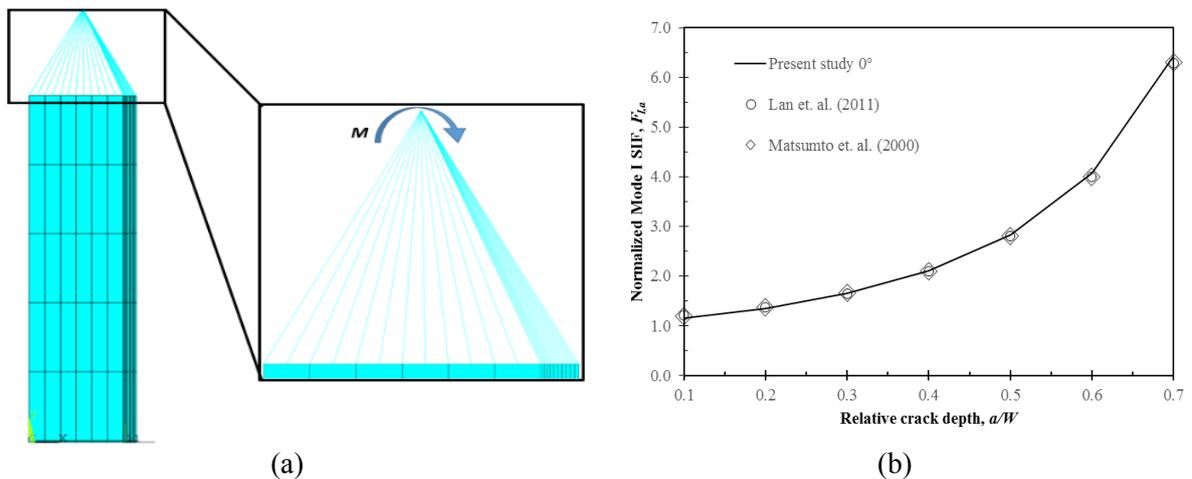


Figure 2 (a) MPC184 element is used for remotely applied bending moment and (b) Validation of the present with the existing models.

Figure 2(a) shows the finite element model of the crack plate. In order to properly manage the mechanical deformation of the plate, it must be constrained properly. At the bottom edge of the plate is fixed in y-direction. This is to ensure that the plate is capable to elongate in both x- and y-directions. In order to prevent model rotation, the plate is also constrained at bottom left point. Tension stress is applied directly at the upper edge while the bending moment is applied remotely through the use multi-point constraint element called MPC184 as shown in Figure 2(a). Both bending moment and tension force are applied to the independent node. Stress intensity factor (SIF) based on the J-integral (domain integral method). In this work, the SIFs are normalized as function of crack length, a and applied stress, σ using the following expression:

$$F = \frac{K}{\sigma\sqrt{\pi a}} \quad (1)$$

where, F is a normalized stress intensity factor, K is a stress intensity factor, σ is an applied stress and a is a crack length. It is a paramount important aspect to validate the present with the existing models (Lan et al. [14] and Matsumto et al. [6]) when the normal crack is used. Figure 2(b) reveals the validation results and it is found that the present model is well agreed with the existing results. In this work, it is also assumed that the normal crack is sufficiently validated the slanted cracks since the solutions of SIFs of slanted cracks are difficult to obtain.

3. Results and Discussion

Normalized SIFs for slanted cracks under tension forces is shown in Figure 3 for both modes I and II. They are plotted against relative crack depth, a/L . In general, the mode I SIFs gradually increased with the increased of a/L . The lowest SIFs are obtained from normal or 90° cracks. It is observed that slightly increased the slanted angle to 5° has significantly increased the SIFs compared with the normal cracks. No significant different of SIFs obtained from the cracks slanted using 5° , 10° , 15° and 20° where the SIFs are almost identical. However, mode I SIFs (Figure 3(a)) decreased as the slanted angles are increased. Similar SIFs trend is observed from the works conducted by Lan et. al. [13] and Matsumto et. al. [6]. In this work, when the slanted angles are increased to up to 45° , the SIFs seemed to approach the SIFs obtained from the normal cracks.

