

Nonlinear problems of stability cylindrical panels with imperfection

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Abstract. The article examines a problem of large deflections and elastic jumps from one equilibrium state to another in a thin-walled structure in the form of a thin elastic cylindrical panel with imperfections in the form of a small-scale additional wave formation. An analytical solution of the problem was obtained and an analysis of the influence of geometric characteristics on the panel deformation process was made.

1. Basic relations

We investigate thin elastic shallow cylindrical panel with imperfections in the form of n half-waves, subjected to a transverse load P (Figure1, a panel without imperfections is shown by dash lines). We believe that the force P is applied with some certain eccentricity ξ_0 and that the wave formation is represented according to the law $a_1(\cos(\pi n \xi/2) - \cos(\pi n/2))$. We search for dependence of the w panel moving due to a transverse linear load P , as well as impact of the wave formation on the critical load.

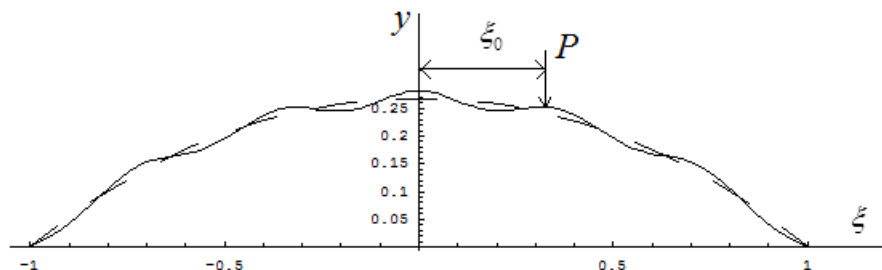


Figure 1. $n=11$, $a_1/a_0=0.1$, $r_0/b=2$.

The following dimensionless values are introduced,

$$\xi = \frac{x}{b}; a = \frac{a_0}{b}; \tilde{a} = \frac{a_1}{b}; q = \frac{Pb^2}{D}; D = \frac{2Eh^3}{3(1-\nu^2)}; w = \frac{W}{b}; \nu = \frac{V}{b},$$



where $2h$ and $2b$ – are thickness and length of the panel; W, V – are moving along the normal and in the tangent directions regarding the panel; E – is a Young module of the material; a_0 – is an arrow of the panel rise; a_1 – is an amplitude of the wave formation.

The equation of the bent axis is represented as [1]:

$$y = \sqrt{\left(\frac{r_0}{k}\right)^2 - \xi^2} - \frac{\sqrt{r_0^2 - k^2}}{k} + \frac{a_1}{k}(\cos(\pi n \xi / 2) - \cos(\pi n / 2)), \quad k = \sqrt{r_0^2 - (r_0 - a_0)^2}, \quad (1)$$

where r_0 – is a radius of the cylindrical panel.

The arch element equilibrium equations take the form [2]:

$$T' - \frac{b}{r}N = 0, \quad N' + \frac{b}{r}T = 0, \quad G' - bN = 0, \quad (2)$$

where dash lines from now on mean a derivative with respect to dimensionless coordinate ξ , T and N – is tangential and transverse forces, G – is a moment of deflection.

Because of the flatness of the arch we can accept [2]:

$$T = \text{const}; \quad \aleph \approx \frac{W''}{b^2} = \frac{w''}{b^2}.$$

It follows from the second and the third equations (2) that:

$$G'' - \mu^2 \frac{D}{r} = 0, \quad (3)$$

where $\mu^2 = -T \frac{b^2}{D}$ – is a dimensionless compressive force.

The equations for each element take the form (3), but for the right and left parts of the panel their solutions will be different because of the load P .

The relation between the bending moment and deflection for the elastic material takes the form:

$$G = -D\aleph = -\frac{D}{b} \ddot{w}.$$

The relation between the curvature of the panel and its deflection:

$$\frac{1}{r} = (y + wb)_{\xi\xi}'' \frac{1}{b^2} = \frac{1}{r_0} + w'' \frac{1}{b}. \quad (4)$$

Substituting the expression for G into the equation of equilibrium (3), we obtain:

$$w'''' + \mu^2 w'' + \mu^2 \left(\frac{b}{r_0} + n^2 \tilde{a} \cos \frac{n\pi\xi}{2} \right) = 0. \quad (5)$$

