Mutual Coupling Compensation on Spectral-based DOA Algorithm

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Abstract. Direction of arrival (DOA) estimation using isotropic antenna arrays are commonly being implemented without considering the mutual coupling effect in between the array elements. This paper presents an analysis of DOA estimation with mutual coupling compensation using a linear antenna array. Mutual coupling effect is represented by mutual coupling coefficients and taken into account when calculating the array output. The mutual coupling compensation technique exploits a banded mutual coupling matrix to reduce the computational complexity. The banded matrix reflects the relationship between mutual coupling effect and the element spacing in an antenna array. The analysis is being carried out using the Capon algorithm, one of spectral-based DOA algorithms, for estimating the DOA of incoming signals. Computer simulations are performed to show the performance of the mutual coupling compensation technique on DOA estimation. Simulation results show that, in term of estimation resolution, the mutual coupling compensation technique manages to obtain a comparable results compared to the case without mutual coupling consideration. However, the mutual coupling compensation technique produces significant estimation error compared to the case without mutual coupling. The study concludes that the banded matrix of mutual coupling coefficients should be properly designed to improve the performance of mutual coupling compensation technique in DOA estimation.

1. Introduction

Array processing is an essential process that is being used to determine the location, strength of incoming signals. The ideal condition in array processing assumes that the elements are isotropic, which ignores the effect of the mutual coupling. In addition, the array manifold also assumes the array geometry and signals are known. However, in a real application the array is affected by mismatch array manifold due to physical conditions such as mutual coupling, phase uncertainties and sensor positioning perturbation [1, 2]. These impairments need additional procedure before signal processing takes place, which is known as array calibration [3-5]. The improvement contributed by the array calibration process has shown a greater importance in array processing [2].

Mutual coupling effect in array processing has been investigated extensively in the last decade to improve the performance of smart antenna system. The mutual coupling effect is caused by the interaction between neighbour elements in an array. The interaction affects the steering vector of each element and also the array manifold. Since the steering vector is the essential information to locate signal

sources, thus the performances of direction of arrival (DOA) algorithms are also affected [6, 7]. Several methods have been proposed to improve the DOA estimation with the presence of mutual coupling. Method of moment is one of early being proposed to calculate and compensate the mutual coupling effect in array processing [8, 9]. However, this meth method requires a priori knowledge of incoming signals such exact number of signals and the DOA. A maximum likelihood based method is also being proposed that exploits the source calibration [10]. It is particularly effective to determine the coupling coefficients, and could be extended to find gain/phase uncertainties and sensor positioning errors.

Another method that also exploits source calibration is being proposed for known location in [11]. The drawback of both methods in [11] is that the source calibration is difficult to be determined in practical applications. Auto-calibration is being proposed in [12, 13] that using an iterative process to calculate both the DOA and coupling coefficients. The advantage of auto calibration is that it does not require the source calibration and thus it could be done offline. Nevertheless, this method is not preferable is a real-time system since it involves many parameters and thus has high computational complexity. Furthermore, it has been reported that this method could take a considerable amount of time in order for the DOA calculation to converge [13]. Another method has been proposed to address the problem of computational complexity [14, 15]. This method is designed specifically for the linear array [14] and circular array [15]. The simplicity in computation is coming from the fact that the mutual coupling is inversely proportional to the element spacing in the array. In other words, the mutual coupling will reduce as the element is further apart. This fact leads to simpler representation of mutual coupling matrix since the mutual coupling is considered as zero after several element spacing.

The focus in this work is to analyse the performance of mutual coupling compensation technique in DOA estimation. This is done through a comparison of DOA estimation between the mutual coupling compensation technique and ideal case that ignore the mutual coupling effect.

2. Mutual coupling compensation

Suppose K narrowband, uncorrelated signals impinging a linear array that has N elements with inter element spacing of 0.5λ, where λ is the wavelength of incoming signals. Each of the kth source has an elevation angle θ_k, $k = 1, 2, \ldots, K$. In ideal condition, the signal output at the ith element at time t can be written as:

$$x_i(t) = \sum_{k=1}^{K} \mathbf{a}_i(\theta_k) s_k(t) + n_i(t) \tag{1}$$

where $s(t)$ is desired signal, and $n(t)$ is Gaussian white noise of zero mean and covariance matrix $\sigma^2 \mathbf{I}$, $\mathbf{I}$ is identity matrix of size N and $\mathbf{a}(\theta_k)$ is the steering vector defined by:

$$\mathbf{a}(\theta_k) = \left[\left(1, \ e^{-j\alpha_2}, \ \ldots, \ e^{-j\alpha_N}\right)\right]^T \tag{2}$$

where $\alpha_k = \dfrac{2\pi d \sin \theta_k}{\lambda}$, $(\ )^T$ is matrix transpose operation, and d is element spacing. In vector notation, (1) can be rewritten as:

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{3}$$

where

$$\mathbf{x}(t) = \left[x_1(t) \ x_2(t) \ \cdots \ x_N(t)\right]^T \tag{4}$$

$$\mathbf{s}(t) = \begin{bmatrix} s_1(t) & s_2(t) & \cdots & s_N(t) \end{bmatrix}^T \tag{5}$$

$$\mathbf{n}(t) = \begin{bmatrix} n_1(t) & n_2(t) & \cdots & n_N(t) \end{bmatrix}^T \tag{6}$$

$$\mathbf{A}(\theta) = \begin{bmatrix} a_1(\theta_k) & a_2(\theta_k) \cdots & a_N(\theta_k) \end{bmatrix}^T \tag{7}$$

In practical applications, there exists interaction between elements and creates mutual coupling effect. Taking this effect into consideration, the steering vectors can be rewritten as the following:

$$\mathbf{a}_m(\theta) = \mathbf{C}\mathbf{a}(\theta) \tag{8}$$

where **C** is the mutual coupling matrix (MCM), and it has Toeplitz property:

$$\begin{bmatrix} 1 & c_1 & \cdots & c_{p-1} & \cdots & c_{N-1} \\ c_1 & 1 & c_1 & \ddots & \ddots & \vdots \\ \vdots & c_1 & \ddots & \ddots & \ddots & c_{p-1} \\ c_{p-1} & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & c_1 \\ c_{N-1} & \cdots & c_{p-1} & \cdots & c_1 & 1 \end{bmatrix} \tag{9}$$

where $c_i = \rho_i e^{j\phi_i}$ is the mutual coupling coefficients with ρ_i and ϕ_i represents amplitude and phase of mutual coupling coefficients respectively. The structure of MCM in (9) can be used to determine the DOA estimation. However, the computational complexity will be significantly high when the number of elements is increased. Using the fact that the mutual coupling effect is inversely proportional to elements spacing, the mutual coupling coefficients can be considered as zero after several element spacing.

Suppose the mutual coupling coefficients are reduced to zero after P element spacing, that is

$$c_i = 0 \qquad P < i < N - 1 \tag{10}$$

Therefore the MCM in (9) can be rewritten in a simpler form as the following:

$$\begin{bmatrix} 1 & c_1 & \cdots & c_{p-1} & & & \\ c_1 & 1 & c_1 & \cdots & c_{p-1} & & 0 \\ \vdots & c_1 & \ddots & \ddots & \cdots & \ddots & \\ c_{p-1} & \cdots & \ddots & \ddots & \ddots & \cdots & \ddots \\ & \ddots & \cdots & \ddots & \ddots & \ddots & \cdots & c_{p-1} \\ & & \ddots & \cdots & \ddots & \ddots & \ddots & \vdots \\ & 0 & & \ddots & \cdots & \ddots & \ddots & c_1 \\ & & & c_{p-1} & \cdots & c_1 & 1 \end{bmatrix} \tag{11}$$

The array manifold in (7) can be rewritten using the modified steering vector in (8):

$$\mathbf{x}_m(t) = \mathbf{CA}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{12}$$

The covariance matrix can be calculated from a limited sample of the modified array output:

$$\mathbf{R}_{xx} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_m(t)\mathbf{x}_m^H(t) \tag{13}$$

where T is the total number of sample output. In Capon algorithm, the DOA can be estimated by minimising the cost function of the following:

$$f(\theta) = \mathbf{a}_m^H(\theta)\mathbf{R}_{xx}^{-1}\mathbf{a}_m(\theta) \tag{14}$$

where $\mathbf{R}_{xx}^{-1}$ denotes the inverse of covariance matrix.

3. Results and discussion

Computer simulations are conducted to analyse the effect of mutual coupling compensation in DOA estimation. All results in this section are obtained using a linear array of eight elements with element separation of 0.5λ. The simulations in this section also assume that the coupling coefficients equal to zero after four element spacing that is $p = 4$. Capon algorithm is used as the DOA estimator throughout in the simulation work. The results are then being compared between without mutual coupling effect and with mutual coupling compensation.

