Profiling procedure for disk cutter to generate the male rotor, screw compressors component, using the “Substitute Family Circle” - graphic method in AUTOCAD environment

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Abstract. This paper proposes a profiling method for the tool which generates the helical groove of male rotor, screw compressor component. The method is based on a complementary theorem of surfaces enveloping - "Substitute Family Circles Method”. The specific theorem of family circles of substitution has been applied using AUTOCAD graphics design environment facility. The frontal view of the male rotor, screw compressor component, has been determinate knowing the transverse profile of female rotor, and using this theorem of "Substitute Family Circle". The three-dimensional model of the rotor makes possible to apply the same theorem, leading to the surface of revolution enveloping the helical surface.
An application will be also presented to determine the axial profile of the disk cutter, numeric and graphics, following the proposed algorithm.

1. Introduction

The helical surface generating, as constituent surfaces of machine bodies (gears, active elements of helical pumps, screw compressors), is done using tools bounded with revolution surfaces, disk cutter type tool. A distinctive problem in these types of tools is to determine the shape of revolution surface that must be conjugated to the helical surface. The mathematical solution to this problem is known as Theorem I Olivier, or Gohman Theorem [1].

Lyukshin [2] has also developed a vectorial solution to the problem, based on decomposition of helical motion in several rotation motions - Nikolaev Theorem. This solution represents the most often form used for solving analytically the problem of profiling the primary surface of the disk cutter, due the ease expressing of the enveloping condition - the condition of coplanarity of three vectors: helix axis, tool axis disk and a vector that connects two points of those axes.

Also have been elaborated specific complementary theorems as: "Substitute Family Circles" and "Minimum Distance Method" as analytical theorems for approaching this issue [3].

Graphic design software products in AutoCAD or CATIA environments allowed approaching the problem graphically [4,5,6]. The graphic solving is very rigorous and intuitive.

2. Screw Compressor Rotors Front View Profile [7]

In the following we are proposing a graphical method developed in AutoCAD, based on the theorem of "Substitute Family Circles", for profiling the disk cutter which generates the male rotor, screw compressor component, first of all for a particular case, but with prospect to generalizing the method to the other helical surfaces (Figure 1).

The mobiles systems $X_2Y_2Z_2$ and $X_1Y_1Z_1$ will be defined, being attached to, respectively, the male rotor and female rotor, in their transverse section, while they will rotate with the global systems $x_1y_1z_1$ and $x_0y_0z_0$.

The female rotor profile is made up of a series of arcs:

$\overset{\frown}{A_2B_2}$ arc, radius r;

$$A_2B_2 = \begin{vmatrix} X_2 = r_2 - r\cos\theta_1; \\ Y_2 = -r\sin\theta_1; \\ Z_2 = 0; \end{vmatrix} \qquad \theta_{1\min} = 0;\ \theta_{1\max} = \left| ar\cos\frac{r + r_0}{2r_2} \right|; \tag{1}$$

$\overset{\frown}{B_2C_2}$ arc, radius r_0;

