

Azimuth DOA Estimation in Y-bend Antenna Array

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Abstract. In smart antenna system, it is extremely crucial to estimate the direction of incoming signals in order to achieve better reception. Reliability of DOA estimation depends on several factors such as the choice of DOA algorithm, size of antenna array as well as array geometry. Therefore, it is particularly desirable to have a configuration of antenna array that could produce an accurate azimuth estimation. In this work, a new planar array is proposed to address the problem of azimuth estimation. This is achieved by having a flexible element position on the x-y plane that improves the steering vector, hence significantly enhances the accuracy of DOA estimation. Besides, a fair distribution of the antenna elements on the x-y plane also helps to eliminates estimation failure in the azimuth range between 240° and 360°. A comparison study between the proposed array and V-shape array is performed in order to gauge the performance of the proposed array in DOA estimation. Simulation results show that the proposed array has acquired better estimation resolution than V-shape array. On top of that, the proposed array has reduced estimation error in V-shape array. It is concluded that the proposed array has shown potential as an excellent choice of antenna array geometry for smart antenna system.

1. Introduction

Direction of arrival (DOA) estimation for smart antenna system has been investigated extensively in the last six decades and has been applied in various fields including wireless communication, radar, sonar and audio processing [1-4]. The role of DOA estimation in smart antenna system is essential since it helps to estimate the direction of the incoming signal. The estimated signal direction will be used to point the array beam towards the estimated direction. There are several factors that contribute to the performance of DOA estimation such as the number of element in the array, spacing between elements, size of the array and shape of array [5]. Linear array provides a simple form of antenna array, and it also straightforward to be analysed. However, its capability is limited to one-dimensional DOA estimation, which is only for elevation angle estimation. In order to have both azimuth and elevation angles estimation, a two-dimensional array is needed. Thus, there are several initial configuration of planar arrays have been proposed in the literature such rectangular, triangular and circular arrays [6-10]. Among these initial proposals, circular array provides a unique property of ability to cover azimuth angle in a uniform manner. In general, the computational complexity of DOA estimation is significantly affected by the array geometry due to the increase in the dimensionality of two-dimensional DOA estimation problem [11]. Therefore, vast efforts have been emerged to reduce the computational complexity as well as to simplify the estimation algorithm.



In recent years, new geometries of planar arrays formed with two or more linear arrays with specified configurations have been reported in the literature. These planar arrays are proposed to improve the performance of two-dimensional DOA estimation and have received considerable attention. Cross arrays provide excellent estimation resolution for planar arrays, but their performance is limited to low number of elements [12]. The development in cross arrays has opened a path to other planar array configurations that could work for conventional spectral-based DOA algorithms. Among recent planar arrays called Y-shape arrays improve the elevation estimation at end-fire, which leads to uniform elevation estimation [13]. However, the angle coverage is slightly narrower compared to the cross arrays. A practical array design of quarter-wave monopole has also been proposed utilising geometry of the Y-shape, which shown estimation error below 2° [14].

Another array variation, known as L-shape, also offers wide-angle coverage with significant accuracy [15, 16]. L-shape arrays are hugely popular since they offer the best estimation accuracy among the planar arrays. Nonetheless, this array is prone to estimation failure at the elevation angle of 70° - 90° [15]. The estimation failure in L-shape arrays is improved by double-L arrays with the cost of a higher number of elements [17]. The double L-shape array helps to eliminate the problem of estimation failure in the L-shaped array for azimuth angle [18]. The only notable drawback of the double L-shape array is that it needs 30% more elements compared with the L-shaped array. The additional elements in the double L-shape array also means the computational load is more complex compared with the L-shape array [19]. Oval arrays are also being proposed to improve the estimation accuracy in circular arrays. It has been reported that the oval arrays could improve the estimation resolution and estimation accuracy by 50% and 75% respectively compared to circular array [20].