2. Solutions

The general solution of the equation (5) takes this form

$$w = B_1 \cos \mu\xi + B_2 \sin \mu\xi + B_3 \xi + B_4 + f(\xi),$$

$$f(\xi) = 4(-\mu^2) \left(\frac{a \cos \frac{\pi\xi}{2}}{\pi^2 - 4\mu^2} + \frac{\tilde{a}}{n^2 \pi^2 - 4\mu^2} \cos \frac{n\pi\xi}{2} \right), \quad (6)$$

or the matrix form:

$$w = [B]\{X\} + f(\xi);$$

$$[B] = [B_1, B_2, B_3, B_4]; \quad \{X\}^T = \{\cos \mu\xi, \sin \mu\xi, \xi, 1\}. \quad (7)$$

The unknown coefficients B_i are found from the boundary conditions. Let us denote a deflection in the left and right parts from the point of force application P as follows:

$$w_l = [C]\{X\} + f(\xi); \quad w_r = [B]\{X\} + f(\xi). \quad (8)$$

Next, let us write the conditions of docking at the point of force application

$$\begin{aligned} w_l(\xi_0) &= w_r(\xi_0); \quad w_l''(\xi_0) = w_r''(\xi_0); \\ w_l'(\xi_0) &= w_r'(\xi_0); \quad w_l'''(\xi_0) = w_r'''(\xi_0) - q. \end{aligned} \quad (9)$$

Let us rewrite the conditions of docking with $\xi = \xi_0$ in the matrix form

$$\begin{aligned} [H]\{C\} &= [H]\{B\} + \{0, 0, 0, -q\}^T, \quad \{C\} = [C]^T, \quad \{B\} = [B]^T, \\ [H] &= \begin{bmatrix} \cos \mu \xi_0 & \sin \mu \xi_0 & \xi_0 & 1 \\ -\mu \sin \mu \xi_0 & \mu \cos \mu \xi_0 & 1 & 0 \\ -\mu^2 \cos \mu \xi_0 & \mu^2 \sin \mu \xi_0 & 0 & 0 \\ \mu^3 \sin \mu \xi_0 & -\mu^3 \cos \mu \xi_0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

The relation between the coefficients for deflections w_l and w_r will take the form

$$\{C\} = \{B\} + [H]^{-1} \{0, 0, 0, -q\}^T. \quad (10)$$

As it follows from the relation (10), it is necessary to find only the last column of the matrix $[H]^{-1}$.

Let us write it for $\{Z\}$. Then

$$[H]\{Z\} = \{0, 0, 0, 1\}^T. \quad (11)$$

The solution of the equation (11) takes the form

$$\{Z\} = \left\{ \frac{\sin \mu \xi_0}{\mu^3}, -\frac{\cos \mu \xi_0}{\mu^3}, \frac{1}{\mu^2}, -\frac{\xi_0}{\mu^2} \right\}^T.$$

It follows from (10) that

$$[C] = [B] + [F], \quad [F] = [-\sin \mu \xi_0, \cos \mu \xi_0, -\mu, \mu \xi_0] q / \mu^3. \quad (12)$$

In order to get a connection between the power q and the force of compression μ^2 it is necessary to find one more equation. For this purpose we substitute the expression for ε , expressed in terms of displacement w и v , into the relation of Hook's law, connecting the force μ^2 and the tangential deformation ε . From the obtained equation we can find v for the left and right parts. The sought equation regarding q and μ will be a condition of equality of the displacement value v at the point of force application q , calculated according to the formulas for the right and left parts.

Given the Hook's law $T = K\varepsilon = 3D/h^2\varepsilon$, we obtain:

$$\varepsilon = -\mu^2 \frac{h^2}{3b^2}. \quad (13)$$

The deformation ε is expressed in terms of w, v according to the formula

$$\varepsilon = \frac{dv}{d\xi} - \frac{b}{r_0} w + \frac{1}{2} (\dot{w})^2.$$

Hence, for the left and right sides, taking into account (13) we obtain the expression for v :

$$\begin{aligned} v_l &= -\mu^2 \frac{h^2}{3b^2} (\xi + 1) + \int_{-1}^{\xi} \frac{b}{r_0} w_l d\xi - \int_{-1}^{\xi} \frac{1}{2} (\dot{w}_l)^2 d\xi; \\ v_r &= -\mu^2 \frac{h^2}{3b^2} (\xi - 1) + \int_{-1}^{\xi} \frac{b}{r_0} w_r d\xi - \int_{-1}^{\xi} \frac{1}{2} (\dot{w}_r)^2 d\xi. \end{aligned} \quad (14)$$

Here we have the boundary conditions $v(\pm 1) = 0$ in mind. The condition of docking at the point of force application takes the form

$$v_l(\xi_0) - v_r(\xi_0) = 0. \quad (15)$$

After substitution of w_l and w_r from (8) we obtain a quadratic equation regarding q :

$$\eta q^2 + \chi q + \tau = 0, \quad (16)$$

By setting different values for μ , we obtain the appropriate values for q . According to the formulas (8), (12) we can find displacements at any point.