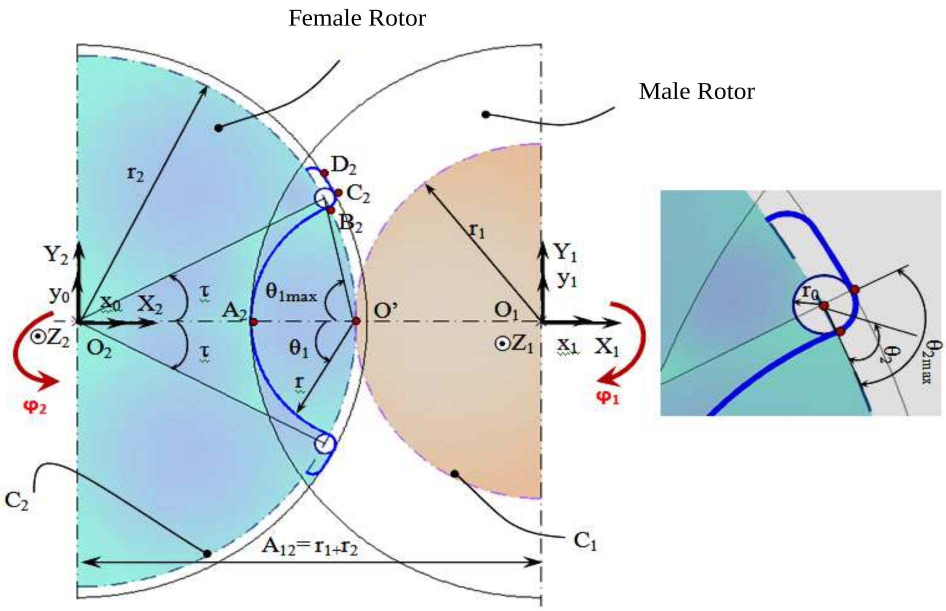

Figure1.The frontal geometry of rotors, screw compressor component [7];
C_1 and C_2 are centrodes in rolling motion, associated to transverse sections of the rotors

$$B_2C_2 = \begin{vmatrix} X_2 = r_2 cos\tau + r_0 cos(\theta_{\max 1} - \theta_2); \\ Y_2 = -r_2 sin\tau - r_0 sin(\theta_{\max 1} - \theta_2); \\ Z_2 = 0; \end{vmatrix} \tag{2}$$

$$\theta_{2\min} = 0;\ \theta_{2\max} = (\pi - \theta_{1\max}); \tag{3}$$

$\overset{\frown}{C_2D_2}$ arc, radius $r_2 + r_0$;
Knowing the absolute movements:

$$\begin{vmatrix} x_2 = \omega_3^T(\varphi_2) \cdot X_2; \\ x_{1`} = \omega_3^T(-\varphi_1) \cdot X_1; \end{vmatrix} \tag{4}$$

and the relative position of global systems,

$$x_1 = x_2 + \begin{Vmatrix} A_{12} \\ 0 \end{Vmatrix}, \tag{5}$$

it will be determine the relative motion

$$X_1 = \omega_3(-\varphi_1)\left[\omega_3^T \cdot X_2 + \begin{Vmatrix} A_{12} \\ 0 \end{Vmatrix}\right]. \tag{6}$$

This motion will determine the envelopes of profiles made up of arcs $\overset{\frown}{A_2B_2}, \overset{\frown}{B_2C_2}, \overset{\frown}{C_2D_2}$, representing the female rotor profile. One circle belongs to the "Substitute Circles Family" (Figure 2) is described by equations as follow:

$$C_i \left| \begin{aligned} X &= -R_{r_1}\sin\varphi_1 + r_i \sin\beta_i; \\ Y &= R_{r_1}\cos\varphi_1 - r_i\cos\beta_i, \end{aligned} \right. \tag{7}$$

r_i , β_i si φ_1 variable.

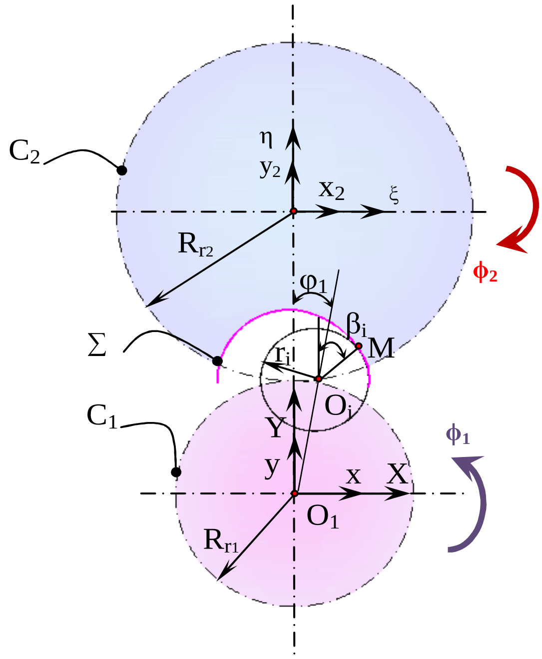

Figure 2.Circle of substitute circles family; generated profile ∑; reference systems