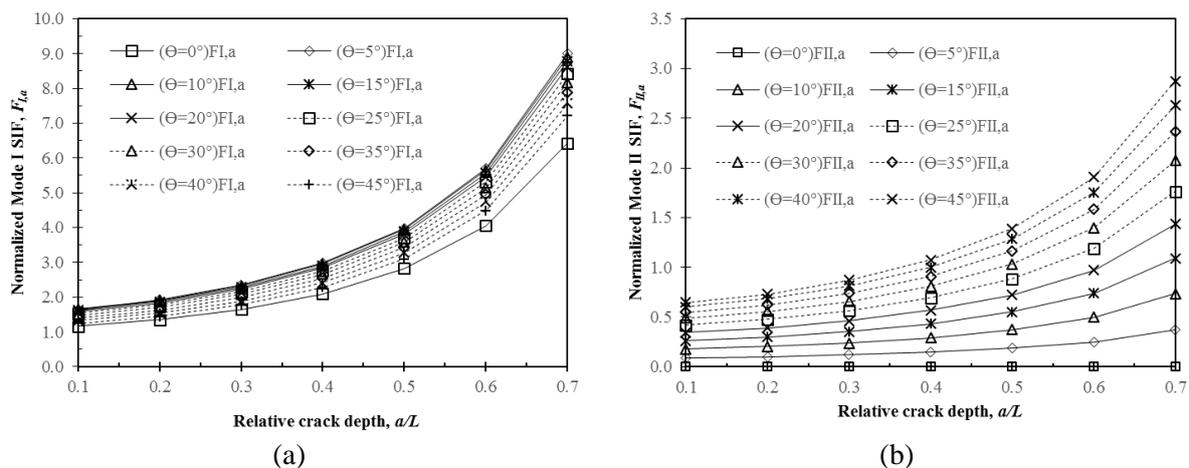


Figure 3 Effect of relative crack depth on the stress intensity factor of (a) mode I and (b) mode II under tension stress.

Figure 3(b) shows the mode II SIFs plotted against the relative crack depth, a/L for different slanted angles. It is observed similarly with mode I SIFs, mode II SIFs are also increased when a/L increased since the ligament of the plate decreased when crack length increased and therefore increasing the stress intensity around the crack tip. It can also be seen that mode II SIFs increased when slanted angles are increased. Contrarily with mode I SIFs where the SIFs decreased when higher slanted angles are used. Figure 4 reveals the effect of slanted angles on the mode I SIFs for different relative crack length. As expected longer the crack length higher the SIFs can be obtained. However, it is interested to note that slightly slanted the crack for an example 5° has the great effect on the mode I SIFs. It is also observed that the maximum SIF occurred when 5° angle is used. However when the angles increased, their effect are diminished where the SIFs reduced to approach the SIFs for normal cracks.

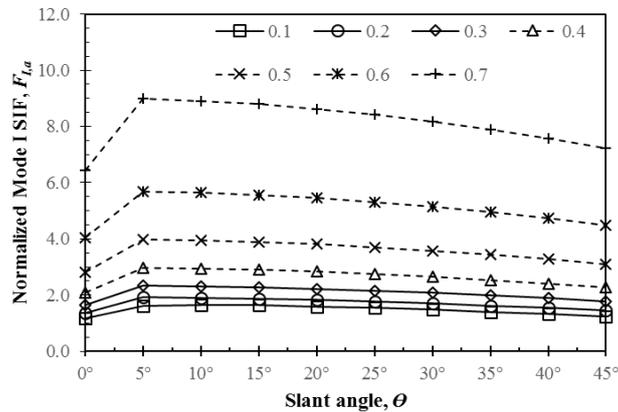


Figure 4 Effect of slanted angles on the SIFs when relative crack depth are varies under tension stress.

