

# Mechanical device for determining the stiffness and the viscous friction coefficient of shock absorber elements modelled by bond graph

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**Abstract.** The present paper presents a mechanical device for the assessment of the fundamental parameters of a shock absorber: the spring stiffness and the viscous friction coefficient, without disassembling the absorber. The device produces an oscillatory motion of the shock absorber and can measure its amplitude and angular velocities. The dynamic model of the system, consisting of the mechanical device and the shock absorber, is performed by using the bond-graph method. Based on this model, the motion equations are obtained, which by integration lead to the motion law. The two previously mentioned parameters are determined by using this law and the measured values of two amplitudes and of their corresponding angular velocities. They result as solutions of a system of two non-linear algebraic equations.

## 1. Introduction

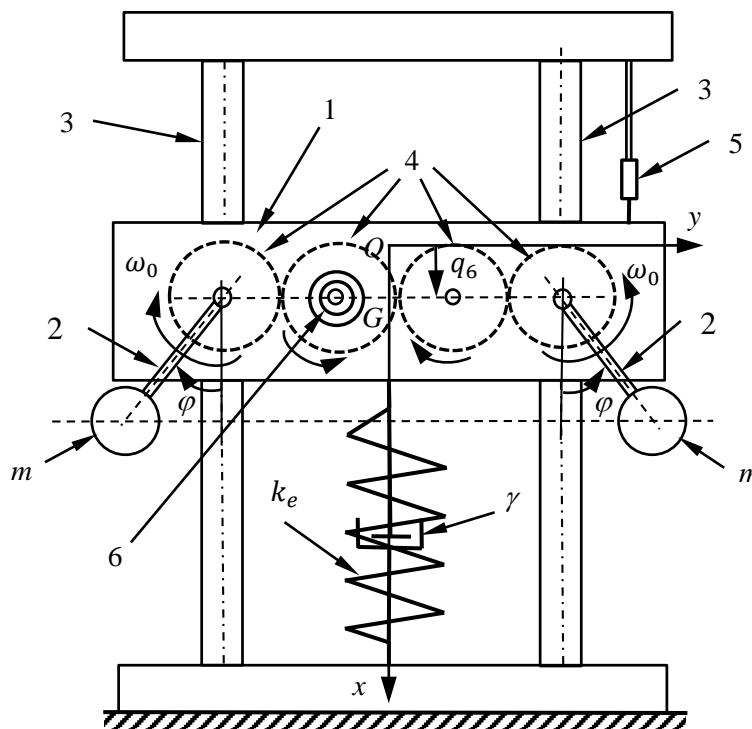
The shock absorbers are used in various applications, especially in the automotive industry. The main parts of a shock absorber are a spring and a dumper. Consequently, the most important parameters of such a device are the spring stiffness and the viscous friction coefficient. These parameters are usually determined separately for the spring [1], [2], [3], and for the dumper [4], [5], [6], [7], by using appropriate theoretical methods or devices. There are papers which study both parameters [8], [9], [10]. Some papers present the two parameters assessment by using the force provided by the force sensor located at the upper end of the shock absorber. This force is the sum of the force developed by the spring and by the dumper and cannot be used for determining these parameters. The paper presents a mechanical device which can provide both parameters by exciting the shock absorber with an inertial mechanism. The mechanism consists of two identical eccentric masses rotating with the same constant angular velocity in opposite directions. The mechanism is fixed at the upper end of the shock absorber, which is in a vertical position. The force of inertia of these two masses induces an oscillatory vertical motion of the shock absorber. The motion equation of the whole device is determined by using the bond graph method [11], [12], [13]. It is necessary to measure two values of the amplitude at corresponding angular velocities, in order to determine the spring stiffness and the viscous friction coefficient of the dumper. Finally, the required parameters of the shock absorber are obtained by resolving an algebraic system of two equations.



The device is very useful in industry, because it provides a very affordable method for determining the parameters of shock absorbers.