3. Results and discussion

Farther, let us consider a case of panel rigid packing. The boundary conditions are written through a deflection:

$$w_l(-1) = 0; w_r(1) = 0; w_l'(-1) = 0; w_r'(1) = 0. \quad (17)$$

Then the coefficients, included into the expression for the deflection (7) are derived from the system (17) and takes the form:

$$\begin{aligned} \{B\} = \frac{q}{2\mu^3} & \left\{ - \frac{\left(-q + 2\frac{\mu^2}{r_0} + q\cos(\mu(1+\xi_0)) \right) \csc(\mu)}{2q} + \frac{8a_1 m \mu^4 \csc(\mu) \sin\left(\frac{n\pi}{2}\right)}{\pi q (m^2 \pi^2 - 4\mu^2)}, \right. \\ & \frac{-\mu\xi_0 - \mu\cos(\mu(1+\xi_0)) + \sin(\mu(1+\xi_0))}{2\mu\cos(\mu) - 2\sin(\mu)}, \frac{\mu(\mu(1+\xi_0))\cos(\mu) - \sin(\mu) - \sin(\mu\xi_0)}{2\mu\cos(\mu) - 2\sin(\mu)}, \\ & \left(32a_1 \mu^5 \cos\left(\frac{n\pi}{2}\right) + \pi^2 (m^2 \pi^2 - 4\mu^2) \left(-\mu \left(q - \frac{\mu^2}{r_0} + q\xi_0 \right) + q\sin(\mu(1+\xi_0)) \right) + \right. \\ & \left. + \pi \cot(\mu) \left(\pi (m^2 \pi^2 - 4\mu^2) \left(-q + 2\frac{\mu^2}{r_0} \right) + \right. \right. \\ & \left. \left. \pi q (m^2 \pi^2 - 4\mu^2) \cos(\mu(1+\xi_0)) - 16a_1 n \mu^4 \sin\left(\frac{n\pi}{2}\right) \right) \right) \left(2\pi^2 q (m^2 \pi^2 - 4\mu^2) \right)^{-1} \left. \right\}. \end{aligned} \quad (18)$$

Let us consider the case of the centrally applied force under $\xi_0 = 0$, given that the loading is rigid, providing the symmetrical deformation.

Figures 2–3 shows the dependences $q = q(\mu)$ and $q = q(w_0)$, $w_0 = w(0)$. Hereinafter, the dash lines show the dependences for the cases in which there are no imperfections. We can see that $q > 0$ all the time, it means that after the loss of panel stability it is necessary to apply an additional load in order to keep it back.

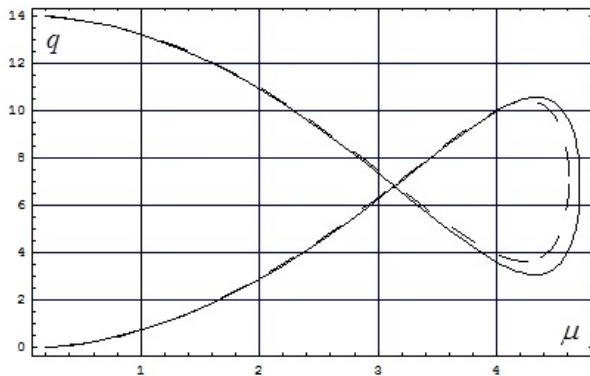


Figure 2. $n=7$, $a_1/a_0 = 0.1$, $r_0/b = 2$, $\xi_0 = 0$.

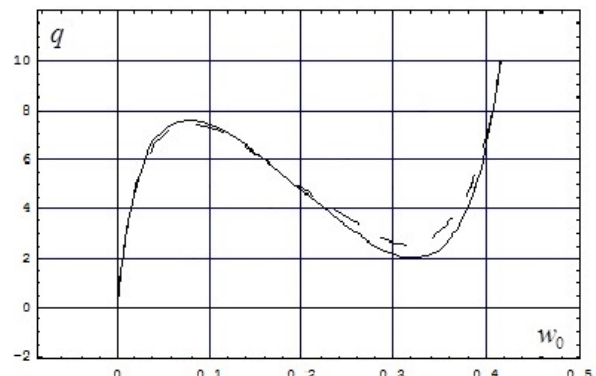


Figure 3. $n=7$, $a_1/a_0 = 0.1$, $r_0/b = 2$, $\xi_0 = 0$.

