IOP Conf. Series: Materials Science and Engineering **158** (2016) 012035 doi:10.1088/1757-899X/158/1/012035

The algorithm for investigation a structure of composite materials

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Abstract. This paper presents a model of deformation of hyperelastic reinforced composite. The constitutive equations are derived using the free strain energy. The numerical algorithm to solve such problems is compiled. A computational algorithm is based on a finite element method.

1. Introduction

In different industries to increase structural strength often use composite materials. Besides, in biomechanics tissues can be described as composite materials with specific properties [13, 14]. Also modelling of mechanical behavior of composite materials is important in contact mechanics [7]. There are many different methods of calculating the structure of these materials [6, 27–30]. It is a promising improvement and development of new techniques. In this paper, an algorithm for calculating a composite material structure is considered with hyperelastic properties. Many papers are devoted to research of hyperelastic isotropic continuum [1–5, 8–12, 25, 26]. They lay down on the basis of this paper. In the first part is considered the free strain energy function for hyperelastic reinforced composite. The basic physical relations are constructed. A computational algorithm is described. The second part is devoted to finite element discretization.

2. Constitutive relations

Hyperelastic reinforced composite material is characterized by a free energy function strain in the form [20–22]:

$$\mathbf{\Psi}(\mathbf{C},\mathbf{A}) = \frac{\lambda}{2}J_1^2 + \mu_t J_2 + \alpha J_3 J_1 + 2(\mu_l - \mu_t)J_4 + \frac{\beta}{2}J_3^2.$$

There $\lambda, \mu_l, \mu_t, \alpha, \beta$ – the mechanical characteristics of the material determined from the experimental [23, 24]. Where invariants are defined as follows:

$$J_1 = \frac{1}{2}tr\left[\mathbf{C}-\mathbf{I}\right],\ J_2 = \frac{1}{4}tr\left[(\mathbf{C}-\mathbf{I})^2\right],\ J_3 = \frac{1}{2}tr\left[\mathbf{A}\cdot(\mathbf{C}-\mathbf{I})\right],\ J_4 = \frac{1}{4}tr\left[\mathbf{A}\cdot(\mathbf{C}-\mathbf{I})^2\right].$$

They depended on the measure of the left Cauchy–Green deformation tensor and the structural tensor of $\mathbf{A}$, which describes the position of the fibers in the material. At the initial time:

$$\mathbf{A}_0 = \vec{a}_0 \otimes \vec{a}_0,$$

$\vec{a}_0$ – the preferential fiber orientation vector in the reference configuration. After deformation, the fibers change their direction, and the tensor structure will

$$\mathbf{A} = \vec{a} \otimes \vec{a},$$

where $\vec{a}$ – the preferential fiber orientation vector in the deformation configuration.

To create a physical relationship is used equation of the form [21, 22, 27]:

$$\delta\mathbf{\Psi} = \frac{1}{2}\mathbf{S}\cdot\cdot\delta\mathbf{C}.$$

This implies that the second Piola–Kirchhoff stress tensor is defined as follows:

$$\mathbf{S} = 2\frac{\partial\mathbf{\Psi}}{\partial\mathbf{C}} = \frac{1}{2}\Big[\lambda tr\left(\mathbf{C}-\mathbf{I}\right)\cdot\mathbf{I} + 2\mu_t\left(\mathbf{C}-\mathbf{I}\right) + \alpha\Big(tr\Big(\mathbf{A}\cdot\left(\mathbf{C}-\mathbf{I}\right)\Big) + tr\Big(\left(\mathbf{C}-\mathbf{B}\right)\cdot\mathbf{A}\Big)\cdot\mathbf{I}\Big) +$$
$$+2\left(\mu_l - \mu_t\right)\Big(\left(\mathbf{C}-\mathbf{I}\right)\cdot\mathbf{A} + \mathbf{A}\cdot\left(\mathbf{C}-\mathbf{I}\right)\Big) + \beta tr\Big(\mathbf{A}\cdot\left(\mathbf{C}-\mathbf{I}\right)\cdot\mathbf{A}\Big)\Big].$$