## 2. The mechanical device

The mechanical device consists in a box (1), having mass  $M$  (Fig.1). It can slide on two parallel columns (3) and it is supported by a shock absorber, which is made of a spring of stiffness  $k_e$  and a dumper, whose viscous friction coefficient is  $\gamma$ . The shock absorber is fixed according to the code [14]. Additional masses can be added to the box (1), when a pre-load is required.



**Figure 1.** The mechanical device

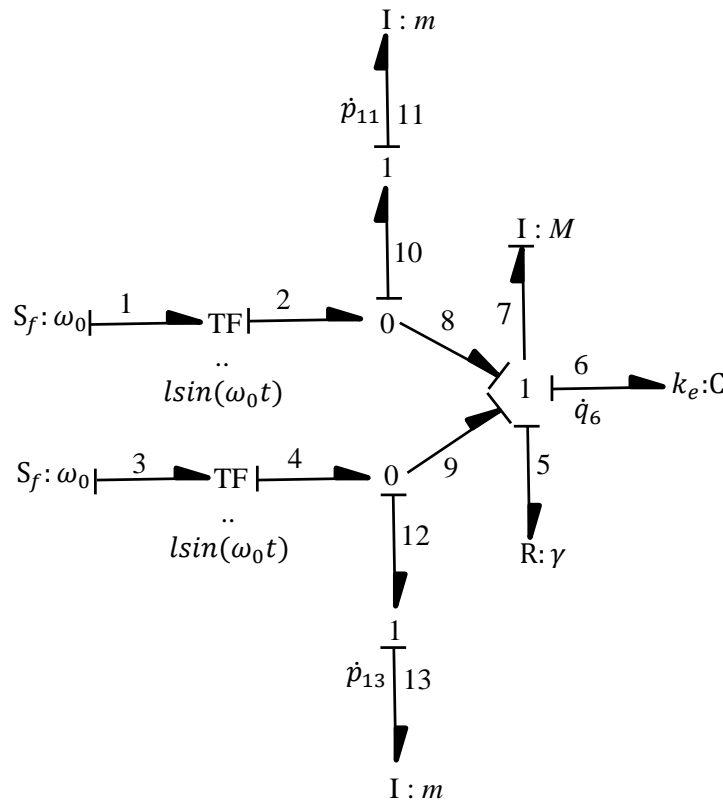
A mechanical system, consisting of four identical gear wheels (4), driven by an electric motor with variable angular velocity, is located inside the box (1). The motor set in rotational motion two rods (2),  $l$  in length. The rods, which have a negligible mass, rotate with the same, constant angular velocity  $\omega_o$ , in opposite directions, starting from the vertical position ( $\varphi=0$ ). The rods angle of rotation is  $\varphi = \omega_o t$ . Each rod it is provided with a mass  $m$  at its end. When the rods rotate, the box (1) has a vertical oscillatory motion, induced by the inertial force of the two masses. For the assessment of spring stiffness and of viscous friction coefficient, it will be shown that is necessary to measure the amplitude of the oscillatory motion for mass (1) and the angular velocity for bars (2). The amplitude measurement is performed by using a displacement sensor (5) of type F38000204, with a travel of 102.87mm and a linearity of 0.1%. For the angular velocity measurement, a sensor, resistant to vibrations and shocks, of type TQG19E (30g for vibration and 100g for shocks) is recommended. The electric motor and the sensor for angular velocity measurement are located on the axis of one of wheels (4) and they are both denoted by (6) in figure 1.

