

Investigation of oscillations of the elastic bodies with joined a mass–spring–damper systems

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Abstract. The problem of free oscillations of elastic systems joined with a simple mechanical system consisting of mass, springs and dampers is considered. The differential equation is obtained. The solution of the received equation is based on series. The oscillation of rectangular plate is considered.

1. Introduction

The work is devoted to the analysis of vibrations of elastic systems with joined mass, springs and dampers at the point. These problems are relevant at modern technology, since real shell structures are characterized by inclusions of masses, springs or dampers. The effect of joined masses, springs or dampers is expressed in the appearance of qualitatively new types of dynamic deformations that is a result of the excitation and interaction of the various bending shapes of the elastic systems.

2. Mathematical model

Consider the problem of free oscillations of elastic systems connected at a finite number of points with a simple mechanical system consisting of mass (M), springs (S) and dampers (D) (Figure 1).

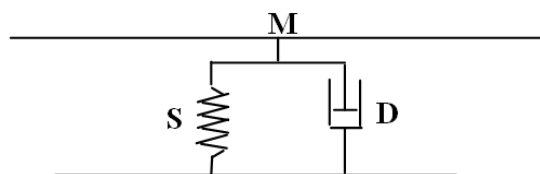


Figure 1. Free oscillations of elastic systems.

Give the differential equation which describes the free vibrations of elastic systems (beams, plates, shells) connected with MSD-systems:

$$L(w) + aL_1(\ddot{w}) + \sum_j L_2(b\ddot{w} + c\dot{w} + ew)\delta(P - P'_j) = 0. \quad (1)$$

Here $w(P, t)$ – the displacements, L , L_1 , L_2 – static operators, a , b , c , e – constants, δ – Dirac delta function, P is an arbitrary point, P'_j is the point of joined of MSD systems.



Model (1) is constructed on the basis of the classical Kirchhoff theory. Takes into account only the transverse component of the inertia forces are not taken into account the internal friction in the material. The characteristics of all MSD systems are the similar. To complete this mathematical model to the equation (1) it is necessary to add appropriate boundary conditions.

We rewrite (1) in the form

$$L(w) + aL_1(\ddot{w}) = - \sum_j [L_2(b\ddot{w} + c\dot{w} + ew)]_{P=P'_j} \delta(P - P'_j). \quad (2)$$

And we will seek its solution in the class of functions describing the harmonic damped oscillations. To do this, put

$$w(P, t) = W(P) \exp(i\omega^0 t) \quad (3)$$

where i is the imaginary unit, $\omega^0 = \omega + i\mu$, ω – frequency oscillations, μ – damping factor. Substituting (3) into (2), we get

$$L(w) - a(\omega^0)^2 L_1(W) = - \sum_j [b\ddot{w} + c\dot{w} + ew]_{P=P'_j} [L_2(W)] \delta(P - P'_j). \quad (4)$$

Decompose the function $W(P)$ and Dirac delta function $\delta(P - P'_j)$ in the system of orthogonal functions $F_{mn}(P)$ describing the forms of natural oscillations of elastic systems:

$$\begin{aligned} W(P) &= \sum_{m,n} W_{mn} F_{mn}(P), \\ \delta(P - P'_j) &= \sum_{m,n} \Delta_{mn}(P'_j) F_{mn}(P). \end{aligned} \quad (5)$$

After substituting (5) into (4) we have

$$A_{mn} a [\omega_0^2 - (\omega^0)^2] W_{mn} = \sum_j [b(\omega^0)^2 - ic\omega^0 - e] \Delta_{mn}(P'_j) [L_2(W)]_{P=P'_j}$$

from which

$$W_{mn}(P'_j) = \frac{\sum_j [b(\omega^0)^2 - ic\omega^0 - e] \Delta_{mn}(P'_j) [L_2(W)]_{P=P'_j}}{a [\omega_{0mn}^2 - (\omega^0)^2] A_{mn}}. \quad (6)$$

Here ω_{0mn} is the natural frequency of the elastic system, A_{mn} – eigenvalues determined by the equation $L_1(F) - AF = 0$, where $F = F_{mn}$.

Now the expression of the unknown function $W(P)$ can be written in the form

$$W(P) = \sum_{m,n} W_{mn}(P'_j) F_{mn}(P). \quad (7)$$

Where the coefficients $W_{mn}(P'_j)$ is given by (6). Will act on both parts of the equality (7) the operator L_2 . Then

$$L_2(W) = \sum_{m,n} W_{mn}(P'_j) L_2(F_{mn}). \quad (8)$$

Equation (8) should be performed at any point in the region definition of the function $W(P)$. Demand its implementation in the points of attachment of MSD systems, the resulting homogeneous system of linear equations relative to the unknown $[L_2(W)]_{P=P'_j}$:

$$[L_2(W)]_{P=P'_j} - \sum_{j,m,n} \frac{[b(\omega^0)^2 - i c \omega^0 - e] \Delta_{mn}(P'_j) [L_2(F_{mn})]_{P=P'_j} [L_2(W)]_{P=P'_j}}{a[\omega_{0mn}^2 - (\omega^0)^2] A_{mn}} = 0.$$

The condition of existence of nontrivial solutions of this system are the equality to zero of its determinant

$$\det(\omega^0) = 0. \quad (9)$$

Solving equation (9) we will find frequency ω and damping factor μ .

