Numerical study of the three-degreed parametrically excited gyroscopic system

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Abstract. A mathematical model of the rotor vibratory gyroscope was constructed. The operating principle of the device based on the use of the tuned resonance of three-degreed oscillating system. Parametric excitation of a gyroscope is achieved by modulating the angular velocity of rotation of the rotor. The results of numerical calculations allow not only to illustrate the process of operation of the gyroscope, but also to find the values of the parameters under which it can be used as a device of orientation of moving objects.

1. Introduction

The development of inertial navigation tools relates with enhancing their functionality and with improving the accuracy of measurement of angular velocity (0.001-0.01 deg/hour) in conditions of exposure to significant congestion, and with minimizing their dimensions, weight characteristics and energy consumption. Such requirements are fully satisfied in the rotary vibratory gyroscope (RVG). Below we consider the case of parametrically excited three-degreed RVG, kinematic scheme of which [1] is shown in Fig. 1.

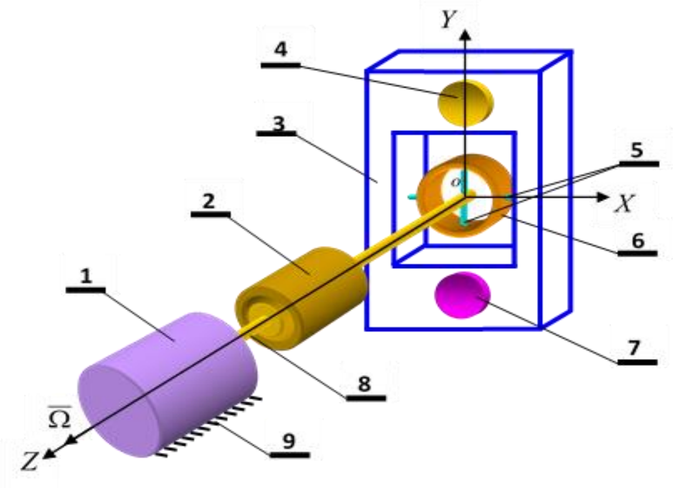

Figure 1. Kinematic scheme of the three-degreed RVG with asymmetrical rotor.

Circuit includes: a sensing element (unbalanced rotor) 3 fixed to the shaft 8 of the drive motor 1 by means of two pairs of torsions 5 and the gimbal 6. The vibration amplitude of the rotor RVG is converted to an electrical signal by the angle sensor 4, the creation of control moments applied to the rotor is effected by the torque sensor 7, controlled by signals carried from the housing 9, by using sine-cosine rotary transformer 2.

Innovative use of the gyroscope, which related to its parametric excitation, imposes special requirements to the dynamics of operation of the device. This, in turn, leads to the necessity of numerical analysis the corresponding gyro system based on a strict mathematical model.

2. The mathematical model of the parametrically excited RVG

Specificity of the modulation of the rotor vibratory gyroscope allows through the use of circuit design, without changing the mechanical design of the circuit, to raise the sensitivity of the instrument to the measured angular velocity by parametric excitation [2]. Parametric excitation of RVG as the oscillatory system is carried out by modulation of the dynamic stiffness of the torsions and gyroscopic moments by modulating the angular velocity of rotation of the rotor. Modulation is implemented by varying within a small range of frequency of the electric actuator, according to the law:

$$\Omega_m(t) = \Omega + \Delta\Omega\cos(\omega_m t) \ , \tag{1}$$

where Ω – the nominal frequency of the motor shaft corresponding to a configured mode of operation of the gyro, $\Delta\Omega$ – frequency deviation, ω_m – frequency modulation.

The current value of the phase of the oscillations of the shaft of the drive motor relative to the nominal value defined by the expression:

$$\phi_m(t) = \int \Omega_m(t)dt = \Omega t + m\sin(\omega_m t) \ , \tag{2}$$

where $m = \Delta\Omega/\omega_m$ – the index of the frequency modulation.

Given the properties of the Bessel functions of the first kind for values of the modulation index $m \leq 0.2$, and the expression (2), periodic functions can be written as:

$$\sin(\phi_m(t)) = \sin(\Omega t) + m(\sin(\Omega + \omega_m)t + \sin(\Omega - \omega_m)t)/2 \ ,$$
$$\cos(\phi_m(t)) = \cos(\Omega t) - m(\cos(\Omega - \omega_m)t - \cos(\Omega + \omega_m)t)/2 \ . \tag{3}$$

To describe the motion of RVG mounted on the moving base, we introduce a coordinate system $0\xi^*\eta^*\zeta^*$ with the origin at the center of mass of the gyro rotor.

