IOP Conf. Series: Materials Science and Engineering **158** (2016) 012073 doi:10.1088/1757-899X/158/1/012073

Transient reaction of an elastic half-plane on a source of a concentrated boundary disturbance

A S Okonechnikov[1], D V Tarlakovski[1, 2], A N Ul'yashina[1] and G V Fedotenkov[1]

[1]Moscow Aviation Institute (National Research University), Russian Federation
[2]Lomonosov Moscow State University, Moscow, Russian Federation

E-mail: oas.mai@mail.ru

Abstract. One of the key problems in studying the non-stationary processes of solid mechanics is obtaining of influence functions. These functions serve as solutions for the problems of effect of sudden concentrated loads on a body with linear elastic properties. Knowledge of the influence functions allows us to obtain the solutions for the problems with non-mixed boundary and initial conditions in the form of quadrature formulae with the help of superposition principle, as well as get the integral governing equations for the problems with mixed boundary and initial conditions. This paper offers explicit derivations for all non-stationary surface influence functions of an elastic half-plane in a plane strain condition. It is achieved with the help of combined inverse transform of a Fourier-Laplace integral transformation. The external disturbance is both dynamic and kinematic. The derived functions in $x\tau$-domain are studied to find and describe singularities and are supplemented with graphs.

1. Problem Definition

The problem under investigation concentrates focuses on distribution of non-stationary boundary disturbance in a homogeneous isotropic elastic half-plane.

Let us introduce a coordinate system Oxz so that Oz axis is directed in the depth of the half-plane, while Ox axis corresponds with the edge of the half-plane $z=0$.

We will use dimensionless quantities system (apostrophes mark the dimensionless quantities and will be omitted in future)

$$x'=\frac{x}{L},\ z'=\frac{z}{L},\ \tau=\frac{c_1 t}{L},\ u'=\frac{u}{L},\ w'=\frac{w}{L},\ \varphi'=\frac{\varphi}{L^2},\ \psi'=\frac{\psi}{L^2},\eta=\frac{c_1}{c_2},$$
$$c_R'=\frac{c_R}{c_1},\ \sigma_{mn}'=\frac{\sigma_{mn}}{\lambda+2\mu},\ \kappa=\frac{\lambda}{\lambda+2\mu}=1-\frac{2}{\eta^2},\ c_1=\sqrt{\frac{\lambda+2\mu}{\rho}},\ c_2=\sqrt{\frac{\mu}{\rho}},\qquad (1)$$

where L – reference length, τ and t – dimensionless and dimensional time, u, w – components of displacement vectors, which match with the direction of Ox and Oz axes correspondingly; $\sigma_{mn}\,(m,n=1,3)$ – non-zero components of stress tensor, φ, ψ – scalar and vector potentials of elastic displacements; c_1, c_2 – velocities of tension and shear waves; c_R – velocity of Raleigh waves, λ, μ – Lame parameters; ρ – continuum density.

Problem definition includes [1]:

– equation of motion (dot accent represents a time derivative from now on)

$$\ddot{\varphi} = \Delta\varphi,\ \eta^2\ddot{\psi} = \Delta\psi,\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}; \tag{2}$$

– relations between displacement vector and stress tensor components with other potentials:

$$u_1 = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z}, u_3 = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x}, \tag{3}$$

$$\begin{gathered}\sigma_{11} = \frac{\partial^2\varphi}{\partial x^2} + (\kappa - 1)\frac{\partial^2\psi}{\partial x\partial z} + \kappa\frac{\partial^2\varphi}{\partial z^2}, \sigma_{13} = \frac{1}{\eta^2}\left(2\frac{\partial^2\varphi}{\partial x\partial z} - \frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial x^2}\right),\\ \sigma_{33} = \frac{\partial^2\varphi}{\partial z^2} + (1 - \kappa)\frac{\partial^2\psi}{\partial x\partial z} + \kappa\frac{\partial^2\varphi}{\partial x^2};\end{gathered} \tag{4}$$

– initial conditions:

$$\left.\varphi\right|_{\tau=0} = \left.\dot{\varphi}\right|_{\tau=0} = 0,\ \left.\psi\right|_{\tau=0} = \left.\dot{\psi}\right|_{\tau=0} = 0; \tag{5}$$

– boundary conditions:

– no-disturbance in the infinity conditions

$$\varphi = O(1),\ \psi = O(1),\ \text{при}\ r \to \infty,\ r = \sqrt{x^2 + z^2}; \tag{6}$$

– generalized conditions on the $z = 0$ boundary:

$$\begin{gathered}\left.(\alpha_1 u_1 + \beta_1\sigma_{13})\right|_{z=0} = \delta(x)\delta(\tau)\delta_{1l},\\ \left.(\alpha_3 u_3 + \beta_3\sigma_{33})\right|_{z=0} = \delta(x)\delta(\tau)\delta_{3l},\end{gathered} \tag{7}$$

where $\delta(\bullet)$ – Dirac delta function, δ_{ml} – Kronecker symbol, $m, l = 1,3.$

Combinations of various values of $\alpha_m, \beta_m\ (m = 1,3)$ parameters describe all the possible boundary conditions, while $\alpha_m^2 + \beta_m^2 \neq 0$.

At $\alpha_1 = \alpha_3 = 1,\ \beta_1 = \beta_3 = 0$ the first boundary problem can be observed.

At $\alpha_1 = \alpha_3 = 0,\ \beta_1 = \beta_3 = 1$ – we get the second boundary problem.

Displacements $u_m = G_{ml}^p(x,\tau)$ and stress $\sigma_{mk} = \Gamma_{mkl}^p(x,\tau)$, $(p = 1,2;\ m,k,l = 1,3)$ as solutions of problems (2)–(7) with $z = 0$ will be called non-stationary surface influence functions of an elastic half-plane.

2. Construction of Surface Influence Functions

Let us apply Laplace transform over τ time to (2)–(7) and Fourier transform over x coordinate:

$$\frac{\partial^2\varphi^{FL}}{\partial z^2} - k_1^2(q^2, s^2)\varphi^{FL} = 0, \frac{\partial^2\psi^{FL}}{\partial z^2} - k_2^2(q^2, s^2)\psi^{FL} = 0, \tag{8}$$

$$k_m(q,s) = \sqrt{q + \eta_m^2 s},\ \ \eta_1 = 1,\ \ \eta_2 = \eta,$$

$$\begin{gathered}u_1^{FL} = -iq\varphi^{FL} - \frac{\partial\psi^{FL}}{\partial z}, u_3^{FL} = \frac{\partial\varphi^{FL}}{\partial z} - iq\psi^{FL},\\ \sigma_{11}^{FL} = -iqu_1^{FL} + \kappa\frac{\partial u_3^{FL}}{\partial z}, \sigma_{33}^{FL} = \frac{\partial u_3^{FL}}{\partial z} - iq\kappa u_1^{FL},\\ \eta^2\sigma_{13}^{FL} = \frac{\partial u_1^{FL}}{\partial z} - iqu_3^{FL},\end{gathered} \tag{9}$$

$$\varphi^{FL} = O(1), \psi^{FL} = O(1)\ \text{at}\ \ z \to \infty, \tag{10}$$

$$\begin{gathered}\left.(\alpha_1 u_1^{FL} + \beta_1\sigma_{13}^{FL})\right|_{z=0} = \delta_{1l},\\ \left.(\alpha_3 u_3^{FL} + \beta_3\sigma_{33}^{FL})\right|_{z=0} = \delta_{3l}.\end{gathered} \tag{11}$$

" FL " index of function identifies its Laplace and Fourier transform, q, s – Fourier and Laplace transformation parameters correspondingly.

From (8)–(11) in the case of the first boundary problem ($p=1,\ \alpha_1=\alpha_3=1,\ \beta_1=\beta_3=0$) and the second one ($p=2,\ \alpha_1=\alpha_3=0,\ \beta_1=\beta_3=1$) we can obtain

$$\begin{gathered} G_{1l}^{pFL}(q,s)=G_{11}^{pFL}\delta_{1l}+G_{13}^{pFL}(q,s)\delta_{3l},\ G_{3l}^{pFL}(q,s)=G_{31}^{pFL}(q,s)\delta_{1l}+G_{33}^{pFL}(q,s)\delta_{3l}, \\ \Gamma_{11l}^{pFL}(q,s)=\Gamma_{111}^{pFL}(q,s)\delta_{1l}+\Gamma_{113}^{pFL}(q,s)\delta_{3l},\ \Gamma_{33l}^{pFL}(q,s)=\Gamma_{331}^{pFL}(q,s)\delta_{1l}+\Gamma_{333}^{pFL}(q,s)\delta_{3l}, \\ \Gamma_{13l}^{pFL}(q,s)=\Gamma_{31l}^{pFL}(q,s)=\Gamma_{131}^{pFL}(q,s)\delta_{1l}+\Gamma_{133}^{pFL}(q,s)\delta_{3l}, \end{gathered} \tag{12}$$