### 3. The bond-graph model of the mechanical device

The bond-graph model of the mechanical device is pictured in Fig.2. It contains two sources of flow, corresponding to the rods angular velocities,  $\omega_o$ . The inertial force of masses  $m$ , which induces the system motion, is oriented in the positive direction of  $Ox$  axis and its velocity is  $\omega_o l \sin(\omega_o t)$ . The transition from the angular velocity,  $\omega_o$ , to the velocity of the inertial force is modeled by introducing a transformer defined by the parameter  $l \sin(\omega_o t)$ . Then, a 0-junction, which distributes the inertial force power to masses  $m$  and  $M$ , is introduced. A 1-junction is associated to each mass. A capacitor, corresponding to the spring, a dissipative element corresponding to the dumper and an inertial element corresponding to mass  $M$  are attached to 1-junction for mass  $M$ .

As a consequence of the causal assignment, two elements in integral causality (the spring and mass  $M$ ), and two elements in derivative causality (the two masses  $m$ ) result.

The bond-graph model leads to the following system of differential-algebraic equations:



**Figure 2.** The bond graph model

$$\dot{p}_7 = -k_e q_6 + \gamma \frac{p_7}{M} + \dot{p}_{11} + \dot{p}_{13} \quad (1)$$

$$\dot{q}_6 = \frac{p_7}{M} \quad (2)$$

$$p_{11} = m \omega_o l \sin(\omega_o t) - m \frac{p_7}{M} \quad (3)$$

$$p_{13} = m \omega_o l \sin(\omega_o t) - m \frac{p_7}{M} \quad (4)$$

For finding the motion equation of mass  $M$  in terms of displacement variable  $q_6$ , equations (2), (3) and (4) are differentiated and introduced in equation (1).

If variable  $q_6$  is denoted by  $x$ , the following motion equation of mass  $M$  results:

$$\ddot{x}(M + 2m) + \gamma\dot{x} + k_e x = 2m\omega_0^2 l \cos(\omega_0 t) \quad (5)$$

which for integration is written in the form:

$$\ddot{x} + \gamma(M + 2m)^{-1}\dot{x} + k_e(M + 2m)^{-1}x = 2m\omega_0^2 l(M + 2m)^{-1} \cos(\omega_0 t) \quad (6)$$

#### 4. Stationary response assessment

The solution of the homogeneous equation:

$$\ddot{x} + \gamma(M + 2m)^{-1}\dot{x} + k_e(M + 2m)^{-1}x = 0 \quad (7)$$

is a non-periodic motion or an oscillatory motion but, in both circumstances, this component of the general solution tends to 0.

A particular solution of equation (6) is chosen of the following shape:

$$x_p = D \sin(\omega_0 t) + E \cos(\omega_0 t) \quad (8)$$

that is the stationary response of mass  $M$  motion.

$C$  and  $D$  are constants, which are determined by imposing relation (8) as solution of equation (6).

The two constants have the values:

$$D = 2ml\omega_0^2 \left( \frac{\gamma}{M+2m} \right) \left( (M + 2m) \left[ \left( \frac{k_e}{M+2m} - \omega_0^2 \right)^2 + \left( \frac{\gamma\omega_0}{M+2m} \right)^2 \right] \right)^{-1} \quad (9)$$

$$E = 2ml\omega_0^2 \left( \frac{k_e}{M+2m} - \omega_0^2 \right) \left( (M + 2m) \left[ \left( \frac{k_e}{M+2m} - \omega_0^2 \right)^2 + \left( \frac{\gamma\omega_0}{M+2m} \right)^2 \right] \right)^{-1} \quad (10)$$

The final shape of the stationary response is:

$$x_p = \sqrt{D^2 + E^2} \sin(\omega_0 t + \theta) = A \sin(\omega_0 t + \theta) \quad (11)$$

where

$$A = \sqrt{D^2 + E^2} \text{ and } \tan(\theta) = \frac{E}{D} \quad (12)$$

The amplitude of the stationary response has the form:

$$A = (2ml\omega_0^2)(M + 2m)^{-1} \left[ \left( \frac{k_e}{M+2m} - \omega_0^2 \right)^2 + \left( \frac{\gamma\omega_0}{M+2m} \right)^2 \right]^{-1/2} \quad (13)$$

The amplitude of the stationary response depends on  $\omega_0$ , but

$$\lim_{\omega_0 \rightarrow \infty} A = \frac{2ml}{M+2m} \quad (14)$$

which demonstrates that the amplitude depends no more on the spring stiffness and on the dumper viscous friction coefficient, when  $\omega_0$  has great values.