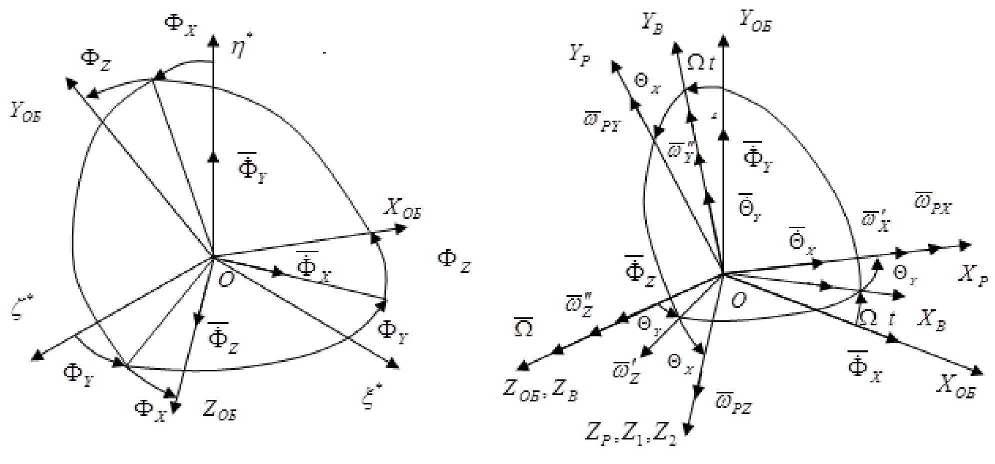

Figure 2. Coordinate systems

With a movable base we can tie the coordinate system $OX_{об}Y_{об}Z_{об}$, the axis $Z_{об}$ of which coincides with the axis of rotation of the rotor RVG (Fig. 2a). The motion of the base of the device will considered as known, i.e. in each instant of time is known the orientation of the coordinate system $OX_{об}Y_{об}Z_{об}$ relative to the inertial $0\xi^*\eta^*\zeta^*$, and the projection $\dot{\Phi}_x, \dot{\Phi}_y, \dot{\Phi}_z$ of the vector of absolute angular velocity on the base axis system $0\xi^*\eta^*\zeta^*$ are given functions of time. In addition to the above systems are required two systems of axes $OX_{в}Y_{в}Z_{в}$, $OX_pY_pZ_p$ associated respectively with the shaft of a drive motor and with the principal axes of inertia of the rotor (Fig. 2b).

The point of origin of the system $OX_{в}Y_{в}Z_{в}$, $OX_pY_pZ_p$ lies in the center of mass of the EGR and its position relative to the system $OX_{об}Y_{об}Z_{об}$ is successive turns in the positive direction on appropriate angles Ωt for $OX_{в}Y_{в}Z_{в}$ and θ_x, θ_y for $OX_pY_pZ_p$.

For deriving the equations of motion of a gyroscopic system will use the variational principle of Ostrogradsky – Hamilton (see for example [3]), from which follow equations

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i \,, \tag{4}$$

where T is the kinetic energy of the considered gyroscopic system; q_{i} – generalized coordinates defining the position of all points of the system in space, Q_{i} – generalized forces acting on the system; i – the number of generalized coordinates describing the degrees of freedom of the considered electromechanical system.

Choose as generalized coordinates the angles of rotation of the rotor $\theta_x = \beta(t)$ and $\theta_y = \alpha(t)$, uniquely determining its position in the coordinate system of the shaft, and as generalized forces – damping moments of moments of elasticity of the torsions.

The differential equations of motion for the case of constant angular velocity of rotation of the base, after the procedure of factorization [4], subsequent linearization using the Jacobian matrix will be of the form

$$A\ddot{\beta} + \mu\dot{\beta} + \big(k + (C - B)\Omega(\Omega + 2m\omega_m\cos(\omega_m t))\big)\beta - (A + B - C)(\Omega + m\omega_m\cos(\omega_m t))\dot{\alpha} =$$
$$= (A + C - B)\Omega\big(\dot{\Phi}_x\sin(\Omega t) - \dot{\Phi}_y\cos(\Omega t)\big),$$

$$B\ddot{\alpha} + \mu\dot{\alpha} + \big(k + (C - A)\Omega(\Omega + 2m\omega_m\cos(\omega_m t))\big)\alpha + (A + B - C)(\Omega + m\omega_m\cos(\omega_m t))\dot{\beta} =$$
$$= (C + B - A)\Omega\big(\dot{\Phi}_x\cos(\Omega t) + \dot{\Phi}_y\sin(\Omega t)\big), \tag{5}$$

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where A, B, C are respectively the equatorial and polar moments of inertia of the rotor, μ is the coefficient of viscous friction of the material of torsions, k – torsional stiffness of the torsions, $(C-A)\Omega^2$ and $(C-B)\Omega^2$ is the dynamic stiffness of the respective axes of suspension.