where

$$G_{ml}^{1FL}(q,s)=\delta_{ml}\ (m,l=1,3),$$

$$\Gamma_{111}^{1FL}(q,s)=\frac{iq\left[\eta^2\kappa s^2-2R_1(q^2,s^2)\right]}{\eta^2R_1(q^2,s^2)},\ \Gamma_{333}^{1FL}(q,s)=\frac{s^2k_2(q^2,s^2)}{R_1(q^2,s^2)},$$

$$\Gamma_{133}^{1FL}(q,s)=-\frac{iq\left[2R_1(q^2,s^2)+\eta^2s^2\right]}{\eta^2R_1(q^2,s^2)},\ \Gamma_{131}^{1FL}(q,s)=\frac{s^2k_1(q^2,s^2)}{R_1(q^2,s^2)},$$

$$\Gamma_{13l}^{1FL}(q,s)=\Gamma_{31l}^{1FL}(q,s),\ \Gamma_{331}^{1FL}(q,s)=-\Gamma_{133}^{1FL}(q,s),\ \Gamma_{113}^{1FL}(q,s)=\kappa\Gamma_{333}^{1FL}(q,s),\ \ (l=1,3).$$

$$\Gamma_{m3l}^{2FL}=\delta_{ml}\ \ (m,l=1,3),$$

$$G_{11}^{2FL}(q,s)=-\frac{\eta^4k_2(q^2,s^2)s^2}{R_2(q^2,s^2)},\ G_{33}^{2FL}(q,s)=-\frac{\eta^4k_1(q^2,s^2)s^2}{R_2(q^2,s^2)},$$

$$G_{13}^{2FL}(q,s)=-G_{31}^{2FL}(q,s)=-\frac{i\eta^2q\left[\eta^2s^2+2R_1(q^2,s^2)\right]}{R_2(q^2,s^2)},$$

$$\Gamma_{111}^{2FL}(q,s)=-\frac{2iq(1+\kappa)}{\eta^2}G_{11}^{2FL}(q,s),$$

$$\Gamma_{113}^{2FL}(q,s)=-\frac{\left(\kappa\eta^2s^2-2q^2\right)\left(\eta^2s^2+2q^2\right)+4q^2k_1(q^2,s^2)k_2(q^2,s^2)}{R_2(q^2,s^2)},$$

$$R_1(q,s)=q-k_1(q,s)k_2(q,s),\ \ R_2(q,s)=(\eta^2s+2q)^2-4qk_1(q,s)k_2(q,s),$$

$$k_1(q,s)=\sqrt{q+s},\ k_2(q,s)=\sqrt{q+\eta^2s}.$$

3. Influence Functions in $x\tau$- Domain

As transformed functions in (12) are homogenous functions for defining the corresponding functions in $x\tau$-domain, we should use an algorithm of combined Fourier-Laplace inverse transform [5]

Among the functions in qs-domain that we study, the most interesting are $\Gamma_{mkl}^{1FL}(q,s),\ G_{ml}^{2FL}(q,s),$ $(m,k,l=1,3)$, as well as $\Gamma_{113}^{2FL}(q,s)$. The remaining components, according to(12), are represented in $x\tau$-domain in a where they corresponds to boundary conditions in (7).

By introducing a variable substitution $\lambda=q/s$, we represent the Fourier-Laplace transformations of the functions that we are looking to obtain as:

$$\begin{gathered} \Gamma_{mkl}^{1FL}(q,s)=\gamma_1^L(s)h_{mkl}^1(\lambda),\ G_{ml}^{2FL}(q,s)=g_2^L(s)h_{ml}^2(\lambda), \\ \Gamma_{113}^{2FL}(q,s)=\gamma_2^L(s)h_{113}^2(\lambda),\ \left(m,k,l=1,3\right), \end{gathered} \tag{13}$$