#### 5. Experimental assessment of spring stiffness and dumper viscous friction coefficient

The experimental assessment of these two parameters is based on the measuring of two amplitudes  $A_1$  and  $A_2$ , corresponding to the angular velocities  $\omega_1$  and  $\omega_2$ , both much lower than the limit value, given by equation (14).

The following system of equations, whose unknowns are  $k_e$  and  $\gamma$ , is obtained:

$$A_1 = \frac{2ml}{M+2m} \omega_1^2 \left[ \left( \frac{k_e}{M+2m} - \omega_1^2 \right)^2 + \left( \frac{\gamma\omega_1}{M+2m} \right)^2 \right]^{-1/2} \quad (15)$$

$$A_2 = \frac{2ml}{M+2m} \omega_2^2 \left[ \left( \frac{k_e}{M+2m} - \omega_2^2 \right)^2 + \left( \frac{\gamma \omega_2}{M+2m} \right)^2 \right]^{-1/2} \quad (16)$$

In order to resolve the system, the following notations are performed:

$$L = \frac{2ml}{M+2m}, P = \frac{k_e}{M+2m}, \text{ and } Q = \frac{\gamma}{M+2m} \quad (17)$$

The new shape of the system is:

$$A_1 = L \omega_1^2 [(P - \omega_1^2)^2 + Q^2 \omega_1^2]^{-1/2} \quad (18)$$

$$A_2 = L \omega_2^2 [(P - \omega_2^2)^2 + Q^2 \omega_2^2]^{-1/2} \quad (19)$$

and the unknowns become  $P$  and  $Q$ .

Unknown  $P$  expression is:

$$P = \omega_1 \omega_2 \left[ 1 + \frac{L^2}{\omega_2^2 - \omega_1^2} \left( \frac{\omega_1^2}{A_1^2} - \frac{\omega_2^2}{A_2^2} \right) \right]^{1/2} \quad (20)$$

and, based on relations (17), it results:

$$k_e = (M + 2m) \omega_1 \omega_2 \left[ 1 + \frac{L^2}{\omega_2^2 - \omega_1^2} \left( \frac{\omega_1^2}{A_1^2} - \frac{\omega_2^2}{A_2^2} \right) \right]^{1/2} \quad (21)$$

In the same manner, unknown  $Q$  expression is:

$$Q = \left( \frac{L^2}{A_1^2} \omega_1^2 - \frac{(P - \omega_1^2)^2}{\omega_1^2} \right)^{1/2} \quad (22)$$

and, based on the same relations (17),  $\gamma$  results:

$$\gamma = (M + 2m) \omega_1 \left[ \frac{L^2}{A_1^2} - \left( \frac{P}{\omega_1^2} - 1 \right)^2 \right]^{1/2} \quad (23)$$

## 6. Conclusions

The shock absorbers have as main components a spring and a damper. The parameters of these two elements are: the spring stiffness and the damper viscous friction coefficient. The magnitude of these parameters is of a great interest for two main reasons. The first one is the possibility of checking if they coincide with the designed values. The other one is the necessity of using their values in the mathematical model of the mechanical device, the shock absorber belongs to.

The mechanical device presented in the work permits the assessment of the stiffness of a spring and the viscous friction coefficient of a damper, as elements of the shock absorber, without disassembling it.

The mathematical model of the mechanical device is constructed by using the bond-graph method. This mathematical model provides the relations for computing the shock absorber parameters, based on the measurements of two oscillatory motion amplitudes and of the corresponding angular velocities. They both have to be much lower than a limit computed value.

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