Another planar arrays known as V-shape arrays have a large aperture that lead to wide angle estimation [21, 22]. The uniqueness of their structure is that none of their element is positioned along the x-y axes. This feature makes the V-shape arrays the only array that has flexible axes for their elements. Nevertheless, the drawback of this array is its poor resolution and high estimation error. Therefore, the purpose of this paper is to propose a new planar array to address the problem of azimuth estimation in V-shape array, notably the estimation resolution and estimation error. Flexible element position used in the proposed array improves the steering vector, which governs the accuracy of DOA estimation. On top of that, a fair distribution of array elements on the x-y plane will help to reduce estimation failure in the V-shape array. The proposed array is evaluated by simulations, demonstrating better estimation resolution and accuracy compared with the V-shape array.

2. Data Model

The structure of the proposed array, we term as Y-bend, is shown in figure 1. It is composed of three linear arrays on the x-y plane. Two linear arrays, U and V , are on both sides of the y-axis, with an element-dependent angle separating between them, denoted as α . Another linear array, W , is placed exactly along the y-axis. Each linear array consists of N elements; therefore the proposed array would have a total of $3N-2$ elements. Each element in the sub-arrays is separated by spacing d , and the element at the origin of coordinates x_0 is the reference one for each sub-array. The element spacing between the elements x_0 , y_1 and z_1 of these sub-arrays is also d . The angle α separating the U and V arrays is measured between the two sub-arrays and defined as follows:

$$\alpha = 2 \times \arctan \left(\frac{\sqrt{N^2 + 3}}{2N} \right) \quad (1)$$

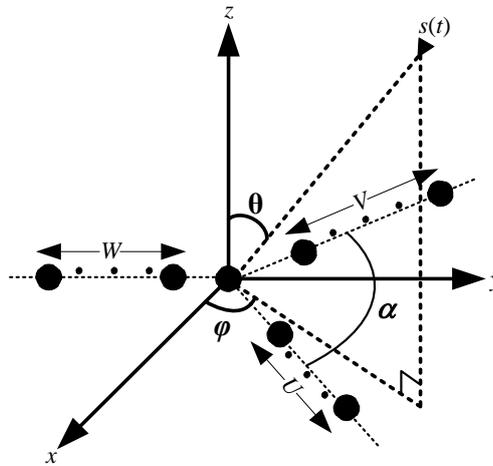


Figure 1. The structure of Y-bend antenna array.

Suppose that there are K uncorrelated narrowband signal sources of wavelength λ impinging the array from far-field points. Each of the k^{th} source has the corresponding azimuth angle, φ_k , and elevation angle, θ_k , $k = 1, 2, \dots, K$, where the elevation angles are measured clockwise relative to the z axis and the azimuth angles are measured counter-clockwise relative to the x axis. The received signal at array U , V , and W can be represented as follows:

$$x_U(t) = \mathbf{A}_U s(t) + n(t) \tag{2}$$

$$x_V(t) = \mathbf{A}_V s(t) + n(t) \tag{3}$$

$$x_W(t) = \mathbf{A}_W s(t) + n(t) \tag{4}$$

where $s(t)$ is the source signal, and $n(t)$ is the Gaussian white noise signal of zero mean and variance σ^2 . \mathbf{A}_U , \mathbf{A}_V and \mathbf{A}_W are the steering matrices defined as:

$$\mathbf{A}_U = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j\gamma_1} & e^{-j\gamma_2} & \dots & e^{-j\gamma_K} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-jN\gamma_1} & e^{-jN\gamma_2} & \dots & e^{-jN\gamma_K} \end{bmatrix} \tag{5}$$

$$\mathbf{A}_V = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j\beta_1} & e^{-j\beta_2} & \dots & e^{-j\beta_K} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-jN\beta_1} & e^{-jN\beta_2} & \dots & e^{-jN\beta_K} \end{bmatrix} \tag{6}$$

$$\mathbf{A}_w = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j\mu_1} & e^{-j\mu_2} & \dots & e^{-j\mu_K} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-jN\mu_1} & e^{-jN\mu_2} & \dots & e^{-jN\mu_K} \end{bmatrix} \quad (7)$$

$$\gamma_k = -\frac{2\pi d \sin \theta_k \sin\left(\phi_k + \frac{\alpha}{2}\right)}{\lambda} \quad k = 1, 2, \dots, K \quad (8)$$

$$\beta_k = \frac{2\pi d \sin \theta_k \sin\left(\phi_k - \frac{\alpha}{2}\right)}{\lambda} \quad k = 1, 2, \dots, K \quad (9)$$

$$\mu_k = -\frac{2\pi d \sin \theta_k \sin \phi_k}{\lambda} \quad k = 1, 2, \dots, K \quad (10)$$

where λ is the wavelength of incoming signal, and d is the spacing between elements expressed in term of λ .