where

$$g_2^L(s)=1/s,\ \gamma_1^L(s)=s,\ \gamma_2^L(s)=1,\ h_{11}^2(\lambda)=-\frac{\eta^4 k_2(\lambda^2,1)}{R_2(\lambda^2,1)},\ h_{33}^2(\lambda)=-\frac{\eta^4 k_1(\lambda^2,1)}{R_2(\lambda^2,1)},$$

$$h_{13}^2(\lambda)=-h_{31}^2(\lambda)=-\frac{i\eta^2\lambda\left[\eta^2+2R_1(\lambda^2,1)\right]}{R_2(\lambda^2,1)},\ h_{111}^1(\lambda)=\frac{i\lambda\left[\kappa-2R_1(\lambda^2,1)\right]}{\eta^2 R_1(\lambda^2,1)},$$

$$h_{333}^1(\lambda)=\frac{\kappa k_2(\lambda^2,1)}{R_1(\lambda^2,1)},\ h_{133}^1(\lambda)=-\frac{i\lambda\left[2R_1(\lambda^2,1)+\eta^2\right]}{\eta^2 R_1(\lambda^2,1)},\ h_{131}^1(\lambda)=\frac{k_1(\lambda^2,1)}{R_1(\lambda^2,1)},$$

$$h_{113}^2(\lambda)=\left[\frac{\left(\kappa\eta^2-2\lambda^2\right)\left(\eta^2+2\lambda^2\right)+4\lambda^2 k_1(\lambda^2,1)k_2(\lambda^2,1)}{R_2(\lambda^2,1)}\right],$$

$$h_{13l}^1(\lambda)=h_{31l}^1(\lambda),\ h_{331}^1(\lambda)=-h_{133}^1(\lambda),\ h_{113}^1(\lambda)=\kappa h_{333}^1(\lambda),\ \ (l=1,3).$$

Considering that $g_2(\tau)=H(\tau),\ \dot{g}_2(\tau)=\delta(\tau)$, where $H(\bullet)$ – Heaviside step function, as well as the properties of delta function, we obtain expressions for the analytical form [6] of $G_{ml}^2(q,s)$ function:

$$\hat{G}_{ml}^2(z,\tau)=-\frac{1}{2\pi}\dot{g}_2(\tau)*\theta_{ml}(z,\tau)=-\frac{1}{2\pi}\theta_{ml}(z,\tau),\ (m,l=1,3)$$
$$z=x+iy,\ \theta_{ij}(z,\tau)=h_{ml}\left[\lambda(z,\tau)\right]\dot{\lambda}. \tag{14}$$

Where $\lambda=\tau/iz$, $\mathrm{Re}\lambda<0$ при $(y\to+0)$; $\mathrm{Re}\lambda>0$ при $(y\to-0)$, «$*$» symbol identifies a convolution of the functions over time.

The function in the $x\tau$-domain can be depicted using the equation [7]:

$$G_{ml}^2(x,\tau)=\lim_{y\to+0}\hat{G}_{ml}^2(z,\tau)-\lim_{y\to-0}\hat{G}_{ml}^2(z,\tau)\ \ (m,l=1,3) \tag{15}$$

As $\gamma_1(\tau)=\dot{\delta}(\tau),\gamma_2(\tau)=\delta(\tau)$, the corresponding functions in the $x\tau$-domain will assume the following forms: $\Gamma_{mkl}^{1FL}(q,s)\ (m,k,l=1,3),\ \Gamma_{113}^{2FL}(q,s)$ according to (14) should hold generalized first- and second-order derivatives correspondingly. To avoid the differentiation procedure, which can lead to complex analytical expressions for the influence functions in $x\tau$-domain, we will find the $x\tau$-domain representations of primitives of corresponding functions:

$$\Gamma_{mkl}^1(x,\tau)=\frac{d^2}{d\tau^2}\tilde{\Gamma}_{mkl}^1(x,\tau),\ \ \Gamma_{113}^2(x,\tau)=\frac{d}{d\tau}\tilde{\Gamma}_{113}^2(x,\tau), \tag{16}$$

where

$$\tilde{\Gamma}_{mkl}^1(x,\tau)=\lim_{y\to+0}\hat{\Gamma}_{mkl}^1(z,\tau)-\lim_{y\to-0}\hat{\Gamma}_{mkl}^1(z,\tau),\ \tilde{\Gamma}_{113}^2(x,\tau)=\lim_{y\to+0}\hat{\Gamma}_{113}^2(z,\tau)-\lim_{y\to-0}\hat{\Gamma}_{113}^2(z,\tau),$$