Figure 5 shows the stress intensity around crack tip of different slanted cracks. Figure 5(a) reveals the crack behaviour for normal crack. Since the problem is symmetrical only half of crack is modelled. It is observed the stress distribution is also symmetrical where the crack face is almost parallel with the symmetrical plane. On the other hand, Figures 5(b) and 5(c) indicate the crack deformations when the cracks are slanted at 25° and 45°, respectively. Once the angles are introduced, the stress intensity around the crack tip is disturbed where the bottom side of the crack the stress distribution is almost similar with the normal cracks. However at the upper side, the stress intensity seemed to shear the crack to the right of the plate and thus creating mode II SIFs. The shear severity increased as the slanted angles increased and therefore increasing the mode II SIFs.

Figure 6 shows the effect of relative crack depth, a/L on the modes I and II SIFs under remote bending moment, respectively. In general both modes of SIFs increased as crack length are increased due to the fact that the ligament cross-section decreased in length and therefore increasing the stress intensities. Figure 6(a) indicates the effect of a/L on the mode I SIFs when slanted angles are varied. In comparison, the SIFs obtained from the normal cracks are also presented. It is found that the introduction of slanted angles have decreased the SIFs significantly. Under bending moment, there is insignificant effect of slanted angles on the mode I SIFs. However, the introduction of slanted angles have slightly decreased the mode I SIFs. Figure 6(b) reveals the influence of relative crack depth on the mode II SIFs. It is found that the introduction of slanted angles are strongly affected the mode II SIFs where increasing the angles also increase the SIFs. It is also observed that for relatively shallow cracks ($a/L \leq 0.4$), the a/L has insignificant role on the mode II SIFs. This is due to the ligament of the plate is strong enough to resist the sliding deformation of the cracks.

Figure 7 reveals the effect of slanted angles on the SIFs when relative crack length, a/L are varied for modes I and II SIFs, respectively. In this work, 0° crack is defined as a normal crack where the crack faces are perpendicular to the axial forces. For the relatively shallow cracks ($a/L \leq 0.4$), the effect of slanted angles on the both modes I and II SIFs are insignificant. Increasing the crack length, slightly increased the SIFs however the increments can be ignored if compared with other SIFs obtained using $a/L \leq 0.4$.