In this paper, the following assumptions on the data model are being made:

- The array is calibrated, and the sensor spacing d of each sub-array and that between the sensors and satisfy the condition $0 < d < \frac{\lambda}{2}$ for avoiding angle ambiguity problems.
- For the simplicity of theoretical performance analysis, the incident signals $s_k(n)$ are temporally complex white Gaussian random processes with zero mean δ and variance given by $E[s_k(n)s_k^*(n)] = r_s \delta_n$ where $E[\cdot]$, $(\cdot)^*$, and δ_n denote the statistical expectation, complex conjugate operation and Kronecker delta.
- The additive noises are temporally and spatially complex white Gaussian random processes with zero-mean and the covariance matrices are statistically independent each other. The additive noise at each element of sub-arrays is uncorrelated with the incident signals.
- The number of incident signals k is known or is estimated by the existing number detection techniques in advance, and it satisfies the condition of $k < N$ for an array of N elements.

In this paper, the problem of estimating the azimuth angle is dealt in a computationally effective way. Furthermore, the statistical performance of DOA estimation using the proposed antenna array will be analysed with the available array data. A spectral-based DOA algorithm will be employed to perform the DOA estimation. This spectral-based algorithm is chosen since subspace-based DOA algorithm is computationally intensive and time-consuming.

3. Results and discussion

In this section, the performance of DOA estimation using the proposed array is verified through simulation works. The proposed array is evaluated using eight elements in each linear array, spacing between elements is 0.5λ . In the simulations, a spectral-based DOA algorithm, Capon, is being employed to carry out estimation analysis using the proposed array. Two fundamental parameters are being investigated; the estimation resolution and estimation error. In both cases, comparison study of DOA estimation performance is set out between the proposed array and the V-shaped array.

3.1. Estimation resolution

Resolution of estimation is a measurement of minimum separation required to distinguish two incoming signals. Higher estimation resolution means two signals separated by smaller angular gap can still be distinguished. The estimation resolution is examined with the number of snapshots being 100, and SNR level being 0 dB. There are four cases being considered in this study, and in each case there are two signal sources present. Table 1 shows the DOA of signal sources for four different cases and the estimation results are illustrated in figure 2.

Table 1. DOA of signal sources for estimation resolution analysis.

Case	DOA of signal sources
Case I	S1 ($\varphi = 65^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 75^\circ$, $\theta = 90^\circ$)
Case II	S1 ($\varphi = 120^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 130^\circ$, $\theta = 90^\circ$)
Case III	S1 ($\varphi = 200^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 205^\circ$, $\theta = 90^\circ$)
Case IV	S1 ($\varphi = 330^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 338^\circ$, $\theta = 90^\circ$)