$$\hat{\Gamma}_{mkl}^1(z,\tau)=-\frac{1}{2\pi}\theta_{mkl}(z,\tau),\ \hat{\Gamma}_{113}^2(z,\tau)=-\frac{1}{2\pi}\theta_{113}(z,\tau),\ \theta_{mkl}(z,\tau)=h_{mkl}\left[\lambda(z,\tau)\right]\dot{\lambda},$$

$$\theta_{113}(z,\tau)=h_{113}\left[\lambda(z,\tau)\right]\dot{\lambda},\ z=x+iy,\ (m,k,l=1,3).$$

Here derivatives are understood in the generalized sense.

This is possible thanks to two reasons. First, the differentiation impairs the properties of the functions in the $x\tau$-domain we are looking for, as they might get singularities that are impossible to integrate. Second, the influence functions we are looking for are not the ultimate goal. They serve as the equation kernels of the corresponding governing integral operators or equations in the process of solving particular problems. Consequently, while solving we can always remove the time derivatives from these kernels, with, for example, integration by parts.

Let us find the limit values of the functions which form (15)–(16):

$$\lambda_0(x,\tau)=\lim_{y\to\pm0}\lambda(z,\tau)=-\frac{i\tau}{x} \tag{17}$$

$$\lim_{y\to\pm 0} k_m(\lambda^2,1)=\begin{cases} k_{m0}(x^2,\tau^2) & \text{at} \left|\tau/x\right|<\eta_m \\ \mp sign\, x\, k_{m0}(x^2,\tau^2) & \text{at} \left|\tau/x\right|\ge\eta_m \end{cases} \tag{18}$$
$$k_{m0}(1,\tau/x)=\sqrt{\eta_m^2-\tau/x},$$

By substituting (17)–(18) into (14)–(15) we will obtain the function in the $x\tau$-domain with different values of $\tau/|x|$:

At $\tau/|x|<1$

$$G^p_{ml}(x,\tau)=\Gamma^p_{mkl}(x,\tau)=0\ \ (m,k,l=1,3). \tag{19}$$

At $1<\tau/|x|<\eta$

$$\begin{aligned}
&G^2_{ml}(x,\tau)=G^{21}_{ml}(x,\tau),\\
&G^{21}_{11}(x,\tau)=\frac{4\eta^2\tau^2\left(\tau^2-\eta^2x^2\right)}{\pi P_3(x^2,\tau^2)}\sqrt{\tau^2-x^2},\ G^{21}_{33}(x,\tau)=\frac{\eta^2\left(\eta^2x^2-2\tau^2\right)^2}{\pi P_3(x^2,\tau^2)}\sqrt{\tau^2-x^2},\\
&G^{21}_{13}(x,\tau)=-G^{21}_{31}(x,\tau)=-\frac{2\eta^2\tau}{\pi}\left[\frac{\left(\eta^2x^2-2\tau^2\right)\sqrt{\eta^2x^2-\tau^2}\sqrt{\tau^2-x^2}}{P_3(x^2,\tau^2)}\right],\\
&\Gamma^{21}_{111}(x,\tau)=-\frac{2iq\beta_3(1+\kappa)}{\eta^2}G^{21}_{11}(x,\tau),\\
&\tilde\Gamma^{21}_{113}(x,\tau)=\frac{2\tau^2\left(\eta^4x^4+2\kappa\eta^4x^4-4\kappa\eta^2x^2\tau^2-4\tau^4\right)\sqrt{\eta^2x^2-\tau^2}}{\eta^2x^4\pi P_3(x^2,\tau^2)}\sqrt{\tau^2-x^2},\\
&\tilde\Gamma^{11}_{111}(x,\tau)=\frac{\tau\kappa\sqrt{\eta^2x^2-\tau^2}}{\pi Q_3(x^2,\tau^2)}\sqrt{\tau^2-x^2},\quad \tilde\Gamma^{11}_{333}(x,\tau)=\frac{\left(\eta^2x^2-\tau^2\right)}{\pi Q_3(x^2,\tau^2)}\sqrt{\tau^2-x^2},\\
&\tilde\Gamma^{11}_{133}(x,\tau)=-\frac{\tau\sqrt{\eta^2x^2-\tau^2}}{\pi Q_3(x^2,\tau^2)}\sqrt{\tau^2-x^2},\ \tilde\Gamma^{11}_{131}(x,\tau)=-\frac{\tau^2\sqrt{\tau^2-x^2}}{\pi Q_3(x^2,\tau^2)},\\
&\tilde\Gamma^{11}_{13l}(x,\tau)=\tilde\Gamma^{11}_{31l}(x,\tau),\ \tilde\Gamma^{11}_{331}(x,\tau)=-\tilde\Gamma^{11}_{133}(x,\tau),\ \tilde\Gamma^{11}_{113}(x,\tau)=\kappa\tilde\Gamma^{11}_{333}(x,\tau),\ \ (l=1,3).
\end{aligned} \tag{20}$$

