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Evaluation of the stress-strain state of a one-dimensional heterogeneous porous structure

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Abstract. The paper deals with the problem of determining the stress-strain state of the distal part of the pelvic girdle bones. The area was modeled using a rod loaded by a compressive force and was described by physical relations linking the stress-strain tensor through the elastic constants, the fabric tensor, and the solid volume fraction of the material. Taking into account the law of porosity variation, we considered the problem of evaluating the stress-strain state depending on the nature of the porous structure, and the relationship of the structure with mechanical macroparameters. In this work, we present the results of calculations for a single load, construct the diagrams for the components of the strain tensor, and carry out an assessment of deformations for various system parameters. To evaluate the macroparameters, we built the dependence of the Poisson ratio of the material on the rotation angle α and the pore ellipticity parameter λ. The sensitivity of the deformations to the elastic constants was also estimated.

1. Introduction

When solving the problems of determining the stress-strain state (SSS) of porous structures, it is necessary to take into account structural peculiarities of the material [1–5]. Today, when solving various problems of bone biomechanics [6–8], structures are commonly described by means of the fabric tensor. In this case, it is assumed that the fabric tensor is a quadratic form that describes the shape of a pore. Some physical relations have currently been built which connect the stress-strain tensor through the elastic constants, the fabric tensor, and the solid volume fraction in the material [9]. Analysis of the SSS of the distal part of the pelvic girdle bones is one of the urgent problems in bone biomechanics. In this work, the distal area is modeled using a rod loaded by a compressive force and described by the relations mentioned above. It is important to evaluate SSS taking into account the nature of the porous structure and the relationship of this structure with easily measurable mechanical macroparameters.

2. Materials and Methods

Let us consider a block diagram loaded by the compressive force P (see figure 1). Assume that it is uniformly filled with pores having the porosity ν_0. The pores have an elliptical shape with the fixed semi-radii and form an angle $\pi/2$-α with the line of action of the applied force. Let us formulate the SSS problem for this rod. Obviously, the constant tension is retained in each section:

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$$\sigma_i = \frac{P}{A}, \tag{1}$$

where A is the cross-section area.

For description of the porous structure, we will use the formalism associated with the fabric tensor. Then we will supplement the system by introducing physical relations in the form [9-12]:

$$\begin{aligned} \tilde{\sigma} &= (g_1 + g_2 e) tr\tilde{\varepsilon} \cdot E + (g_3 + g_4 e)\tilde{\varepsilon} + g_5(\tilde{\varepsilon}\tilde{K} + \tilde{K}\tilde{\varepsilon}) + \\ &+ g_6\left(tr(\tilde{K}\tilde{\varepsilon}) \cdot E + tr\tilde{\varepsilon} \cdot \tilde{K}\right), \vec{x} \in \bar{S}, t \geq 0, \end{aligned} \tag{2}$$

where e is the variation of the solid bone volume fraction, $\tilde{K}$ is the fabric tensor deviator, g_i are the elastic constants.

The fabric tensor is by definition normalized in such a way that

$$Tr\tilde{H} = 1,$$

its deviator is defined by the formula

$$\tilde{K} = \tilde{H} - \frac{1}{3}\tilde{E}.$$

As a result in this case we obtain

$$Tr\tilde{K} = 0.$$

The law of porosity variation was used in the following form:

$$\nu = \frac{\nu_0 + \sigma_i\left({}^{1}\!/_{3K_{sp}} - {}^{1}\!/_{3K_s} \right)}{1 + {}^{\sigma_i}\!/_{3K_{sp}}} \tag{3}$$

where K_{sp} is the bulk modulus of elasticity of the skeleton with pores, K_s is the bulk modulus of elasticity of the skeleton, ν and ν_0 are the actual and initial porosities respectively, and σ_i is the compressive stress.

The following parameters were introduced during the research:

$$n = \frac{K_{sp}}{K_s}; \lambda = \frac{r_1}{r_2},$$

where n characterizes the volume liquid loss of the material, and λ is the pore ellipticity.

In this case the fabric tensor can be written in the following form:

$$\tilde{H} = \begin{pmatrix} \lambda \cdot h & 0 \\ 0 & h \end{pmatrix}$$

Rotation of the tensor H is implemented by means of multiplication by a rotation matrix, wherein the rotation angle α is a system parameter.

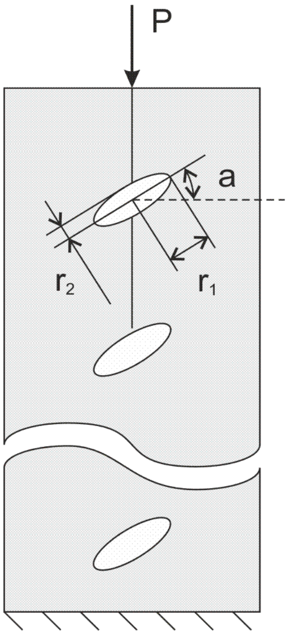

Figure 1. Computational scheme.