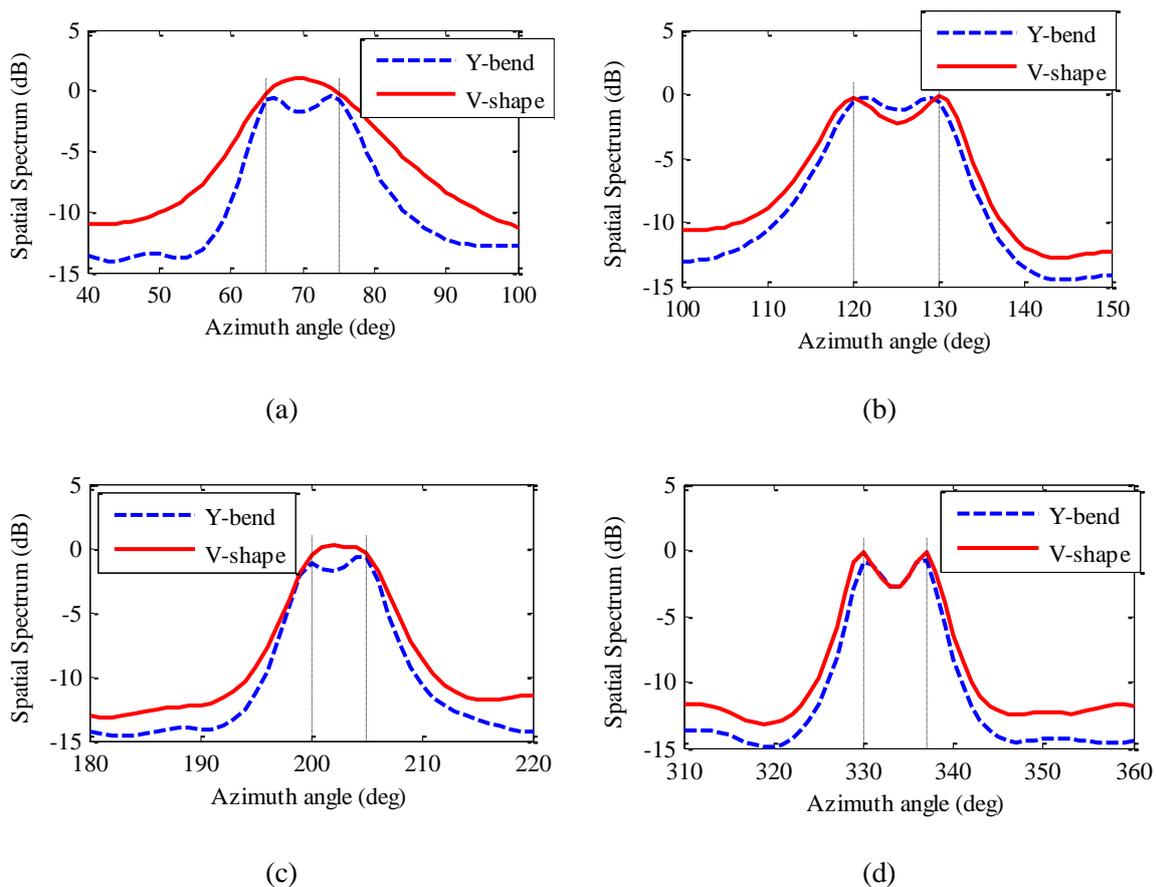


Figure 2. Azimuth estimation of two signal sources (a) S1 ($\varphi = 65^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 75^\circ$, $\theta = 90^\circ$), (b) S1 ($\varphi = 120^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 130^\circ$, $\theta = 90^\circ$), (c) S1 ($\varphi = 200^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 205^\circ$, $\theta = 90^\circ$), and (d) S1 ($\varphi = 300^\circ$, $\theta = 90^\circ$) and S2 ($\varphi = 338^\circ$, $\theta = 90^\circ$).

Simulation results in figure 2(a) and figure 2(c) show that the proposed array has better estimation resolution than V-shape array when the signal sources are in the range of $0^\circ < \varphi < 90^\circ$ and $180^\circ < \varphi < 270^\circ$. However, as shown in figure 2(b) and figure 2(d), the estimation resolution of the proposed array and the V-shape array are comparable when the signal sources in the range of $90^\circ < \varphi < 180^\circ$ and $270^\circ < \varphi < 360^\circ$.

3.2. Estimation error

Estimation error occurs when there are more than two signals impinging on the array. It represents how much the estimation DOA deviates from the true DOA. A Monte Carlo simulation of root-mean-square error (RMSE) is presented and compared with V-shape array. In this case, it is assumed that there are two signal sources present at S1 ($\varphi = 65^\circ, \theta = 90^\circ$) and S2 ($\varphi = 75^\circ, \theta = 90^\circ$). The RMSE is calculated as:

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T \left\{ \left(\phi - \hat{\phi}_t \right)^2 \right\}} \quad (11)$$

where $\hat{\phi}_t$ is the estimated azimuth angle ϕ at t^{th} trial, and the number of trial, T , being 500.