At $\tau/|x|>\eta>1$

$$\begin{aligned}
&G^2_{ml}(x,\tau)=G^{23}_{ml}(x,\tau)\\
&G^{23}_{11}(x,\tau)=\frac{\eta^2R_{22}(x^2,\tau^2)}{\pi P_3(x^2,\tau^2)}\sqrt{\tau^2-\eta^2x^2},G^{23}_{33}(x,\tau)=\frac{\eta^2\mathrm{R}_{21}(x^2,\tau^2)}{\pi P_3(x^2,\tau^2)}\sqrt{\tau^2-x^2},\\
&G^{23}_{13}(x,\tau)=G^{23}_{31}(x,\tau)=0,\\
&\Gamma^{23}_{111}(x,\tau)=-\frac{2iq\beta_3(1+\kappa)}{\eta^2}G^{23}_{11}(x,\tau),\ \tilde\Gamma^{23}_{113}(x,\tau)=0,\\
&\tilde\Gamma^{13}_{333}(x,\tau)=\frac{\sqrt{\tau^2-\eta^2x^2}R^1_{21}(x^2,\tau^2)}{\pi Q_3(x^2,\tau^2)},\ \tilde\Gamma^{13}_{131}(x,\tau)=\frac{\sqrt{\tau^2-x^2}R^1_{21}(x^2,\tau^2)}{\pi Q_3(x^2,\tau^2)},\\
&\tilde\Gamma^{13}_{111}(x,\tau)=\tilde\Gamma^{13}_{133}(x,\tau)=\tilde\Gamma^{13}_{331}(x,\tau)=0,\ \tilde\Gamma^{13}_{113}(x,\tau)=\kappa\tilde\Gamma^{13}_{333}(x,\tau),\\
&\tilde\Gamma^{11}_{13l}(x,\tau)=\tilde\Gamma^{11}_{31l}(x,\tau),\ \ (l=1,3),
\end{aligned} \tag{21}$$

where

$$Q_3(x,\tau)=R_{21}^1(x,\tau)R_{21}^2(x,\tau),$$
$$R_{21}^1(x,\tau)=-\left(k_{10}(x,\tau)k_{20}(x,\tau)+\tau\right),\quad R_{22}^1(x,\tau)=\tau-k_{10}(x,\tau)k_{20}(x,\tau),$$
$$R_{21}(x,\tau)=\left(\eta^2x-2\tau\right)^2+4\tau\,k_{10}(x,\tau)k_{20}(x,\tau),\ R_{22}(x,\tau)=\left(\eta^2x-2\tau\right)^2-4\tau\,k_{10}(x,\tau)k_{20}(x,\tau),$$
$$R_{21}(x,\tau)R_{22}(x,\tau)=\eta^2xP_3(x,\tau).$$

Taking (19)–(21) into consideration, we rewrite the derivations for the influence functions:

$$\begin{aligned}
&G_{ml}^2(x,\tau)=\sum_{r=1}^{2}G_{ml}^{2r}(x,\tau)H\left(\tau-\eta_r|x|\right),\\
&G_{ml}^{22}(x,\tau)=G_{ml}^{23}(x,\tau)-G_{ml}^{21}(x,\tau),\ (m=l=1,3),\\
&G_{ml}^2(x,\tau)=G_{ml}^{21}(x,\tau)H\left(\tau-|x|\right),\ \ (m,l=1,3;m\neq l)\\
&\Gamma_{111}^2(x,\tau)=-\frac{2iq\beta_3(1+\kappa)}{\eta^2}G_{11}^2(x,\tau),\\
&\tilde{\Gamma}_{113}^2(x,\tau)=\tilde{\Gamma}_{113}^{21}(x,\tau)H\left(\tau-|x|\right),\\
&\tilde{\Gamma}_{m3l}^1(x,\tau)=\sum_{r=1}^{2}\tilde{\Gamma}_{m3l}^{1r}(x,\tau)H\left(\tau-\eta_r|x|\right),\\
&\tilde{\Gamma}_{m3l}^{12}(x,\tau)=\tilde{\Gamma}_{m3l}^{13}(x,\tau)-\tilde{\Gamma}_{m3l}^{11}(x,\tau),(m=l=1,3),\\
&\tilde{\Gamma}_{111}^1(x,\tau)=\tilde{\Gamma}_{111}^{11}(x,\tau)H\left(\tau-|x|\right),\,\tilde{\Gamma}_{133}^1(x,\tau)=\tilde{\Gamma}_{133}^{11}(x,\tau)H\left(\tau-|x|\right),\\
&\tilde{\Gamma}_{331}^1(x,\tau)=\tilde{\Gamma}_{331}^{11}(x,\tau)H\left(\tau-|x|\right),\,\tilde{\Gamma}_{113}^1(x,\tau)=\kappa\tilde{\Gamma}_{333}^1(x,\tau),\\
&\tilde{\Gamma}_{13l}^1(x,\tau)=\tilde{\Gamma}_{31l}^1(x,\tau),\ \ (l=1,3),
\end{aligned}\tag{22}$$

where

$$G_{11}^{22}(x,\tau)=\frac{\eta^2\left(\eta^2x^2-2\tau^2\right)^2}{\pi P_3(x^2,\tau^2)}\sqrt{\tau^2-\eta^2x^2},\ G_{33}^{22}(x,\tau)=\frac{4\eta^2\tau^2\left(\tau^2-x^2\right)}{\pi P_3(x^2,\tau^2)}\sqrt{\tau^2-\eta^2x^2},$$
$$\tilde{\Gamma}_{131}^{12}(x,\tau)=\frac{\left[\tau^2-\left(R_{21}^1(x^2,\tau^2)\right)\right]\sqrt{\tau^2-x^2}}{\pi Q_3(x^2,\tau^2)},\ \ \tilde{\Gamma}_{333}^{12}(x,\tau)=-\frac{\tau^2\sqrt{\tau^2-\eta^2x^2}}{\pi Q_3(x^2,\tau^2)}.$$

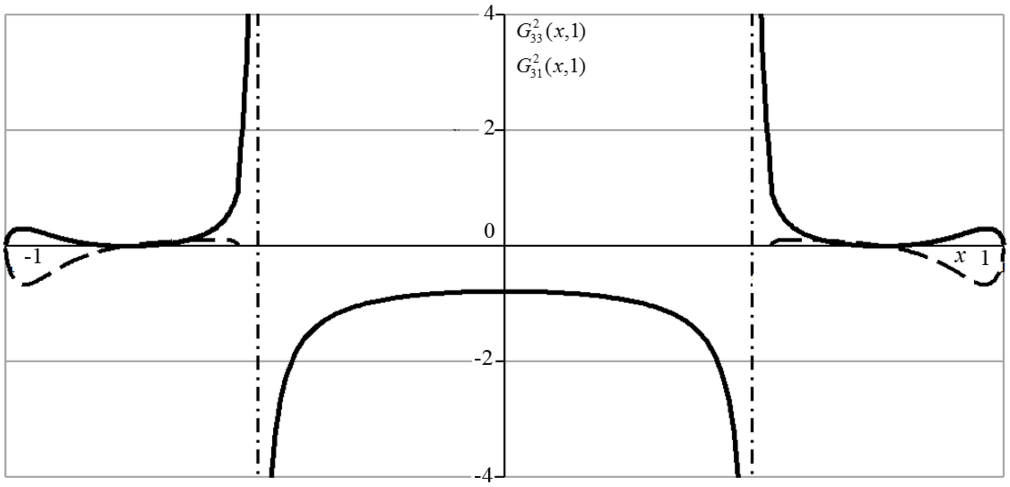

Figure 1 demonstrates influence functions, which describe the normal displacements under the effect of normal (solid line) and tangential (dashed line) load. Dot-and-dash line shows gaps corresponding to the front lines of the Raleigh waves [2].