The circle, center O_i, fulfils the tangency conditions to ∑ profile. The tangency condition between circle C_i and ∑ profile means: conditions of contact points (Equation 8) and common tangent in contact point (Equation 9):

$$\left| \begin{aligned} X(u) &= R_{r_1}\sin\varphi_1 - r_i\sin\beta_i; \\ Y(u) &= R_{r_1}\cos\varphi_1 + r_i\cos\beta_i, \end{aligned} \right. \tag{8}$$

$$\begin{aligned} \dot{X}_u^k &= -r_i\cos\beta_i; \\ \dot{Y}_u^k &= -r_i\sin\beta_i, \end{aligned} \tag{9}$$

The equations assembly (8) and (9) allows us to determinate the next condition, as follow:

$$\dot{Y}_u\sin\varphi_1 - \dot{x}(u)\cos\varphi_1 = \frac{x(u)\dot{X}_u + y(u)\dot{Y}_u}{R_{r_1}}, \tag{10}$$

and

$$tg\beta_i = \frac{Y_u}{\dot{X}_u} \quad , \tag{11}$$

meaning the enveloping condition.

The transposition in rolling motion of the substitute family circles leads to envelope shape:

$$\begin{aligned} \xi &= -r_i\cos(\varphi_1+\varphi_2+\beta_i) - R_{r_2}\cos\varphi_2; \\ \eta &= -r_i\sin(\varphi_1+\varphi_2+\beta_i) + R_{r_2}\sin\varphi_2, \end{aligned} \tag{12}$$

$$R_{r_1}\varphi_1 = R_{r_2}\varphi_2 \,. \tag{13}$$

Equation 13 represents the rolling condition of the two centrodes; τ, r_0, r, r_1, r_2 are known sizes.
It will propose, in the following, a graphical solution to the problem, using the "Substitute Family Circles" method.

3. The male rotor profiling

The substitute family circles method [3] assumes the next definition for the *family circles*: family circles belonging to a compound profile, linked of a couple of centrodes in rolling motion, is the family circles having centres on the associated centrode and in tangency with the compound profile, at the same time, as shown in Figure 3. $\overset{\frown}{A_2B_2C_2}$ is made up of some arcs with centres on centrode C_2 and the substitute family circles linked with centrode C_2 is reduced of two circles of radius r_i and centre O_1', respectively, radius r_0 and centre O_0'.

The substitute family circles for $\overset{\frown}{C_2D_2}$ is made up of circles of radius r_0 and centres on centrode C_2 - circles c_1, c_2..

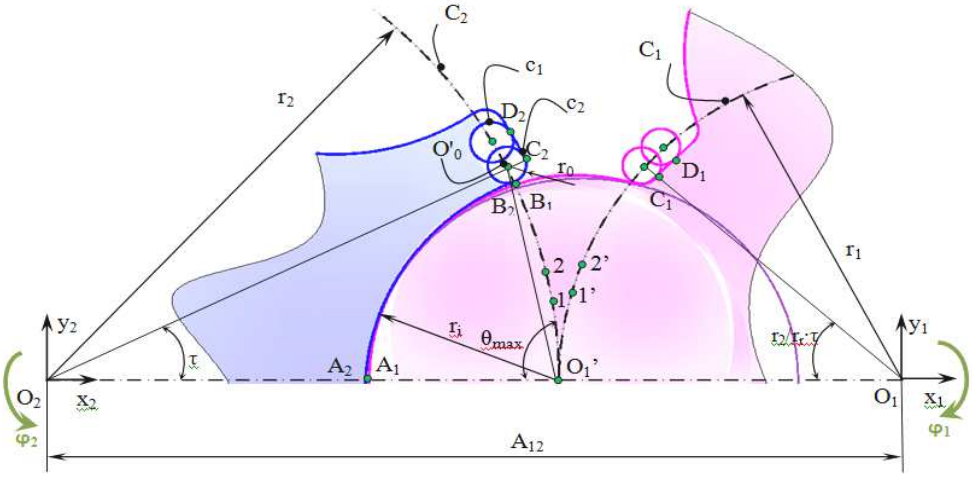

Figure 3.The substitute family circles

In this particular case, the transposed of substitute family circles on centrode C_1, to whom the arcs radius *r*, belong, is made up of arcs, having centres on centrode C_1, in O_1', for $\overset{\frown}{A_1B_1}$ and in O_2', for $\overset{\frown}{BC_1}$, as shown in Figure 4. O_2' will be determinate through condition that arc $\overset{\frown}{O_1'O_0'}$ measured on centrode C_2 to be equal to arc $\overset{\frown}{O_1'O_2'}$ measured on centrode C_1.