The aim of the study is to evaluate deformations for various system parameters. We used the following constant values in the numerical simulation [Cowin, Kichenko]: g_1 = 154.9 GPa, g_2 = 1147 GPa, g_3 = 612.9 GPa, g_4 = 4536 GPa, g_5 = 2384 GPa, g_6 = 510.8 GPa. We also evaluated the solution sensitivity to the deviation from these parameters in case of their variation within 20%.

3. Results and Discussion

We carried out calculations for a single load and constructed diagrams for the strain tensor components (see figure 2) on the λ-α axes. In extreme cases when λ=1 (round pores), the solution coincides with the known one and does not depend on the rotation angle; there are no shear deformations. After analyzing longitudinal deformations, we can note that they reach the largest values when the pore with the bigger radius is directed along the load action line (see figure 2a). The distribution of the transverse deformations is symmetrical relative to the line $\alpha = \pi / 4$, and their absolute value increases when approaching $\alpha = \pi / 4$ (see figure 2b). As far as shear deformations are concerned, they reach the maximum in the case of $\alpha = \pi / 6$ and $\lambda \to 0$ (see figure 2c).

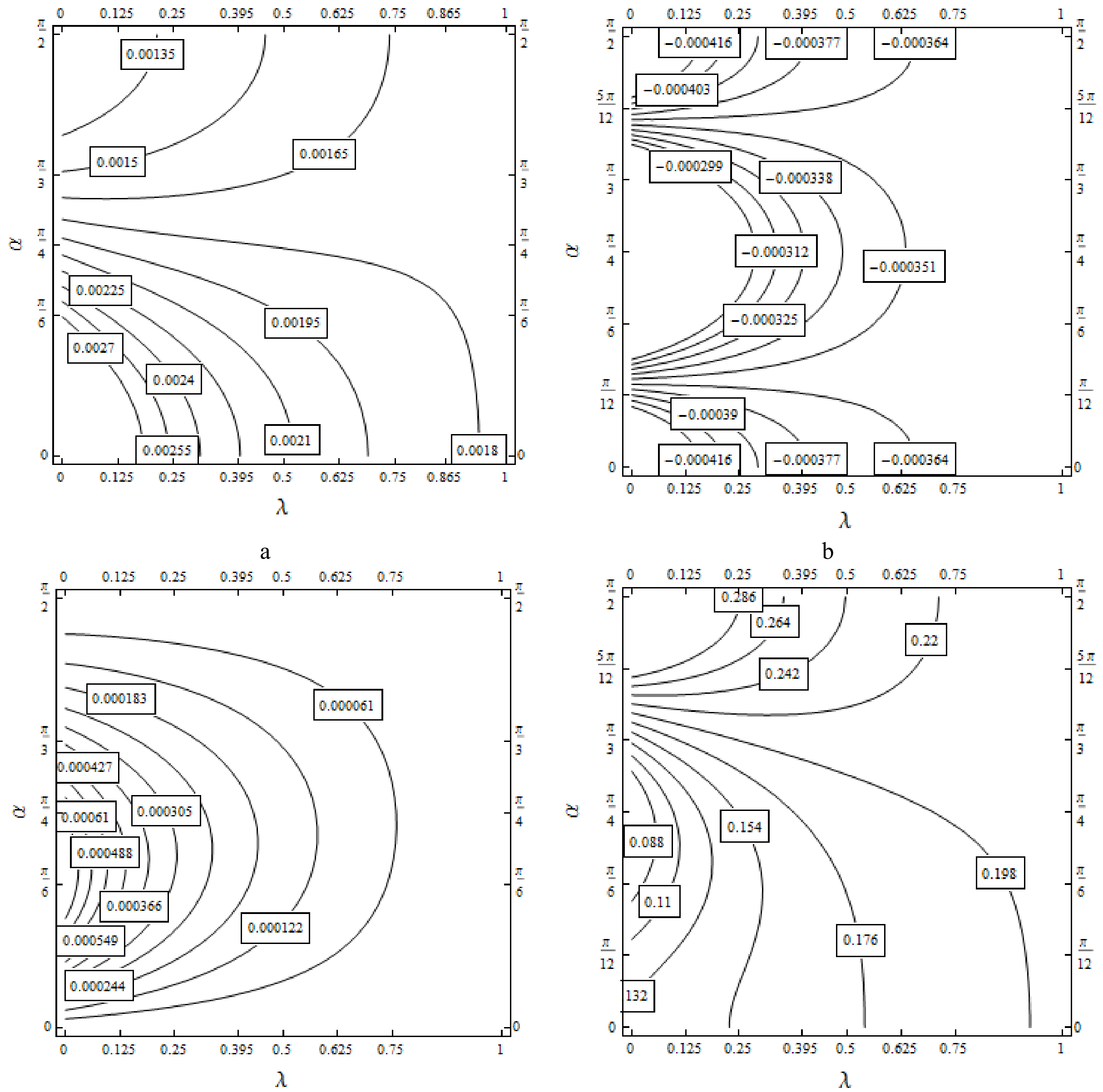

Figure 2. Diagrams for the components of the strain tensor.