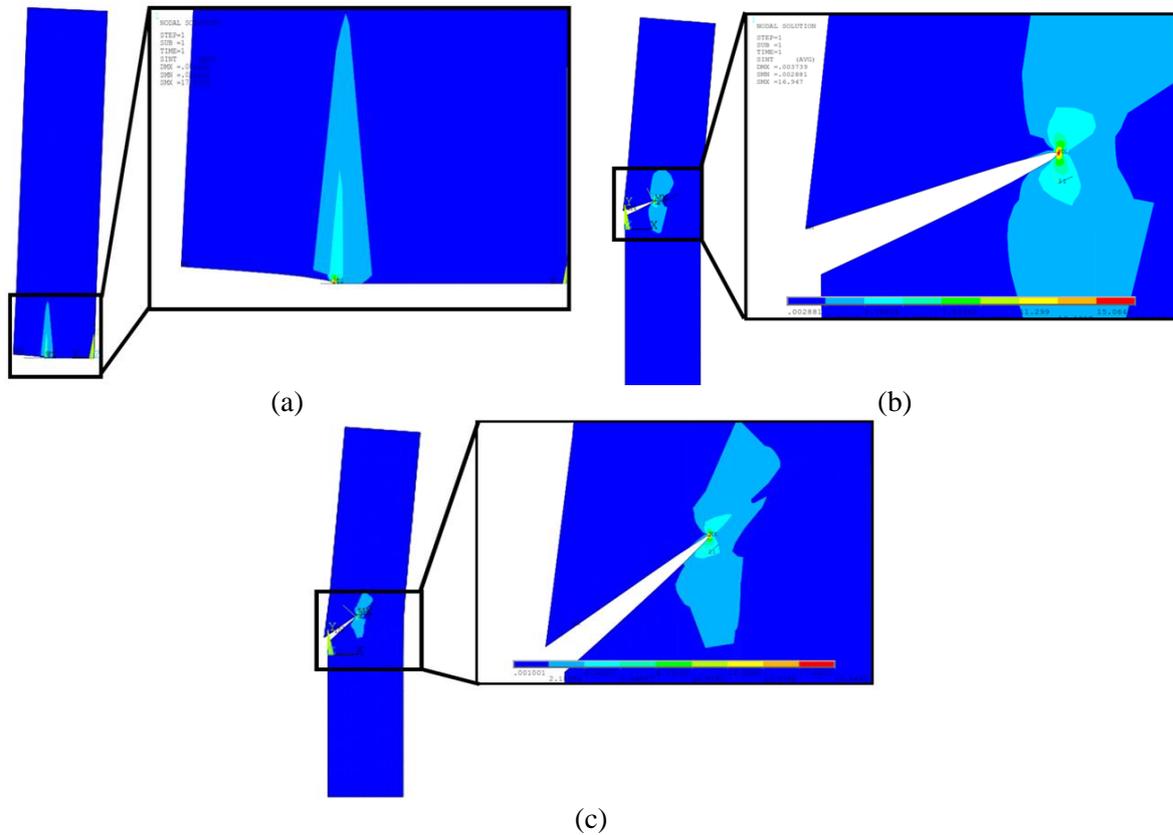


Figure 5 Crack deformation under mode I tension stress of different slanted cracks, (a) 0°, (b) 25° and (c) 45° under tension stress.

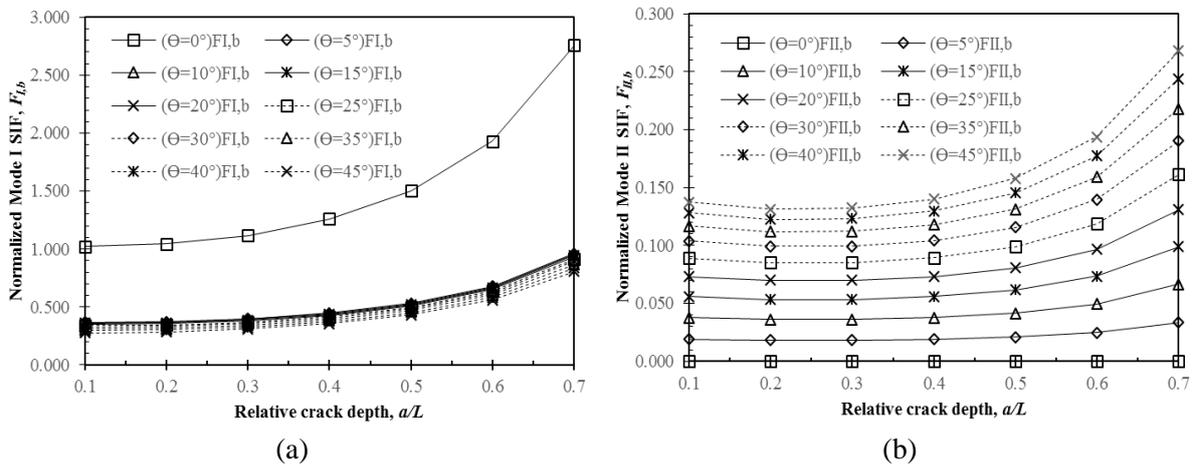


Figure 6 Effect of relative crack depth on the SIFs of (a) mode I and (b) mode II under bending moment.