3.1. Estimation resolution

In this subsection, analysis of estimation resolution is presented, with assumption of signal-to-noise ratio (SNR) at 0 dB, and number of snapshots is 500. In the first case, suppose that there is a signal impinging at -10°. Result of DOA estimation of is shown in figure 1. The result illustrates that the mutual coupling compensation has comparable with the case of without mutual coupling compensation. In the second case, there are two signals impinging at -10° and 40°. The DOA estimation result is shown in figure 2. Similar to the observation in the first case, the mutual coupling compensation approach managed to estimate close to the true DOA.

The results also show the estimation with mutual coupling compensation is comparable to the case of without mutual coupling compensation. Estimation produced through mutual coupling compensation illustrate the significance of mutual coupling effect towards the DOA estimation process. Results in both cases clearly show that the mutual coupling compensation technique managed to counteract the mutual coupling effect in estimation process.
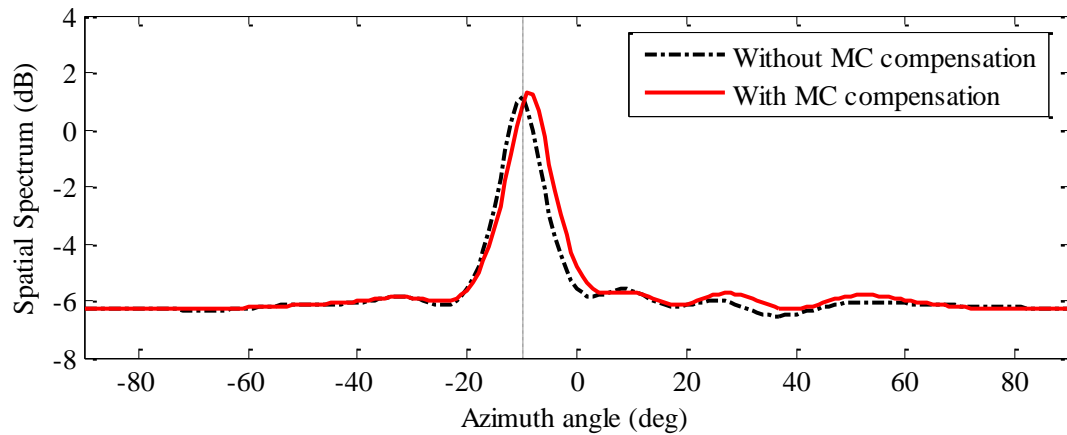
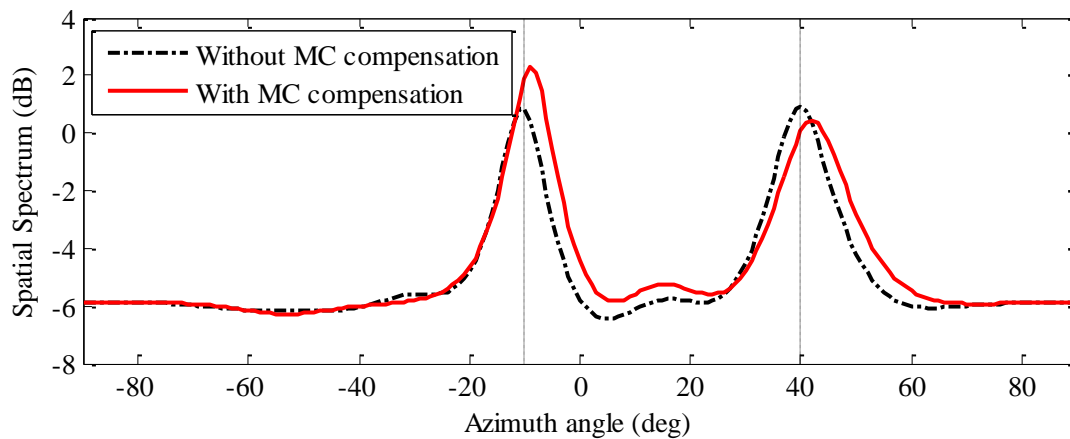**Figure 1.** DOA estimation for signal impinging at -10°.

Figure 2. DOA estimation for signal impinging at -10° and 45°.

3.2. Estimation error

Estimation error measures how much the estimation DOA deviates from the true DOA when there are two or more signals arrived on the array. All the simulation results in this subsection use the root-mean-square error (RMSE) to measure error of estimation, which is given as:

$$RMSE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}\left(\theta - \hat{\theta}\right)^2} \tag{15}$$

where T (=500) is the number of trials, θ is the true DOA and $\hat{\theta}$ is the estimated DOA of the ith trial. It is assumed that the signal is coming from -10° and 40°.