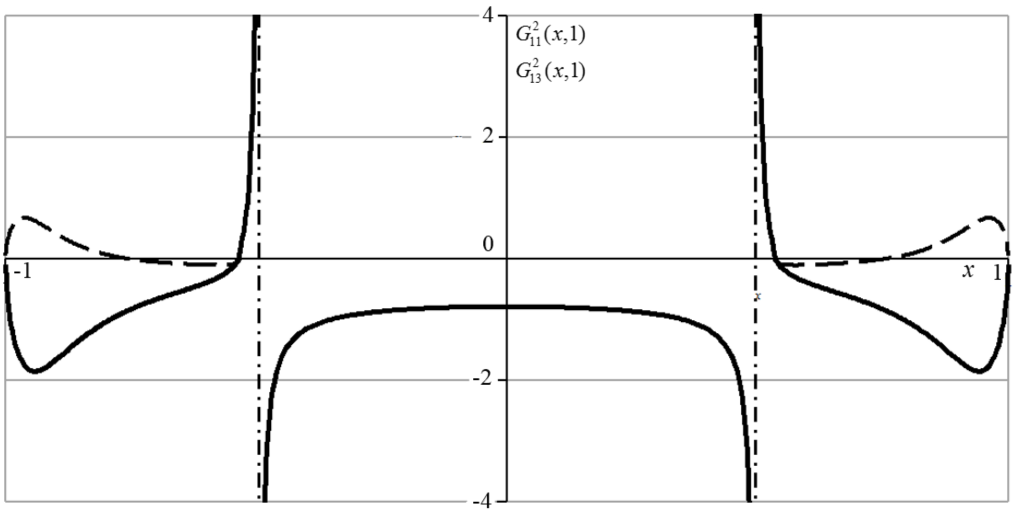

Figure 2 shows influence functions that describe the tangential displacements under the effect of tangential (solid line) and normal (dashed line) loads. Dot-and-dash line, as before, depicts the gaps in the fronts of Raleigh waves.

4. Conclusion

As a result, the influence functions for elastic half-space have been obtained. Some of these functions were applied for solving non-stationary problem of a mobile surface load [3,4], as well as in a series of works on solving the non-stationary contact problems [5-7].

Acknowledgements

The research is completed under the financial support of Russian Fund for Fundamental Research (project 16-08-00260) and grant of President of the Russian Federation for young scientists and postgraduate students (2016-2018).

References

[1] Gorshkov A G, Medvedski A L, Rabinski L N and Tarlakovski D V 2004 Waves in continua *Phizmatlit* (Moscow) 472
[2] Okonechnikov A S, Tarlakovskii D V and Fedotenkov G V 2014 Plane nonstationary problem of motion of the surface load over an elastic half-space *Journal of Mathematical Sciences* **174 (2)** 193-201. DOI 10.1007/s10958-014-2100-z.
[3] Okonechnikov A S, Tarlakovskii D V and Fedotenkov G V 2015 Nestatsionarnoe dvizhenie normal'noi sosredotochennoi nagruzki vdol' granitsy uprugoi poluploskosti *Elektronnyi zhurnal Trudy MAI* **82** https://www.mai.ru/science/trudy/published.php?ID=58453
[4] Mikhailova E Yu and Fedotenkov G V 2011 Nonstationary Axisymmetric Problem of the Impact of a Spherical Shell on an Elastic Half-Space (Initial Stage of Interaction) *Mechanics of Solids* **46** (2) 239-247. DOI: 10.3103/S0025654411020129
[5] Suvorov Ye M, Tarlakovskii D V and Fedotenkov G V 2012 The plane problem of the impact of a rigid body on a half-space modelled by a Cosserat medium *Journal of Applied Mathematics and Mechanics* **76** (5) 511–518 doi: 10.1016/j.jappmathmech.2012.11.015
[6] Tarlakovskii D V and Fedotenkov G V 2014 Two-dimensional nonstationary contact of elastic cylindrical or spherical shells *Journal of Machinery Manufacture and Reliability* **43 (2)** 145-152 doi: 10.3103/S1052618814010178
[7] Gorshkov A G and Tarlakovski D V 1995 Dynamic contact problems with motion boundaries (Moscow: Phizmatlit) 352