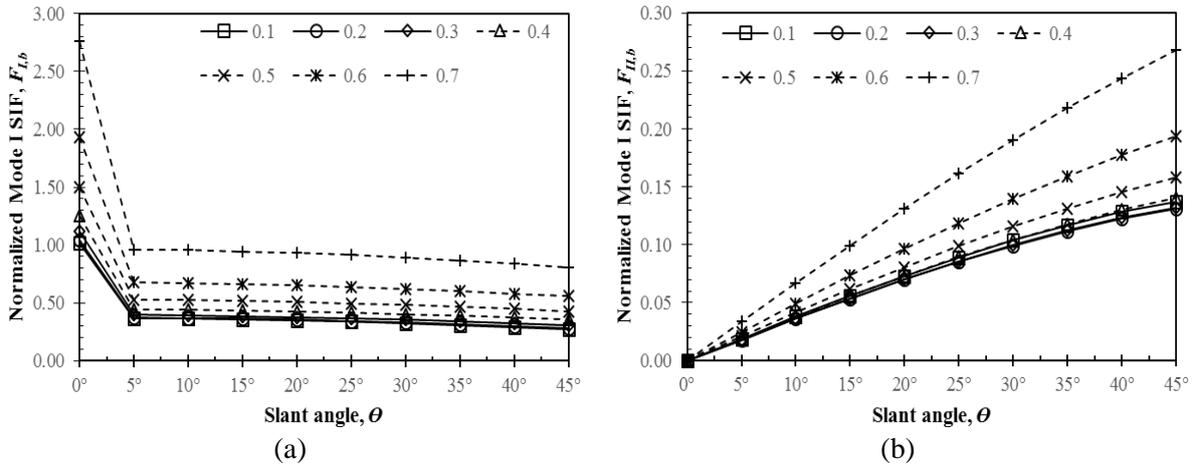


Figure 7 Effect of slanted angles on the SIFs when relative crack depth are varies under bending moment.

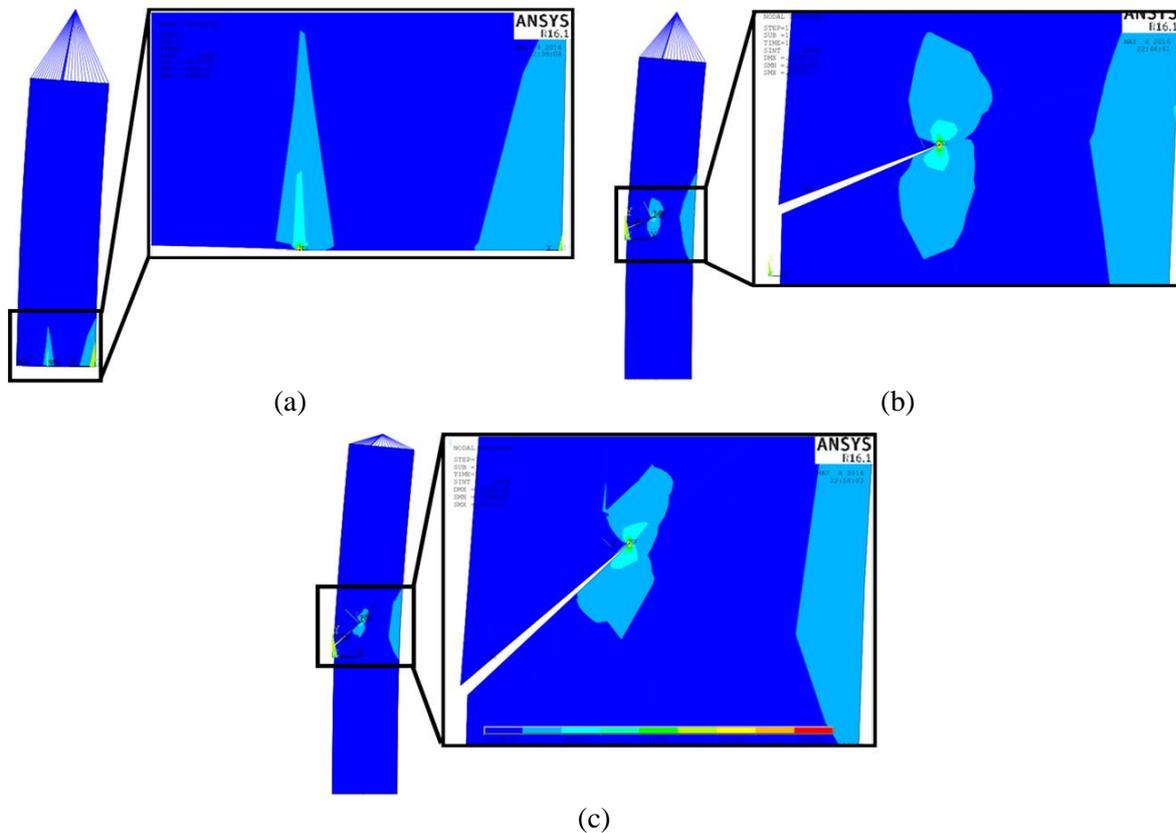


Figure 8 Crack deformation under mode I tension stress of different slanted cracks, (a) 0°, (b) 25° and (c) 45° under bending moment.

