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Decomposition method for zonal resource allocation problems in telecommunication networks

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Abstract. We consider problems of optimal resource allocation in telecommunication networks. We first give an optimization formulation for the case where the network manager aims to distribute some homogeneous resource (bandwidth) among users of one region with quadratic charge and fee functions and present simple and efficient solution methods. Next, we consider a more general problem for a provider of a wireless communication network divided into zones (clusters) with common capacity constraints. We obtain a convex quadratic optimization problem involving capacity and balance constraints. By using the dual Lagrangian method with respect to the capacity constraint, we suggest to reduce the initial problem to a single-dimensional optimization problem, but calculation of the cost function value leads to independent solution of zonal problems, which coincide with the above single region problem. Some results of computational experiments confirm the applicability of the new methods.

1. Introduction

Despite the existence of powerful processing and transmission devices, increasing demand of different telecommunication services and its variability lead to serious congestion effects and inefficient utilization of network resources. This situation forces one to replace the fixed allocation rules with more flexible mechanisms, which are based on proper mathematical models; see e.g. [1]–[3]. In particular, spectrum sharing is now one of the most critical issues in this field and various adaptive mechanisms have been suggested. Most papers are devoted to game-theoretic models and implementation of decentralized iterative methods for finding the Nash equilibrium points or their generalizations; see e.g. [4, 5]. At the same time, various optimization based mechanisms are also suggested; see e.g. [6, 7, 5, 3].

In this paper, we consider some problems of optimal allocation of a homogeneous resource in telecommunication networks such that the income received from users payments is maximized and the implementation costs of the network operator are minimized. We first present an optimization formulation for the case where the network manager aims to distribute some homogeneous resource (bandwidth) among users of one region with quadratic charge and fee functions. These convex quadratic optimization problems can be solved by simple and efficient solution methods. We describe some modifications for this special problem. Next, we consider a more general resource allocation problem for a provider of a wireless communication network divided into zones (clusters); which was formulated as a convex optimization problem in [8, 9]. Now, since the price functions are affine, we obtain again a convex quadratic optimization problem having capacity and zonal balance constraints. Unlike [8, 9], we now suggest to apply the dual Lagrangian method with respect to only capacity constraint. Therefore, we replace the initial problem with a single-dimensional optimization problem, however, calculation of its cost function value requires independent solution of zonal problems. Each of these problems coincides with the above single region resource allocation problem and can be solved by the simple algorithms suggested. In such a way we develop a new dual decomposition approach for solution finding, whose implementation is simpler essentially in comparison with the methods from [8, 9]. We present results of computational experiments which confirm the applicability of the new method.

2. Simple resource allocation model

Let us consider a single telecommunication network with nodes (users). The general problem of a network manager is to find an optimal allocation of a limited homogeneous resource among the users in order to maximize the total payment received from the users and to minimize the total network implementation expenses. That is, x is an unknown quantity of the resource offered by the network, with the capacity bounds $x \in [0,b]$, which yields the network expense (cost of implementation) $u(x)$. Similarly, y_i is the unknown resource offered to user $i \in I$ and $\varphi_i(y_i)$ is the fee (incentive) value paid by node i with the capacity bounds $y_i \in [0,a_i]$, where I is the index set of users. The network manager problem is formulated as follows:

$$\max_{(x,y)\in D} \to \sum_{i\in I}\varphi_i(y_i) - u(x), \tag{1}$$

where $y = (y_i)_{i\in I}$,

$$D = \left\{ (x,y) \ \sum_{i\in I} y_i = x, 0 \le y_i \le a_i, i \in I, 0 \le x \le b \right\}.$$

Suppose that the set D is non-empty, functions $u(x)$ and $-\varphi_i(y_i)$ are convex and quadratic, i.e.,

$$\varphi_i(y_i) = 0.5\alpha_i y_i^2 + \beta_i y_i, \alpha_i < 0, i \in I; u(x) = 0.5\gamma x^2 + \delta x, \gamma > 0.$$

Then (1) is a convex quadratic optimization problem, which can also be treated as a two-side auction models with one trader where all the participants have affine price functions; see [10, 11]. For these problems there exist many rather efficient solution methods; see e.g. [12] and references therein. They are mostly based on duality theory.