$\overset{\frown}{A_1B_1}$ belongs to circle radius „r" centre in O'_1 ,

$\overset{\frown}{BC_1}$ belongs to circle radius "r_0" centre in O'_2.

$\overset{\frown}{B_2B}$ belonging to female rotor profile, will be determinate using point B, which is initially superposed to point B_2, in the relative motion of the two centrodes.

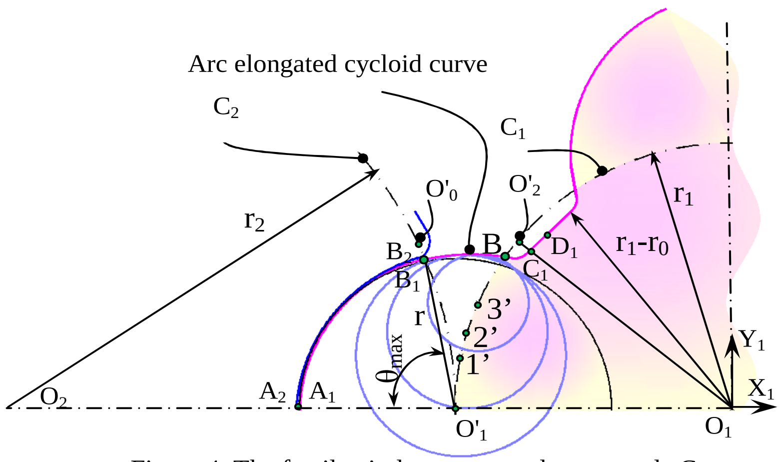

Figure 4. The family circles transposed to centrode C_1

The successive positions of the two centrodes, in the rolling process, and the position of point B, in relation to centrode C_1 attached to male rotor, as well, are represented in Figure 5.

Will be considere a large number of points, on arc $\overset{\frown}{O_1'B}$ angularly equidistant, thus arc $\overset{\frown}{O'_11}$ belonging to centrode C_1, to be equal to arc $\overset{\frown}{O_1'1'}$ belonging to centrode C_2, and so on.

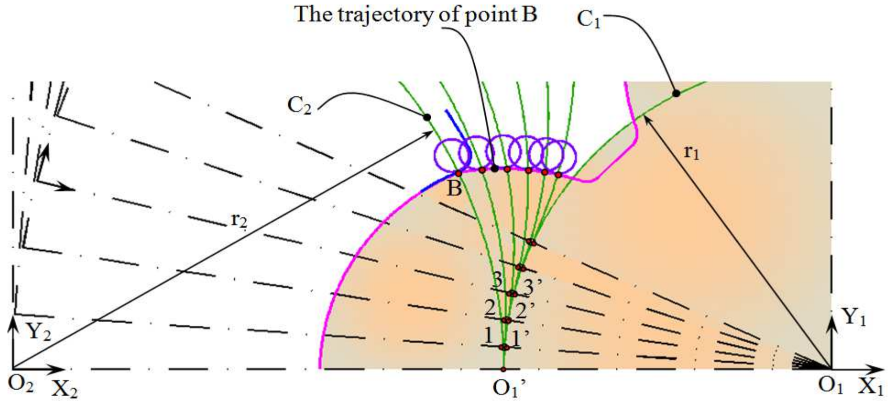

Figure 5. The relative position of centrodes C_1 and C_2 in the rolling process and the trajectory of point B toward the system and X_1 Y_1

The male rotor profile is described by the reference system, whose X_1' axis represents the symmetry axis of the gap between the two successive lobes belonging to male rotor, Figure 6.