RMSE produced by both arrays are compared against three parameters; number of snapshot, SNR and separation of DOA. It is apparent that the number of snapshots does not have a significant effect on estimation error as illustrated in figure 3. Both the proposed array and the V-shape array have their estimation error almost unchanged as the number of snapshots is increased. However, it is apparent that the proposed array has estimation error 90% lower than the V-shape array.

Figure 4 clearly shows that, in general, the proposed array outperforms the V-shape array irrespective of the SNR level. At low SNR, the proposed array has slightly lower estimation error than the V-shape array, which is about 3° . However, as the SNR is improved, the proposed array exhibits higher rate of decrement in the estimation error than the V-shape array. It is estimated that, in high SNR, the proposed array has at least 10 times lower estimation error than the V-shape array. In figure 5, the azimuth of S1 remains and S2 is varied between 70° and 80° . It is noted that the RMSE of the proposed array is significantly lower than V-shape array when the signals are separated by more than 7° . In this comparison, the proposed array has reduced the estimation error in V-shape array by at least 90%.

Finally, the consistency of estimation is shown in figure 6 for 500 trials. It is clear that the azimuth estimation of the proposed array is close to the actual DOA. The distribution of histogram peaks also shows that the estimation is highly consistent throughout the whole trials.

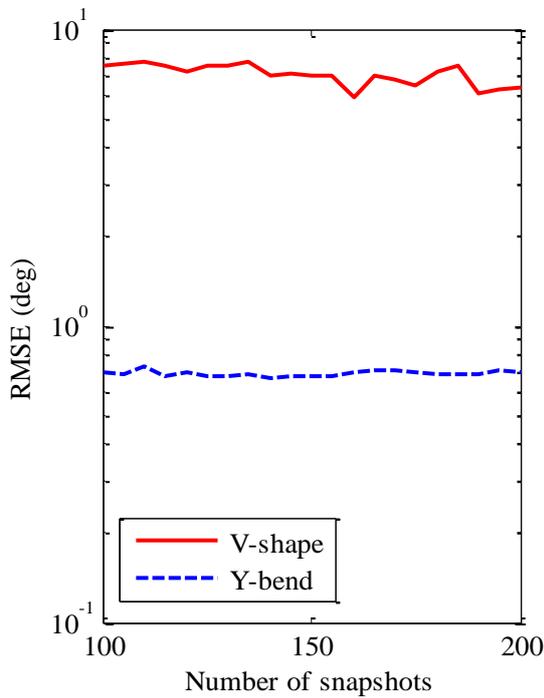


Figure 3. RMSE of DOA estimation for various number of snapshots when SNR being 0 dB. The DOAs are at $(\varphi = 65^\circ, \theta = 90^\circ)$ and $(\varphi = 75^\circ, \theta = 90^\circ)$.

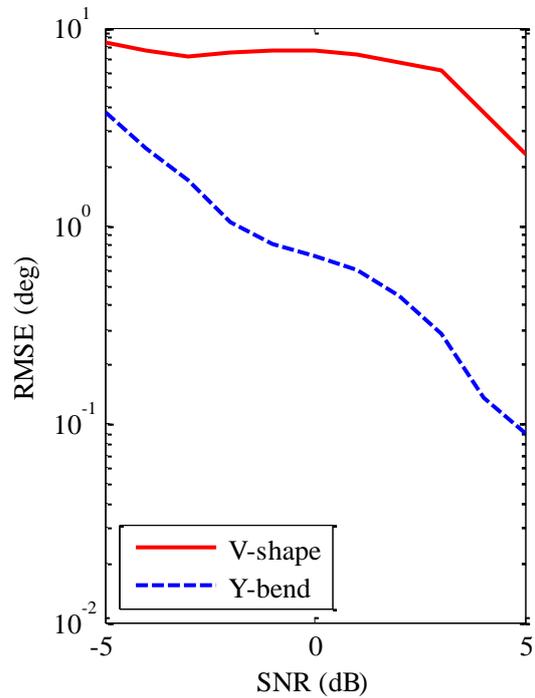


Figure 4. RMSE of DOA estimation for various SNR when number of snapshots being 100. The DOAs are at $(\varphi = 65^\circ, \theta = 90^\circ)$ and $(\varphi = 75^\circ, \theta = 90^\circ)$.