Following this approach, write the Lagrange function of problem (1) with the negative sign:

$$\begin{aligned} M(x,y,p) &= u(x) - \sum_{i\in I}\varphi_i(y_i) - p\left(x - \sum_{i\in I} y_i \right) \\ &= (u(x) - px) - \sum_{i\in I}(\varphi_i(y_i) - py_i) \\ &= (0.5\gamma x^2 + \delta x - px) - \sum_{i\in I}(0.5\alpha_i y_i^2 + \beta_i y_i - py_i) \end{aligned}$$

In order to find a value of the dual cost function

$$\theta(p) = \min_{x\in[0,b],\, y\in[0,a]} M(x,y,p),$$

where $a = (a_i)_{i\in I}$, we have to solve one- dimensional problems:

$$\min_{0\le x_k \le b_k} \to (0.5\gamma x^2 + \delta x - px),$$

and

$$\min_{0\le y_i\le a_i} \to (-0.5\alpha_i y_i^2 - \beta_i y_i + p y_i),$$

for $i \in I$. Solutions of these problems denoted by $x(p)$ and $y_i(p)$, $i \in I$, respectively, are defined uniquely. Set $\tilde{x}(p) = (p-\delta)/\gamma$ and $\tilde{y}_i(p) = -(\beta_i - p)/\alpha_i$, then

$$x(p) = \begin{cases} 0 & \text{if } p \le \delta, \\ b & \text{if } p \ge \delta + \gamma b, \\ \tilde{x}(p) & \text{otherwise}; \end{cases} \quad y_i(p) = \begin{cases} 0 & \text{if } p \ge \beta_i, \\ a_i & \text{if } p \le \beta_i + \alpha_i a_i, \\ \tilde{y}_i(p) & \text{otherwise}; \end{cases} \quad \text{for } i \in I; \tag{2}$$

It follows that the function $\theta(p)$ is concave and differentiable with

$$\theta'(p) = \sum_{i\in I} y_i(p) - x(p).$$

Besides, the one- dimensional dual problem

$$\max_p \to \theta(p)$$

coincides with the simple equation

$$\theta'(p) = 0, \tag{3}$$

where $\theta'(p)$ is non-increasing. If p^* is the solution of (3), then we can find the solution of the initial problem (1) from (2) by setting $p = p^*$.

If we set $p'' = \max_{i\in I} \beta_i$ and $p' = \delta$, then the case $p'' \le p'$ gives immediately the zero solutions in accordance with (2). So we can consider only the non-trivial case where $p' < p''$. Then by (2) we must have $\theta'(p') > 0$ and $\theta'(p'') < 0$. These properties enable us to find a solution of (3) by the simple bisection algorithm, denoted as **Algorithm (BS)**. Given an accuracy $\varepsilon > 0$ and the initial segment $[p', p'']$, we take $\tilde{p} = 0.5(p' + p'')$, calculate $\theta'(\tilde{p})$. Then we set $p' = \tilde{p}$ if $\theta'(\tilde{p}) > 0$ and $p' = \tilde{p}$ otherwise, until $(p'' - p') < \varepsilon$.

We can also utilize various heuristic algorithms. For instance, we describe a simple **Algorithm (SQ)**. Define $I_a = \{i \in I \mid \beta_i > p'\}$, set $y_i^* = 0$ for $i \notin I_a$ and re-arrange the indices in I_a to have the descending order for the values of β_i. Then find two sequential indices i_l and i_{l+1} in I_a such that $\Delta_l < 0$ and $\Delta_{l+1} > 0$, where

$$\Delta_l = \sum_{s=1}^{l} y_{i_s}(\beta_{i_l}) - x(\beta_{i_l}).$$

Then find p^* such that $\theta'(p^*) = 0$ in the segment $[\beta_{i_l}, \beta_{i_{l+1}}]$.

