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The Rack-Gear Tool Generation Modelling. Non-Analytical Method Developed in CATIA, Using the Relative Generating Trajectories Method

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Abstract. The modelling of a curl of surfaces associated with a pair of rolling centrodes, when it is known the profile of the rack-gear's teeth profile, by direct measuring, as a coordinate matrix, has as goal the determining of the generating quality for an imposed kinematics of the relative motion of tool regarding the blank. In this way, it is possible to determine the generating geometrical error, as a base of the total error. The generation modelling allows highlighting the potential errors of the generating tool, in order to correct its profile, previously to use the tool in machining process. A method developed in CATIA is proposed, based on a new method, namely the method of "relative generating trajectories". They are presented the analytical foundation, as so as some application for knows models of rack-gear type tools used on Maag teething machines.

1. Introduction

The method of relative generating trajectories, as complementary method for study of enwrapping surfaces, by rolling method, was presented [1]. A variant of this method is presented in this paper. The variant includes a CATIA developed application, regarding the modelling of generation with rack-gear tool for an ordered curl of surfaces (profiles) associated with a pair of circular centrodes. These centrodes rolls on the rectilinear centrode, associated with the rack-gear tool.

The issue presented in this paper was approached in others forms by Baicu [2], using the 3D modelling, as so as by Berbinschi and all [3], based on the "minimum distance" complementary theorem [5].

Also, the generation modelling is used in order to identify the optimal cutting scheme at generation with hob mill of the involutes teeth by Antoniadis [4].

At the same time, Teodor [6] developed an analytical form of the trajectories method, applied also for generation modelling.

A variant of the "relative generating trajectories" method is presented in this paper, as complementary method for study of the enwrapping surfaces associated with a pair of rolling centrodes.

Obviously, the complementary theorems are alternative ways for express the fundamental theorem of Willis [7].

The graphical method proposed in this paper, applied for modelling of surfaces generated with the rack-gear, is based on the "relative generating trajectories" method. This method determines the trajectories of points belong to the tool's cutting edge in their relative motion regarding the blank.

The graphical expression is based on the specific algorithm, developed for a tool with profile known in discrete form, as an ordered cloud of points. This cloud of points is obtained by measuring of the generating tool.

The graphical method is characterised by simplicity of application because use the CATIA capabilities. The method is very rigorous and intuitive at the same time.

The obtained results for presented applications are compared with results obtained by an analytical method, in order to prove the graphical method's quality.

2. The generation modelling using rack-gear tool

The rolling centrodes, reference systems and profile of the rack-gear tool are presented in figure 1.

They are defined the reference systems:

xy is the global reference system;

XY — the reference system joined with the blank and the C_1 centrode;

$\xi\eta$ — relative reference system joined with the C_2 centrode. The generating rack-gear's profile is joined with the C_2 centrode.

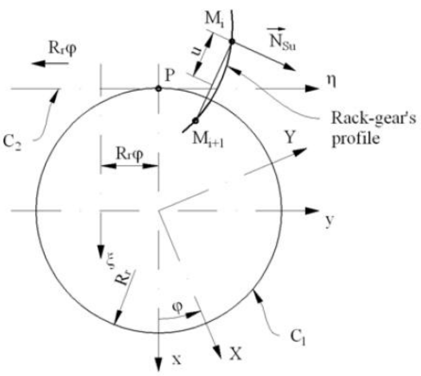

Figure 1. Centrodes and reference systems associated; the profile of rack-gear.

The tool's profile is defined in the $\xi\eta$ reference system, in a discrete form,

$$S = \begin{pmatrix} \xi_1 & \eta_1 \\ \vdots & \vdots \\ \xi_i & \eta_i \\ \vdots & \vdots \\ \xi_n & \eta_n \end{pmatrix}, \; i = 1 \ldots n. \tag{1}$$

For M_iM_{i+1} elementary segment which replace the rack-gear tool, with equations:

$$S_u \begin{vmatrix} \xi = \xi + u \cdot \cos\beta_i; \\ \eta = \eta + u \cdot \sin\beta_i, \end{vmatrix} \tag{2}$$

with,

$$\tan\beta_i = \frac{\Delta\eta}{\Delta\xi}; \; \Delta\xi = \xi_{i+1} - \xi_i; \; \Delta\eta = \eta_{i+1} - \eta_i. \tag{3}$$

For $i = 1 \ldots n$, with n big enough, the profile of the rack-gear tool can be described very rigorous.

The $\vec{N}_{Su}$ normal versor to the elementary profile M_iM_{i+1},

$$\vec{N}_{Su} = \sin\beta_i \cdot \vec{i} + \cos\beta_i \cdot \vec{j} \tag{4}$$

is defined.