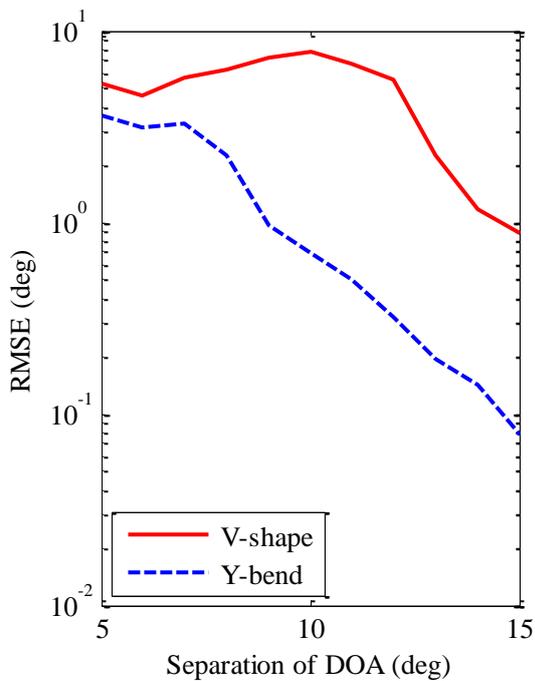


Figure 5. RMSE of DOA estimation for various signal separation when number of snapshots being 100 and SNR being 0dB. The DOA of first signal source at $(\varphi = 65^\circ, \theta = 90^\circ)$ and DOA of second signal source varies in the range $(70^\circ < \varphi < 80^\circ, \theta = 90^\circ)$.

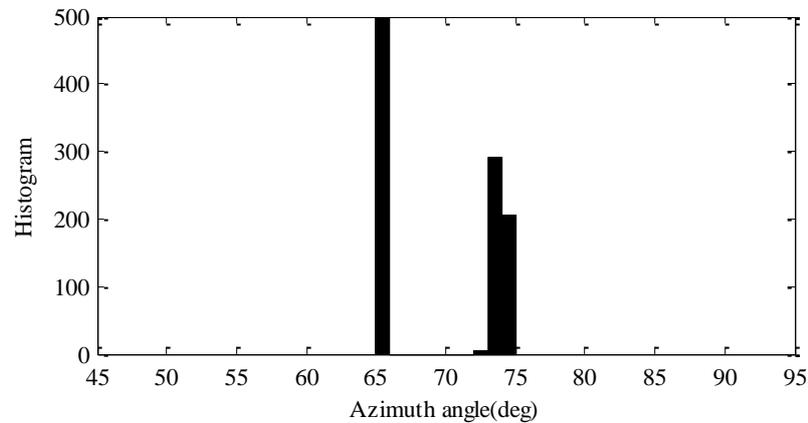


Figure 6. Histogram of azimuth estimation for two signal sources at $(\varphi = 65^\circ, \theta = 90^\circ)$ and $(\varphi = 75^\circ, \theta = 90^\circ)$.

4. Conclusion

A new planar array has been proposed and analysed in this paper to address the problem of azimuth DOA estimation in the V-shape array. The structure of the proposed array enhances the steering vector, hence produces high accuracy of DOA estimation. In addition, fair distribution of antenna element in the x-y plane reduced the possibility of estimation failure. Simulation results show that the proposed array has better estimation resolution than the V-shape array in the azimuth range of $0^\circ < \varphi < 90^\circ$ and $180^\circ < \varphi < 270^\circ$. Furthermore, the proposed array also reduced the estimation error up to 90% in the V-shape array. These results show that the proposed array has the potential to be an excellent choice for planar arrays targeting smart antenna systems.

5. References

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