For evaluating the macroparameters, we built a dependence of the Poisson ratio of the material on the λ-α axes (see figure 2d). It was noted that in the case of circular pores, the obtained dependence does not depend on the parameters n, ν_0, K_s (but obviously depends on the elastic constants g_i) and tend to 0.2. The largest value of the Poisson ratio is achieved when $\alpha \to \pi/2$ and $\lambda \to 0$ is within 0.3–0.35, depending on the initial porosity and elastic constants. Evaluation of sensitivity to the elastic constants showed their underestimation to the value of up to 20% results in the deformation increase by up to 30%. With an increase of the elastic constants by 20%, the deformations decrease by 20%.

4. Conclusion

This study examines the problem of finding the stress-strain state of the distal area in the pelvic girdle bones. The area was modeled by a rod loaded with a compressive force and was described by the physical relations linking the stress-strain tensor through the elastic constants, the fabric tensor, and the solid volume fraction of the material. For the stated law of porosity variation, we considered the problem of evaluating the stress-strain state, depending on the nature of the porous structure and the relationship of the structure with the mechanical macroparameters. The calculation results are given for a single load. We constructed the diagrams for the strain tensor components and made an assessment of deformations for various system parameters. To evaluate the macroparameters, we built the dependence of the Poisson ratio of the material on the rotation angle α and the pore ellipticity parameter λ. The sensitivity of the deformations to the elastic constants was found to be 20–30%.

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References
[1] Berezhnoi D V, Sachenkov A A and Sagdatullin M K 2014 Geometrically nonlinear deformation elastoplastic soil *Applied Mathematical Sciences* **8** 6341-6348 http://dx.doi.org/10.12988/ams.2014.48672
[2] Berezhnoi D V, Sachenkov A A and Sagdatullin M K 2014 Research of interaction of the deformable designs located in the soil *Applied Mathematical Sciences* **8** 7107-7115 http://dx.doi.org/10.12988/ams.2014.49706
[3] Berezhnoi D V and Sagdatullin M K 2015 Calculation of interaction of deformable designs taking into account friction in the contact zone by finite element method *Contemporary Engineering Science*s **8** 1091-1098 http://dx.doi.org/10.12988/ces.2015.58237
[4] Sultanov L U 2015 Numerical Modelling of Deformation of Soil Foundations *Procedia Earth and Planetary Science* **15** 119–124 doi:10.1016/j.proeps.2015.08.029
[5] Davydov R L, Sultanov L U and Kharzhavina V S 2015 Elastoplastic Model of Deformation of Three- Dimensional Bodies in Terms of Large Strains *Global Journal of Pure and Applied Mathematics* **11** 5099-5108
[6] Sachenkov O, Kharislamova L, Shamsutdinova N, Kirillova У and Konoplev Yu 2015 Evaluation of the bone tissue mechanical parameters after induced alimentary Cu-deficiency followed by supplementary injection of Cu nanoparticles in rats *IOP Conference Series: Materials Science and Engineering* **98** 012015 doi:10.1088/1757-899X/98/1/012015
[7] Shigapova F A, Mustakimova M F, Saleeva G T and Sachenkov O A 2015 Definition of the intense and deformable jaw state under the masseters hyper tone *International Journal of Applied Engineering Research* **10** 44711-44714
[8] Galiullin R R, Sachenkov O A, Khasanov R F and Andreev P S 2015 Evaluation of external fixation device stiffness for rotary osteotomy *International Journal of Applied Engineering Research* **10** 44855-44860
[9] Kichenko A A, Tverier V M, Nyashin Y I, Osipenko M A and Lokhov V A 2012 On application of the theory of trabecular bone tissue remodeling *Russian J. of Biomechanics* **16** 46–64
[10] Cowin S C and Benalla M 2011 Graphical illustrations for the nur-byerlee-carroll proof of the formula for the biot effective stress coefficient in poroelasticity *Journal of Elasticity* **104** 133-141 DOI 10.1007/s10659-011-9324-7
[11] Thompson M and Willis J R 1991 A reformation of the equations of anisotropic poroelasticity *Journal of Applied Mechanics* **58** 612-616 doi:10.1115/1.2897239
[12] Rho J Y, Roy M E, Tsui T Y and Pharr G M 1999 Elastic properties of microstructural components of human bone tissue as measured by indentation *J. Biomed. Mater. Res.* **45** 45–48