Figure 4 shows the dependences of the critical load q_* of the type and amplitude of wave formation. Figure 5 represents the dependences of the critical load q_* on the amount of half-waves n . The diagram shows, that under small values of n there is a noticeable change in the critical load. The critical load under sufficiently large n is close to the value for the case in which there are no imperfections.

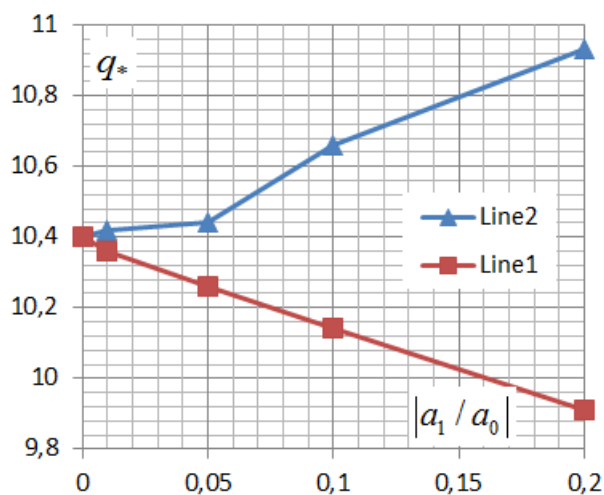


Figure 4. $n=7$, $r_0/b=2$, $\xi_0=0$,
 $a_1/a_0=-0.1$, (Line 1),
 $a_1/a_0=0.1$, (Line 2)

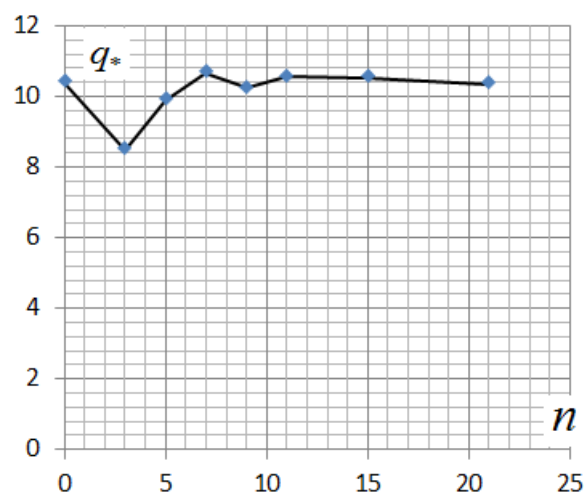


Figure 5. $n=7$, $r_0/b=2$, $\xi_0=0$,
 $a_1/a_0=-0.1$.

The dependences of the critical load on the application point is shown in Figure 6. The forms of the deformed panel in the case of a centrally applied force under the different values of q are shown in Figure 7. Because of the hard boundary conditions, only the symmetric form of loss in stability is realized.

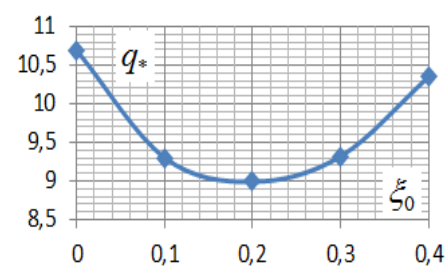


Figure 6. $n=7$, $r_0=2$, $a_1/a_0=-0.1$

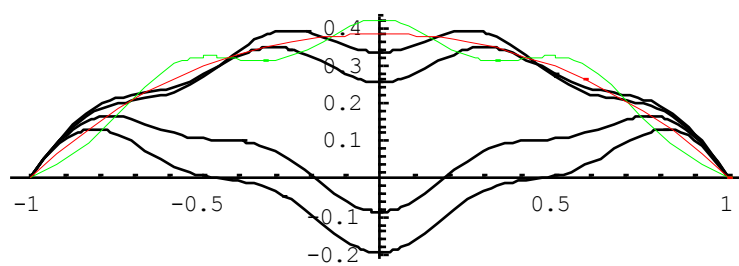


Figure 7. $n=7$, $r_0/b=2$, $\xi_0=0$, $a_1/a_0=-0.1$

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