In this way, the parametrical equations of the normal to the elementary profile, in the point $M_i(\xi\eta)$, are given by

$$\vec{N}_{Su}\left|\begin{array}{l}\xi = \xi_i + u\cdot\cos\beta_i + \lambda\cdot\sin\beta_i;\\ \eta = \eta_i + u\cdot\sin\beta_i + \lambda\cdot\cos\beta_i,\end{array}\right. \tag{5}$$

withλ scalar parameter. The direction of $\vec{N}_{Su}$ versor is given by (5).

In the relative motion between the rack-gear tool and the blank,

$$X = \omega_3(\varphi)[\xi + a];\ a = \begin{pmatrix} -R_r \\ -R_r\cdot\varphi \end{pmatrix}, \tag{6}$$

the normal (6) describes the family:

$$\left(\vec{N}_{Su}\right)_\varphi\left|\begin{array}{l}X = [\xi_i + u\cdot\cos\beta_i + \lambda\cdot\sin\beta_i - R_r]\cdot\cos\varphi + [\eta_i + u\cdot\sin\beta_i + \lambda\cdot\cos\beta_i - R_r\cdot\varphi]\cdot\sin\varphi;\\ Y = -[\xi_i + u\cdot\cos\beta_i + \lambda\cdot\sin\beta_i - R_r]\cdot\sin\varphi + [\eta_i + u\cdot\sin\beta_i + \lambda\cdot\cos\beta_i - R_r\cdot\varphi]\cdot\cos\varphi.\end{array}\right. \tag{7}$$

In equations (7), u and φ are variable parameters.

For $\lambda = 0$, the equations (7) represents the trajectory family of a point from the rack-gear's profile, in the relative motion regarding the blank.

If the goal is to determine the S profile's enwrapping, it is necessary that the normals family $\left(\vec{N}_{Su}\right)_\varphi$ to pass through the gearing pole, P, for various rolling positions [7]. The different rolling positions are obtained for various values of the φ parameter.

If the gearing pole, P, is defined in the XY reference system,

$$P\left|\begin{array}{l}X_P = -R_r\cdot\cos\varphi;\\ Y_P = R_r\cdot\sin\varphi,\end{array}\right. \tag{8}$$

then, from (7) and (8) results the equations assembly:

$$\begin{array}{l}[\xi_i + u\cdot\cos\beta_i - R_r]\cdot\cos\varphi + [\eta_i + u\cdot\sin\beta_i - R_r\cdot\varphi]\cdot\sin\varphi + \lambda\cdot\sin(\varphi+\beta_i) = -R_r\cdot\cos\varphi;\\ -[\xi_i + u\cdot\cos\beta_i - R_r]\cdot\sin\varphi + [\eta_i + u\cdot\sin\beta_i - R_r\cdot\varphi]\cdot\cos\varphi + \lambda\cdot\cos(\varphi+\beta_i) = R_r\cdot\sin\varphi.\end{array} \tag{9}$$

The λ variable scalar is eliminated between the two conditions (9) and the enwrapping condition specifically for the relative generating trajectories is determined. This condition establish the blank's flank as enveloping for the trajectories family for points belong to the rack gear tool's flank, regarding the XY reference system of the blank,

$$\varphi = \frac{(\xi_i + u\cdot\cos\beta_i)\cdot\cos\beta_i - (\eta_i + u\cdot\sin\beta_i)\cdot\sin\beta_i}{-R_r\cdot\sin\beta_i}. \tag{10}$$

The equations assembly:

$$\begin{array}{l}X = (\xi_i + u\cdot\cos\beta_i - R_r)\cdot\cos\varphi + (\eta_i - u\cdot\sin\beta_i - R_r\cdot\varphi)\cdot\sin\varphi;\\ Y = -(\xi_i + u\cdot\cos\beta_i - R_r)\cdot\sin\varphi + (\eta_i - u\cdot\sin\beta_i - R_r\cdot\varphi)\cdot\cos\varphi,\end{array} \tag{11}$$

and the condition (10), determine the enwrapped profile — the analytical model of the blank's generated surface.

3. Application

The involute flank generated with a rack-gear tool is presented in figure 2. In the same figure are presented the pair of rolling centrodes and the reference systems.

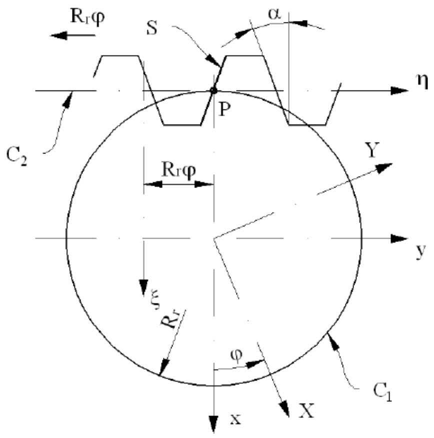

Figure 2. Rack-gear; reference systems.