3. Multi-zonal network problem

Let us consider a more general model where a telecommunication network is divided into several zones (clusters). The problem of a manager of the network is to find the optimal allocation of a limited homogeneous network resource among the zones in order to maximize the total profit containing the total income received from consumers' fees and negative resource implementation costs; see [8, 9].

Let us use the following notation:

- n is the number of zones;
- I_k is the index set of users (currently) located in zone k ($k = 1,\ldots,n$);
- B is the total resource supply (the total bandwidth) for the system (network);
- x_k is an unknown quantity of the resource allotted to zone k with the upper bound b_k and $f_k(x_k)$ is the cost of implementation of this quantity of the resource for zone k ($k = 1,\ldots,n$);
- y_i is the resource amount received by user i with the upper bound a_i and $\varphi_i(y_i)$ is the charge value paid by user i for the resource value y_i.

The network manager problem is the optimization problem involving capacity and balance constraints:

$$\max \to \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k)\right], \tag{4}$$

subject to

$$\sum_{k=1}^{n} x_k \le B; \tag{5}$$

$$\sum_{i\in I_k} y_i = x_k,\ k = 1,\ldots,n; \tag{6}$$

$$0 \le y_i \le a_i,\ i \in I_k,\ 0 \le x_k \le b_k,\ k = 1,\ldots,n. \tag{7}$$

That is, (6) provides the balance for demand and supply in each zone, (7) involves capacity constraints for users and network supply values in each zone, and (5) gives the upper bound for the total resource supply.

In what follows we assume that there exists at least one feasible point satisfying conditions (5)–(7), all the functions $f_k(x_k)$ and $-\varphi_i(y_i)$ are convex and quadratic, i.e.

$$\begin{aligned} &\varphi_i(y_i) = 0.5\alpha_i y_i^2 + \beta_i y_i,\ \alpha_i < 0,\ i \in I_k, \\ &f_k(x_k) = 0.5\gamma_k x_k^2 + \delta_k x_k,\ \gamma_k > 0;\ k = 1,\ldots,n. \end{aligned} \tag{8}$$

This means that (4)–(8) is a convex quadratic optimization problem. However, due to large dimensionality and inexact data one can meet serious drawbacks in solving this problem with usual finite or penalty solution methods. In order to create an efficient method, we have to take into account its separability and apply certain decomposition approach. However, the standard duality approach using the Lagrangian function with respect to all the functional constraints leads to the multi-dimensional dual optimization problem. We will apply another approach, which was suggested in [13]. Let us define the Lagrange function of problem (4)–(7) as follows:

$$L(x,y,\lambda) = \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k)\right] - \lambda\left(\sum_{k=1}^{n} x_k - B\right).$$

We utilize the Lagrangian multiplier λ only for the total resource bound. We can now replace problem (4)–(7) with its dual:

$$\min_{\lambda\ge 0} \to \psi(\lambda), \tag{9}$$

where

$$\psi(\lambda) = \max_{(x,y)\in W} L(x,y,\lambda) = \lambda B + \max_{(x,y)\in W} \sum_{k=1}^{n} \left[\sum_{i\in I_k} \varphi_i(y_i) - f_k(x_k) - \lambda x_k \right],$$

$$W = \left\{ (x,y) \begin{array}{l} \sum_{i\in I_k} y_i = x_k,\, 0 \le y_i \le a_i,\, i \in I_k, \\ 0 \le x_k \le b_k,\, k = 1,\dots,n \end{array} \right\}.$$

By duality (see e.g. [14, 15]), problems (4)–(7) and (9) have the same optimal value. But solution of (9) can be found by one of well-known single-dimensional optimization algorithms; see e.g. [15]. In order to calculate the value of $\psi(\lambda)$ we have to solve the inner problem:

$$\max \to \sum_{k=1}^{n} \left[\sum_{i\in I_k} \varphi_i(y_i) - f_k(x_k) - \lambda x_k \right],$$

subject to

$$\sum_{i\in I_k} y_i = x_k,\, 0 \le y_i \le a_i,\, i \in I_k,$$
$$0 \le x_k \le b_k,\, k = 1,\dots,n.$$