Coordinate transformation between the two systems is described by Equation 14:

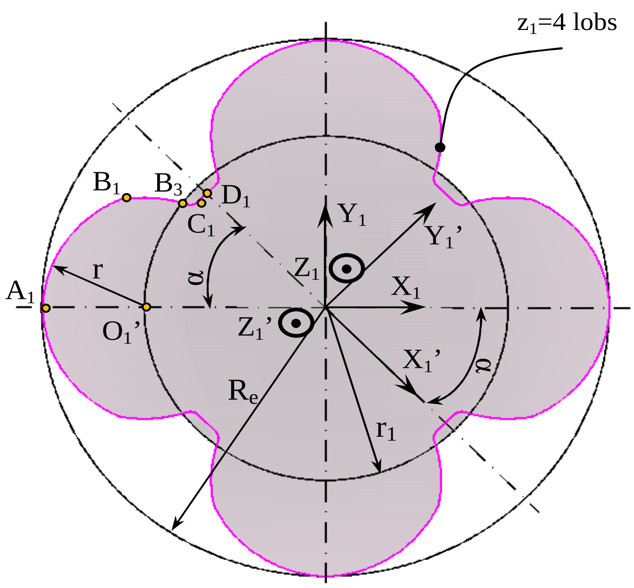

Figure 6. The new reference system X_1'Y_1' of female rotor ($\alpha=45^0$)

$$\begin{Vmatrix} X_1^{'} \\ Y_1^{'} \end{Vmatrix} = \begin{Vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{Vmatrix} \cdot \begin{Vmatrix} X_1 \\ Y_1 \end{Vmatrix} \quad (14)$$

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4. The disk cutter for male rotor

Will be make the solid model of male rotor, Figure 7, reported to the system $X_1'Y_1'Z_1'$, for a rotor left screw, pitch 188.67mm.

The substitution family circles method stipulates that the contact between a helical surface, cylindrical and constant pitch and a revolution surface (the primary peripherical surface of disk cutter) is based on the specific theorem [3], as follows:

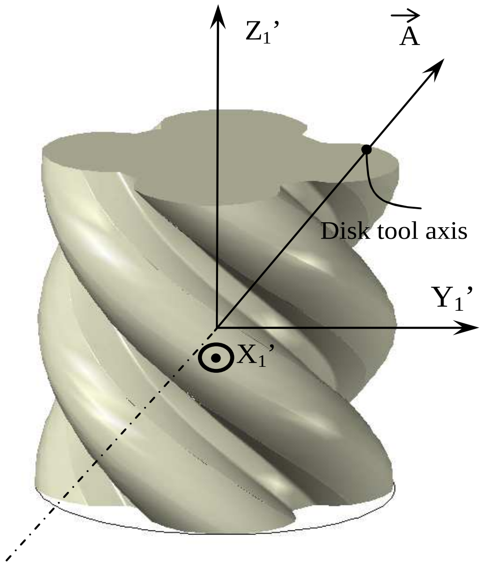

Figure 7. Male rotor 3D solid

The revolution surface (disk cutter), mutually envelope with one helical surface, cylindrical and constant pitch, is the locus made up of family circles, which have the common axis identical with the axis of the future disc cutter; those family circles are tangents to the intersection curves between the helical surface and the orthogonal planes to the disc cutter axis, in transverses planes with the future revolution surfaces axis (Figure 8).