The *S* profile of the generating rack-gear was obtained in discrete form by measuring with a profile projector, see figure 3.

Figure 3. The real rack-gear flank's measuring.

In table 1 are presented the point's coordinates measured on the rack-gear's profile.

Table 1. Coordinates measured on rack-gear profile.

Nr. crt.	X [mm]	Y [mm]
1	0.000	0.000
2	0.150	0.348
3	0.139	0.312
4	0.302	0.723
5	0.690	1.722
6	1.040	2.724
7	1.357	3.721
8	1.708	4.725
9	2.032	5.721
10	2.380	6.724
11	2.742	7.720
12	3.086	8.725
13	3.430	9.721
14	3.783	10.726
15	4.148	11.726
16	4.511	12.722
17	4.660	13.250

The equations assembly (11) and (10) determines the generating profile for the input data: R_r = 50 mm, rolling circle radius; m = 5 mm, modulus and α = 20°.

In figure 4 is presented the generated profile of an involute tooth, the analytical modelled profile.

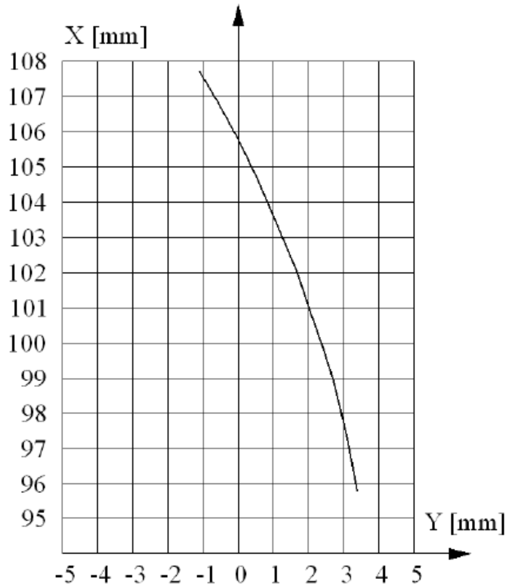

Figure 4. The modelled profile

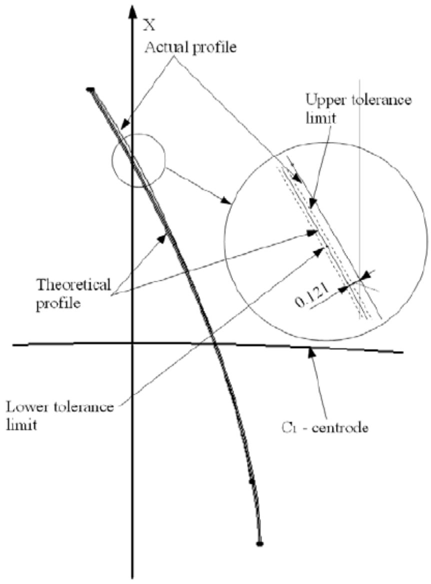

Figure 5. The generated profile

A graphical solution was developed in CATIA design environment.

According to this method, in DMU Kinematics module, the generating process was simulating by rolling for the tooth's flank, using the rack-gear tool.

The rack-gear measured flank was modelled using a spline curve which has as control points those points with coordinates measured using the profile projector.

In the same file was drawn a normal line to the tool's profile (to the spline curve) which was constrained to pass through one of the control points.

By simulating the mechanism function, keeping as fixed element the rack-gear, is found the position when the previously drawn normal line pass through the gearing pole. The intersection point between this normal and the modelled profile will be a point from the generated involute profile.

This last step is repeated for each of the measured points, obtaining points onto the generated profile.

Due to the fact that the deviation of the rack-gear tool's profile exceeds the allowed limits we can expect that the deviations of the tooth's profile exceed the allowed limits too.

In figure 5 are presented the limits obtained for the given input data.

4. Conclusions

The relative generating trajectories method is a complementary method for study of the enwrapping surfaces.

The presented applications refers to conjugated profiles associated with a pair of rolling centrodes — the particular case of the generation with rack-gear regarding the generation starting from the discrete form of the tool's profile.

The graphical solution developed in CATIA use the capabilities of this design environment. The analytical results and those obtained with the graphical algorithm are identical from the technical point of view.

The graphical method is simple, intuitive and easy to apply.

The generating trajectories may highlight the problem linked with the profile's interference at machining.

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Acknowledgments

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