Obviously, this problem decomposes into n independent zonal optimization problems

$$\max \to \left[\sum_{i\in I_k} \varphi_i(y_i) - f_k(x_k) - \lambda x_k \right], \tag{10}$$

subject to

$$\sum_{i\in I_k} y_i = x_k,\, 0 \le y_i \le a_i,\, i \in I_k,$$
$$0 \le x_k \le b_k;$$

for $k = 1,\dots,n$. Each k-th independent zonal problem (10) clearly coincides with problem (1) where

$$\varphi_i(y_i) = 0.5\alpha_i y_i^2 + \beta_i y_i,\, i \in I_k,$$
$$u(x) = f_k(x_k) + \lambda x_k = 0.5\gamma_k x_k^2 + (\delta_k + \lambda) x_k.$$

Therefore, we can find its solution by the algorithms of Section 2.

4. Numerical experiments

The methods were implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable λ were taken as [0,1000]. Values of b_k were chosen by trigonometric functions in [1,11], values of a_i were chosen by trigonometric functions in [1,2]. Value B were taken equal 1000. The number of zones was varied from 5 to 105, the number of users was varied from 210 to 1010. Users were distributed in zones either uniformly or according to the normal distribution.

The coefficients of the functions $f_k(x_k)$ and $\varphi_i(y_i)$ from (8) were taken as

$$\gamma_k = 2(|sin(2k+2)|+1), \delta_k = |cos(k+1)|+3,$$

and

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$$\alpha_i = -6|cos(2i+1)|-6, \beta_i = |sin(i+2)|-1.$$

For all the methods of finding solution of problem (4)–(7) the accuracy of upper dual problem solution were varied from 10^{-1} to 10^{-4}. The accuracy of lower level problem solution in Algorithm (BS), was fixed and equal to 10^{-2}. For each set of the parameters made 1000 tests. Let J denote the total number of users, T_ε the total processor time in seconds. The results of computations are given in Tables 1–3.

Table 1. Results of testing with $J = 510$, $n = 70$.

ε_λ	T_ε: Algorithm (SQ)	T_ε: *Algorithm (BS)*
10^{-1}	0.0183	0.0018
10^{-2}	0.0217	0.0017
10^{-3}	0.0247	0.0020
10^{-4}	0.0289	0.0022

Table 2. Results of testing with $n = 70$, $\varepsilon = 10^{-2}$.

J	T_ε: Algorithm (SQ)	T_ε: *Algorithm (BS)*
210	0.0043	0.0009
310	0.0088	0.0009
410	0.0142	0.0012
510	0.0217	0.0017
610	0.0303	0.0022
710	0.0397	0.0026
810	0.0515	0.0032
910	0.0639	0.0039
1010	0.0791	0.0048

Table 3. Results of testing with $J = 510$, $\varepsilon = 10^{-2}$.

n	T_ε: Algorithm (SQ)	T_ε: *Algorithm (BS)*
5	0.0221	0.0004
15	0.0217	0.0005
25	0.0218	0.0006
35	0.0217	0.0006
45	0.0214	0.0017
55	0.0220	0.0010
65	0.0215	0.0011
75	0.0223	0.0010
85	0.0216	0.0012
95	0.0220	0.0012
105	0.0219	0.0015

As we can see from the results in the tables, in all the cases the suggested methods were rather effective in finding a solution. Moreover, for the same accuracy, both the methods gave the same numbers of upper iterations, so that the main difference was in the processor time which showed that utilization of Algorithm (BS) for inner optimization problems give better performance.

5. Conclusions

We considered several problems of optimal resource allocation in telecommunication networks. We presented simple and efficient solution methods for the case where the network manager aims to distribute some homogeneous resource (bandwidth) among users of one region with quadratic charge and fee functions. Next, we considered a more general problem for a provider of a wireless communication network divided into zones. By using the dual Lagrangian method with respect to the capacity constraint, we suggest to reduce the initial problem to a single-dimensional optimization problem, where calculation of the cost function value leads to independent solution of zonal problems, which coincide with the above single region problem. Some results of computational experiments confirm the applicability of the new methods.

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