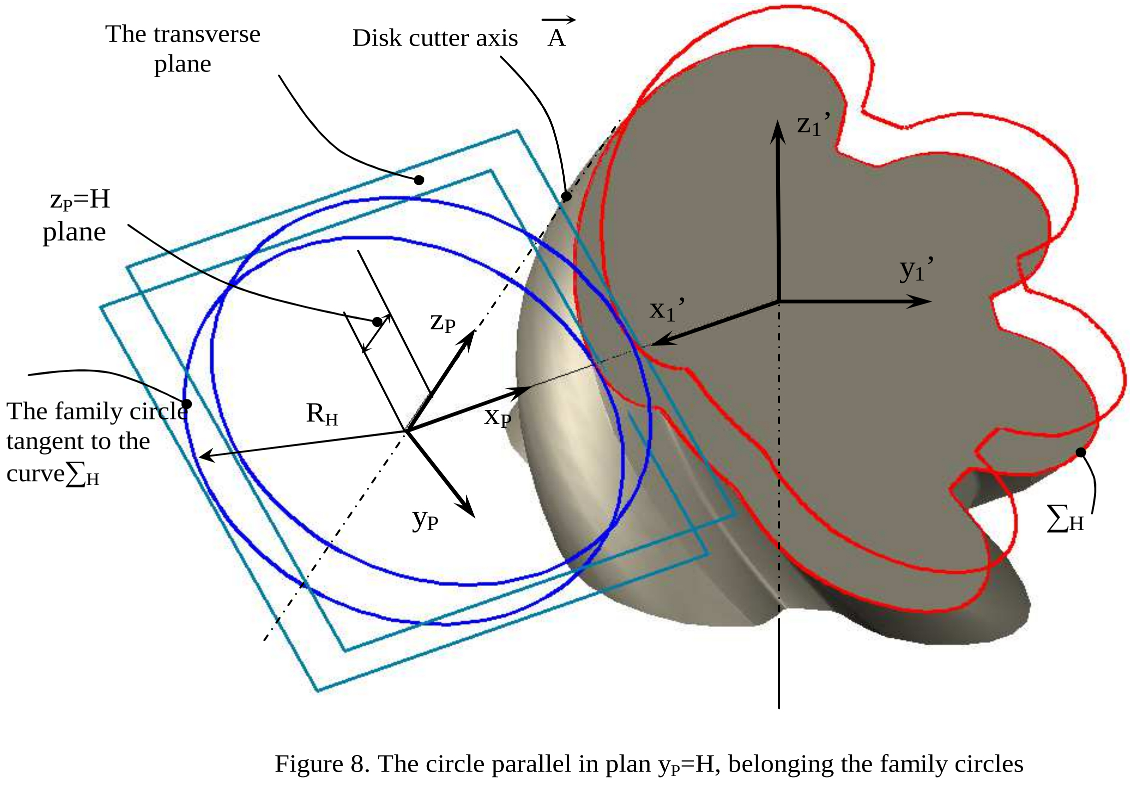

Figure 8. The circle parallel in plan y_P=H, belonging the family circles

The coordinates ensemble (H,R_H) defines the disk cutter axial section, mutually envelope with the helical groove belonging to the generated rotor, for different size of H parameter.

5. Profiling disk cutter generating helical groove of male rotor, using graphical procedure in AUTOCAD environment

Different sizes of H parameter and the coordinates ensemble (H, R_H) determine the axial section of the future disk cutter, which is mutually envelope with the helical grove of male rotor (Figure 9). The centres of the circles radius R_H belong to the tool axis, Z_p and are in tangency with the curves $\sum_H$ – planes curves of the surfaces to generate.

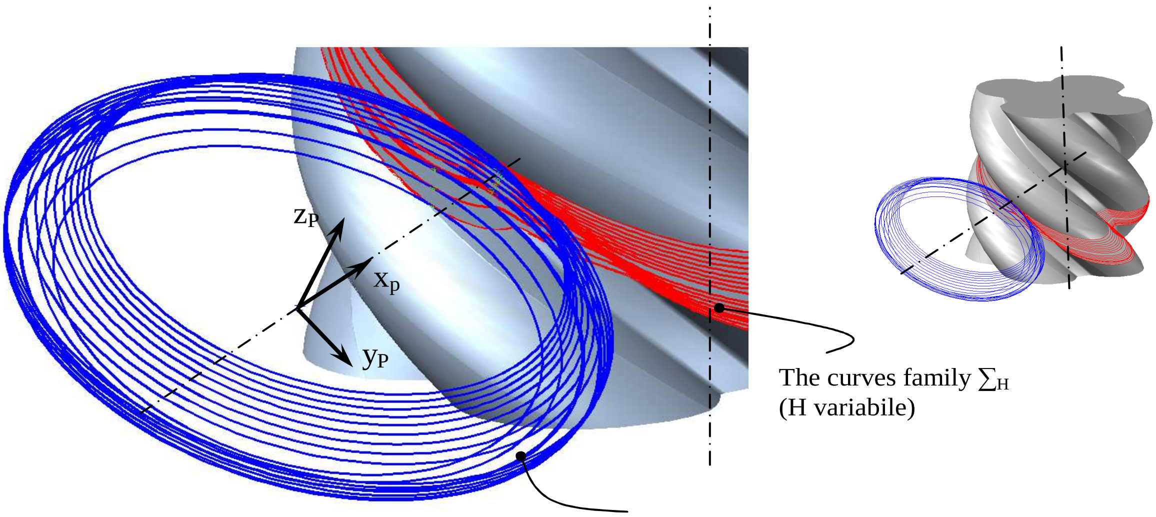

Figure 9. The male rotor solid and "the family of parallel circles" of the revolution surface (disk cutter)