Figure 8 shows the stress distribution around crack tip located at plate edge. The crack was subjected to bending moment. The SIF for slant crack subjected to single load was shown in Figure 7. Slant crack at angle 25° and 45° as shown in Figures 8(b) and 8(c) produces smaller opening while slant crack at angle 0° produces larger crack opening as shown in Figure 8(a). As the slanted angles are increased, the symmetrical pattern of stress intensity distributions are disturbed where the upper distribution is shifted to the right while the lower distribution is then shifted to the left. This is due to the fact that the degree of mode II stress intensity factors increased when the slanted angles are increased.

4. Conclusion

This work numerically presents the slanted crack behavior in plain strain plate under bending and tension forces. In general tremendous amount of works on the normal cracks are available. However, lack number of stress intensity factor solutions are obtained especially when the cracks are slanted. Based on the numerical works, several conclusion can be stated as below:

1. Both under bending and tension forces, stress intensity factor increased as a function of crack length.
2. Under tension force, increasing the slanted angles lowered the stress intensity factors. However, the mode I stress intensity factors obtained from slanted angles are higher than the stress intensity factors for normal cracks. On the other hand, mode II stress intensity factors increased when slanted angles are increased due to the capability of plate to slide along the crack lines.
3. Under bending moment, the effect of slanted angles on the mode I stress intensity factors are insignificant where increasing the slanted angles slightly reduced the stress intensity factors. The effect of slanted angles on mode II stress intensity factors are significant where increasing the slanted angles gradually increased the mode II stress intensity factors.

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References

- [1] Courtin S, Gardin C, Bezine G and Ben Hadj Hamouda H 2005 *Engineering Fracture Mechanics* **72** 2175-2185.
- [2] Guinea GV, Planas J and Elices M 2000 *Engineering Fracture Mechanics* **66** 243-255.
- [3] Newman JC Jr, James MA and Zerbst U 2003 *Engineering Fracture Mechanics* **70** 371-385.
- [4] Albinmousa J, Merah N and Khan SMA 2011 *Engineering Fracture Mechanics* **78** 3300-3307.
- [5] Dempsey JP and Mu Z 2014 *Engineering Fracture Mechanics* **132** 93-103.
- [6] Matsumoto T, Tanaka M and Obara R 2000 *Engineering Fracture Mechanics* **65** 683-702.
- [7] Hammond MJ and Fawaz SA 2016 *Engineering Fracture Mechanics* **153** 25-34.
- [8] Kuang JH and Chen LS 1993 *Engineering Fracture Mechanics* **46**(5) 735-741.
- [9] Ismail AE, Ariffin AK, Abdullah S and Ghazali MJ 2014 *Research Journal of Applied Sciences, Engineering and Technology* **7**(10) 1985-1993.
- [10] Ismail AE, Ariffin AK, Abdullah S and Ghazali MJ 2014 *Jurnal Teknologi (Sciences & Engineering)* **68**(1) 7-18.
- [11] Ismail AE, Ariffin AK, Abdullah S and Ghazali MJ 2012 *Meccanica* **47** 1141-1156.
- [12] Ismail AE, Ariffin AK, Abdullah S and Ghazali MJ 2012 *Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)* **13**(1) 1-8.
- [13] Ismail AE, ALM Tobi and NHM Nor 2015 *AIP Conference Proceedings* 1660 070027.
- [14] Lan X, Noda NA, Zhang Y and Michinaka K 2012 *Acta Mechanica Solida Sinica* **25** 404-416.
- [15] Ismail AR, Azlan MA, Mohd Tobi AL and Ahmad MH 2016 *IOP Conf. Series: Materials Science and Engineering* **152** 012049.