3. Numerical study

Feature of the system of differential equations (5) is the presence of elements associated with periodic change of the gyroscopic and the positional moments:

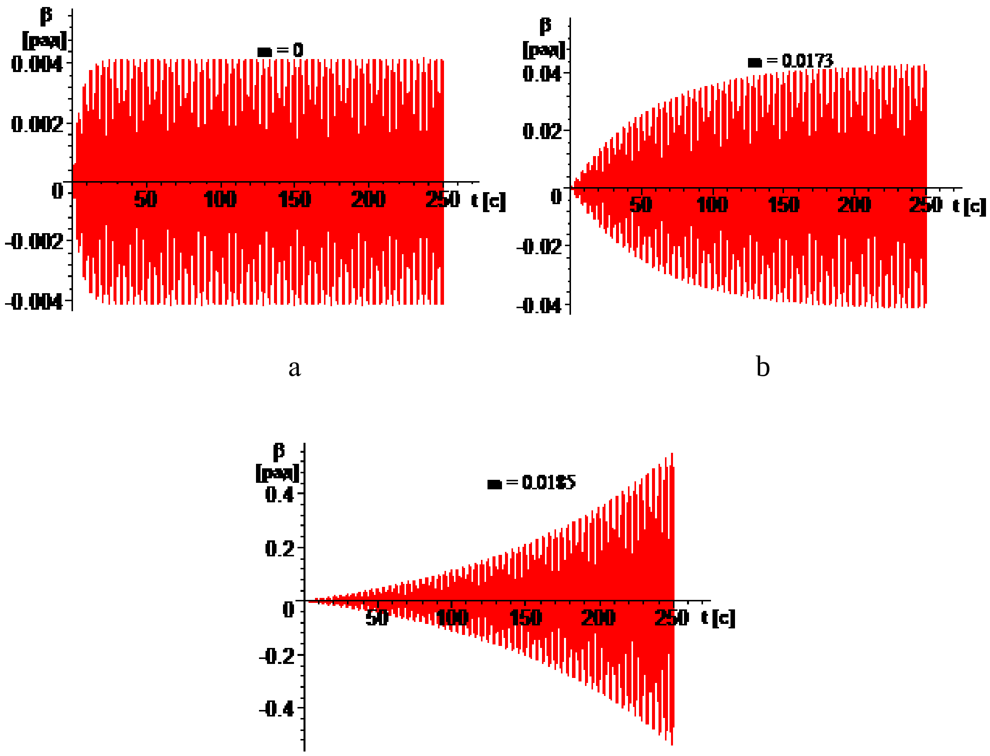
$$(A+B-C)m\omega_m\cos(\omega_m t)\dot{\alpha}A^{-1},\quad (A+B-C)m\omega_m\cos(\omega_m t)\dot{\beta}B^{-1},$$

$$2(C-A)m\omega_m\Omega\cos(\omega_m t)\alpha B^{-1},\qquad 2(C-B)m\omega_m\Omega\cos(\omega_m t)\beta A^{-1}.$$

The presence of such periodically varying energy parameters RVG as gyroscopic moments and dynamic stiffness included in the respective nonhomogeneous differential equations, provides favorable conditions for the parametric excitation of the mechanical contour of the considered gyro with frequency $\omega_m = \Omega$ [5].

In Fig.3 shows the results of numerical calculations carried out using the mathematical package Maple 9 when $\omega_m = \Omega$ (coherent mode) and when $\dot{\Phi}_x = 0.001\ c^{-1}$, $\dot{\Phi}_y = 0$.

Analysis of the obtained results shows that the solution of equations (5) essentially depends on values of the modulation index m. If at small values of m the solution is stable (Fig. 3a, 3b), when m = 0.0185 observed erratic behavior of the gyro (Fig. 3c). The critical value of the modulation index can be found through numerical experiments, and analytically [6].



a b

c

Figure 3. The graphs of the variation of rotor vibration amplitude in time at various values of modulation index *m*.

From the graph in Fig. 3b it follows that the modulation of the angular velocity of the gyro rotor enhances its sensitivity in the steady state (differentiating mode gyro) approximately one order of magnitude compared to the device in which parametric excitation of no (Fig. 3a). This is due to the fact, that the modulation the angular velocity of rotation of the rotor significantly reduces the coefficient of friction μ. This increases the duration of the ramp-up of amplitude, which corresponds to the operation of the device in the mode of measuring the angle of rotation of foundation (integrated mode).

It should be noted, that in the coherent mode, the sensitivity of RVG depends on to the value of the phase shift between the periodically varying dynamic stiffness of the torsions, the gyroscopic moments, which is characteristic of three-degreed gyroscope, and the external gyroscopic moment, creating by the angular velocity of rotation of the base of device.

Studies have shown that when using RVG as the gyrocompass it can be used as the coherent mode and incoherent mode of operation of the device, which is implemented when the condition $\omega_m \approx \Omega$. In the incoherent case, the amplitude of oscillation of the gyro rotor is also increasing, but the waveform is not stored and instead of harmonic oscillations with one spectral component is obtained a complex oscillation consisting of two harmonic component. It may be considered as quasi-harmonic oscillation with periodically variable amplitude.

The results obtained agree well with the experimental data given in [7].

Thus, the mathematical model describing the dynamics of motion parametrically excited by a three-degreed RVG, allows not only to illustrate the process of operation of the gyroscope, but also to find the values of the parameters under which it can be used as a device of orientation of moving objects.

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