To use the incremental method of calculation is necessary to define the stress increments, which are calculated as a material derivative of the stress tensor by time. In the general case we can write [1, 3]:

$$\dot{\mathbf{S}} = 2\frac{\partial^2\mathbf{\Psi}}{\partial\mathbf{C}^2}\cdot\cdot\dot{\mathbf{C}}.$$

After transformations the following expression:

$$\frac{\partial^2\mathbf{\Psi}}{\partial\mathbf{C}^2} = \mathbf{\Lambda}^{\circ} = \frac{1}{2}\left(\lambda\mathbf{I}\otimes\mathbf{I} + 2\mu_t\mathbf{\Lambda} + \alpha(\mathbf{I}\otimes\mathbf{A} + \mathbf{A}\otimes\mathbf{I}) + 2(\mu_l - \mu_t)\mathbf{\Lambda}_A + \beta\mathbf{A}\otimes\mathbf{A}\right),$$

where $\Lambda_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ – components of the 4th order tensor $\mathbf{\Lambda}$, $\Lambda_{A_{ijkl}} = A_{im}\Lambda_{jmkl} + A_{jm}\Lambda_{mikl}$ – components 4th order tensor $\mathbf{\Lambda}_A$, which components depend on the fiber direction.

The equation of principle of virtual power in the initial configuration is written as

$$\int_{V_0}\frac{1}{2}(\mathbf{S}\cdot\cdot\delta\dot{\mathbf{C}})dV_0 = \int_{V_0}\vec{f}_0\cdot\delta\vec{\upsilon}dV_0 + \int_{S_0^\sigma}\vec{t}_{0n}^{\,*}\delta\vec{\upsilon}dS_0$$

where $\vec{\upsilon}$ – velocity vector of a material point; S_0^σ – part of the surface on which are defined forces; $\vec{t}_{0n}^{\,*}$ – vector of surface forces, $\vec{f}_0$ – vector of body forces.

A total Lagrangian formulation is used to solve this problem [27, 28]:

$$\int_{V_0}\frac{1}{2}(\dot{\mathbf{S}}\cdot\cdot\delta\dot{\mathbf{C}} + \mathbf{S}\cdot\cdot\delta\ddot{\mathbf{C}})dV_0 = \int_{V_0}\dot{\vec{f}}_0\cdot\delta\vec{\upsilon}dV_0 + \int_{S_0^\sigma}\dot{\vec{t}}_{0n}^{\,*}\delta\vec{\upsilon}dS_0. \tag{1}$$

3. Finite element discretization

An 8 node isoparametric finite element is used for computer implementation. We introduce approximation geometry and velocities:

$${}^k y^i(\xi^j) = \sum_{t=1}^{8} {}^k y_t^i N_t(\xi^j),\ {}^k\upsilon^i\left(\xi^j\right) = \sum_{t=1}^{8} {}^k\upsilon_t^i N_t\left(\xi^j\right),$$

where $N_t\left(\xi^j\right)=\frac{1}{8}\left(1+\xi_t^1\xi^1\right)\left(1+\xi_t^2\xi^2\right)\left(1+\xi_t^3\xi^3\right)$ – interpolation function, $-1\le\xi^1,\xi^2,\xi^3\le 1$, ${}^k y_t^i$ – coordinates of nodes, $\xi_t^i=\pm 1$ the coordinates of the respective nodes in the local coordinate system, ${}^k\upsilon_t^i$ – velocity of nodes.