RMSE produced by both with and without mutual coupling compensation approaches are compared against number of snapshots. Figure 3 shows the RMSE for both approaches when the SNR is fixed at 0 dB. In general observation, as the number of samples is increased, the performance of approach without mutual coupling is improved. On the other hand, the estimation error in mutual coupling compensation approach is almost constant. It is apparent that the approach without mutual coupling compensation yields one tenth estimation error compared to the approach with mutual coupling compensation.

Figure 4 shows the changes in estimation error for various SNR levels when the number of snapshots is kept at 100. Similar to the observation in figure 3, the approach without mutual coupling compensation produced smaller estimation error as the SNR level improves. In contrast, the estimation error in mutual coupling compensation approach remains almost the same for all SNR levels.

Results shown in figure 3 and figure 4 illustrate that the mutual coupling compensation approach is less effective to counteract the mutual coupling effect when multiple signals are impinging the antenna array. This could be due to an improper design of banded mutual coupling matrix when calculating the array output in estimation process.
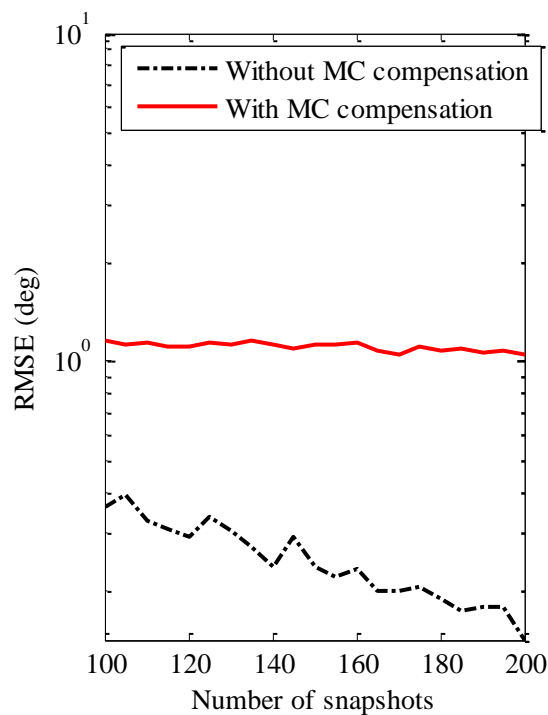
Figure 3. RMSE of DOA estimation for various number of snapshots when SNR being 0 dB. The DOAs are at (θ = -10°) and (θ = 40°).
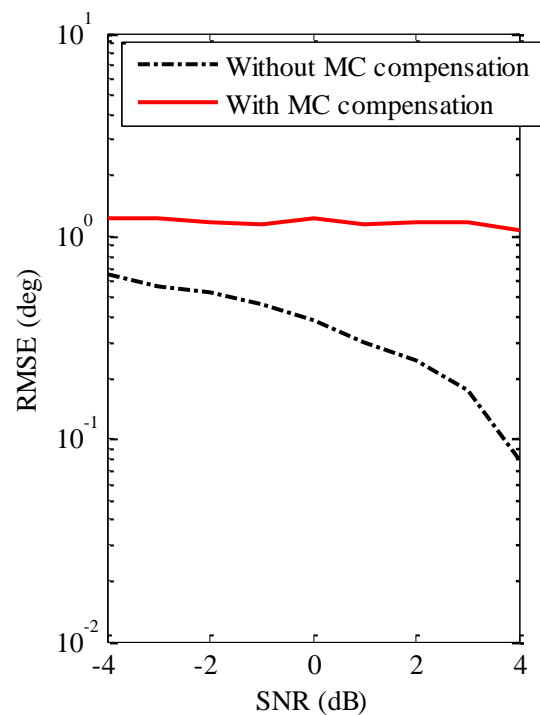
Figure 4. RMSE of DOA estimation for various SNR when number of snapshots being 100. The DOAs are at (θ = 20°) and (θ = 40°).

4. Conclusion

The purpose of this work is to analyse the performance of mutual coupling compensation technique in DOA estimation. A comparison study of DOA estimation between the mutual coupling compensation technique and without the mutual coupling consideration has been carried out. The mutual coupling compensation is done by introducing a matrix of coupling coefficients that represent the mutual coupling between the elements in an array. The matrix of coupling coefficients is formed in such a way that the mutual coupling coefficients become zero after several element spacing to reduce the computational complexity. The matrix is then used to calculate the array output in the DOA estimation. Results show that the mutual coupling compensation technique produced a comparable estimation resolution with the case of without mutual coupling consideration. However, the mutual coupling compensation technique performance is significantly below par compared with the approach of without mutual coupling consideration. These results suggest that a proper design of coupling coefficients matrix is essential to ensure the mutual coupling compensation would lead to accurate DOA estimation.

5. References

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