Consider the male rotor screw compressor component, having dimensions as follow: r_1=32mm; r_2=48mm; Re_1=50mm; Re_2=50mm; i=6/4; r=22mm; r_0=1.8mm [7], $\alpha=45^0$, helical pitch P_E=188.67mm; the axis $\vec{A}$, of the future disk cutter will be determinate being in tangency to the counterclokwise helix.

5.1. *AUTOCAD application description*

It will be considere the 3D solid of the male rotor, shown in Figure 7.

The succession of commands it will be as follow:

-UCS command for rotating the coordinates system with the complement of twist angle of the helix;
-SLICE command for determination the transverse plan;
-SECTION command for definition the intersection surface by the transverse plan;
-UCS command for the new coordinates system in the transverse plan;
-EXPLODE the section defined previously;
-CIRCLE command for determination the circles radius minimal and tangent to the curve.
-Repeat the entire sequence above.

Table1 describes the coordinates of the axial section of the disk tool which generates the helical grove of the male rotor.

Table1. The axial section coordinates of the disk cutter

R[mm]	38.936	41.749	43.768	47.791	48.006	..	39.993	39.252	38.560	38.207
H[mm]	3.300	4.300	5.300	6.300	8.000	..	17.000	18.000	19.000	20.000

Figure 10 describes the shape of revolution surface of the disk tool.

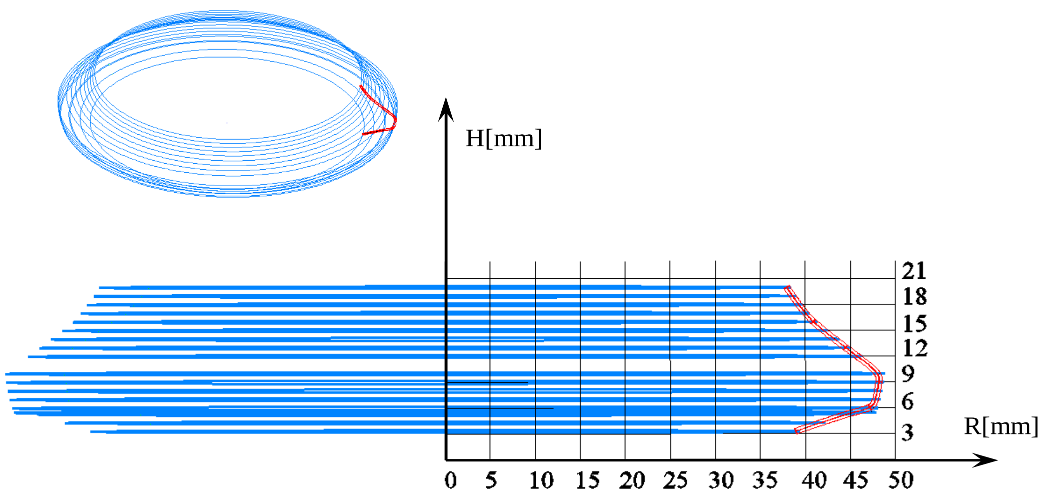

Figure10. The axial section shape of the disk cutter

6. Conclusions

The paper consists in presentation in AUTOCAD environment, regarding a graphical algorithm to profiling the disk cutter which generates the male rotor screw compressor component.

The family circles method has been used, for both, the determination of male rotor transverse profile and for the disk cutter solid generating of helical groove.

The graphical method to express the substitute family circles allows an accurate and fast approach of the mutually envelope profiles, which are in rolling process; this method is also appropriate to shape the revolution surfaces who generate helical surfaces.

For the presented example, the solve of the problem was made up first manually, using AUTOCAD commands. In the future, it can be written a specific code AUTOLISP to allow the automation, after the parameterization of the tool and surfaces dimensions.

The graphical interpretation of points is made with o polyline edited with the option Spline. Once the increasing of number of points, the polyline is more accurate with the theoretical shape of the axial section of the tool.

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