Define the following components of the tensors

$$C^{ij}=\sum_{t,s=1}^{8} y_t^m y_s^m N_{t,i}N_{s,j},\quad F^{ij}=\sum_{t=1}^{8} y_t^i N_{t,j},\quad \dot F^{ij}=\sum_{t=1}^{8}\upsilon_t^i N_{t,j}.$$

For the equation (1) we can write:

$$\begin{aligned}
&\dot{\mathbf{S}}\cdot\cdot\delta\dot{\mathbf{C}}=\dot{\mathbf{S}}\cdot\cdot(\mathbf{F}^T\cdot\delta\dot{\mathbf{F}}+\delta\dot{\mathbf{F}}^T\mathbf{F})=\mathbf{\Lambda}^\circ\cdot\cdot\dot{\mathbf{C}}\cdot\cdot(\mathbf{F}^T\cdot\delta\dot{\mathbf{F}}+\delta\dot{\mathbf{F}}^T\mathbf{F})=\\
&=\mathbf{\Lambda}^\circ\cdot\cdot(\dot{\mathbf{F}}^T\cdot\mathbf{F}+\mathbf{F}^T\cdot\dot{\mathbf{F}})\cdot\cdot(\mathbf{F}^T\cdot\delta\dot{\mathbf{F}}+\delta\dot{\mathbf{F}}^T\cdot\mathbf{F})=\\
&=\Lambda^\circ_{ijkl}\upsilon_t^m(N_{t,l}F_{mk}F_{ri}N_{s,j}+N_{t,l}F_{mk}F_{rj}N_{s,\mathrm{i}}+N_{t,\mathrm{k}}F_{ml}F_{ri}N_{s,j}+N_{t,\mathrm{k}}F_{ml}F_{rj}N_{s,\mathrm{i}})\delta\upsilon_s^r,
\end{aligned}$$

$$\mathbf{S}\cdot\cdot\left(\dot{\mathbf{F}}^T\cdot\delta\dot{\mathbf{F}}+\delta\dot{\mathbf{F}}^T\cdot\dot{\mathbf{F}}\right)=2S_{ij}\upsilon_t^m N_{t,k}\delta\upsilon_s^m N_{s,r}=2\upsilon_t^m S_{ij}\upsilon_t^m N_{t,j}N_{s,i}\delta_r^m\delta\upsilon_s^r$$

since the second Piola–Kirchhoff tensor is a symmetric tensor.

$$\int_{V_0}\dot{\vec f}_0\cdot\delta\vec\upsilon dV_0+\int_{S_0^\sigma}\dot{\vec t}_{0n}^{\,*}\cdot\delta\vec\upsilon dS_0=\left\{\dot{\mathbf{P}}\right\}^T\cdot\left\{\delta\vec\upsilon\right\},$$

$$-\frac{1}{\Delta t}\left\{\int_{V_0}(\mathbf{S}\cdot\cdot\delta\mathbf{C})dV_0-\int_{V_0}\vec f_0^{\,*}\cdot\delta\vec\upsilon\, dV_0-\int_{S_0^\sigma}\vec t_{0n}^{\,*}\cdot\delta\vec\upsilon dS\right\}=\frac{1}{\Delta t}\left\{\mathbf{H}\right\}^T\cdot\left\{\delta\vec\upsilon\right\}.$$

After integration (1) we obtain a system of linear algebraic equations:

$$\left[{}^k\mathbf{K}\right]\left\{\Delta^k\vec u\right\}=\left\{\Delta^k\mathbf{P}\right\}+\frac{1}{\Delta t}\left\{{}^k\mathbf{H}\right\}. \tag{2}$$

Solving equation (2), give the increment of the displacements $\Delta^k\vec u$, ${}^{k+1}y^i={}^k y^i+\Delta^k u^i$ and stresses ${}^{k+1}\mathbf{S}=2\frac{\partial\mathbf{\Psi}}{\partial\,{}^{k+1}\mathbf{C}}$.

4. Conclusion

The paper is constructed procedure of investigation of hyperelastic composites. The constitutive relationships are obtained for composite materials. The computational algorithm is created. A total Lagrangian formulation is used. The basic equations are obtained for computational algorithm. The numerical implementation is based on the finite element method.

Acknowledgments

The reported study was supported by Government of the Republic of Tatarstan research projects No. 15-41-02557 and 15-31-20602.

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