3. Example

Let's consider a rectangular plate with sides a along the x -axis and b on the y pivotally supported on all edges. Combining the direction of the x and y axes with the principal axes, by the usual arguments come to the equation of oscillations of an orthotropic plate with an attached MSD system at point (x', y')

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} + \left[M \frac{\partial^2 w}{\partial t^2} + \alpha \frac{\partial w}{\partial t} + \beta w \right] \delta(x - x', y - y') = 0. \quad (10)$$

That is, in this problem, operators take the form

$$L(w) = D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4}, \quad L_1(w) = 1, \quad L_2(w) = 1.$$

And constants $a = \rho h$, $b = M$, $c = \alpha$, $e = \beta$, where M is the magnitude of the concentrated mass, α – viscosity damper coefficient, β – spring stiffness, ρ is the density of plate, h – plate thickness.

We write in accordance with boundary conditions of the orthogonal function

$$F_{mn}(P) = \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (11)$$

and

$$\Delta_{mn}(P') = \frac{4}{ab} \sin \frac{m\pi x'}{a} \sin \frac{n\pi y'}{b} \quad (12)$$

Then the frequency equation in this case will take the form of

$$\sum_m \sum_n 4 \frac{[M(\omega^0)^2 - i c \omega^0 - \beta] F_{mn}(x', y')}{M_p [p_{mn}^2 - (\omega^0)^2]} = 1. \quad (13)$$

where $M_p = abh\rho$ – mass of the plate, and p_{mn} – frequency oscillation, defined as

$$p_{mn}^2 = \frac{\pi^4 a}{b^3 M_p} D_2 \left[\frac{D_1}{D_2} \left(\frac{mb}{a} \right)^4 + 2 \frac{D_3}{D_2} n^2 \left(\frac{mb}{a} \right)^2 + n^4 \right] \quad (14)$$

The equation (13) is an algebraic equation with respect to unknown ω^0 , generally speaking, infinitely high degree. In specific calculations, there is a natural truncation of the series, standing in the left side of the equation, and how it will end.

Let us turn to dimensionless quantities

$$p_{mn}^* = \frac{p_{mn}}{p_{11}}, \quad \omega^* = \frac{\omega}{p_{11}}, \quad M^* = \frac{M}{M_p}, \quad \beta^* = \frac{\beta}{M_p p_{11}^2}.$$

Then the equation (2.4) takes the form

$$\sum_m \sum_n 4 \frac{[M^*(\omega^*)^2 - iq\omega^* - \beta^*] F_{mn}^2(x', y')}{[(p_{mn}^*)^2 - (\omega^*)^2]} = 1. \quad (15)$$

The solution of the reduced equation (15), we obtain the frequency spectrum of the vibrations of the plate.

Fixing such parameters

$$\frac{D_1}{D_2} = 20, \quad \frac{D_3}{D_2} = 2,196, \quad \frac{a}{b} = 1, \quad x' = 0,2a, \quad y' = 0,2b$$

and having considered the various cases of MSD systems, we present graphs of the $\omega^* = \omega^*(q)$, characterizing, respectively, the oscillation frequency and the viscosity of the damper.

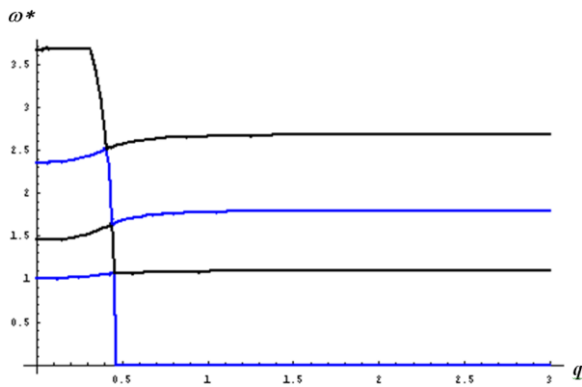


Figure 2. Graphics frequency–viscosity, $M^* = 0, \beta^* = 0$.

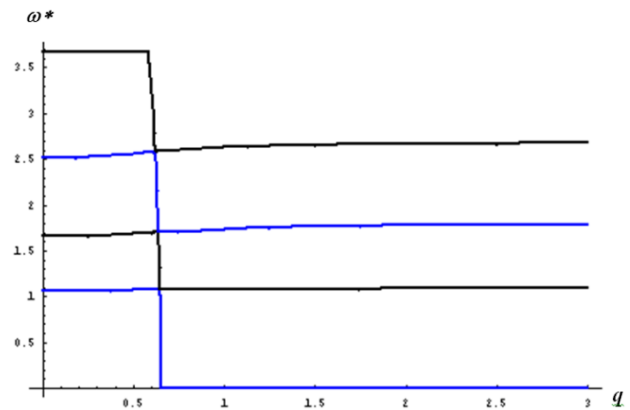


Figure 3. Graphics frequency–viscosity, $M^* = 0, \beta^* = 1$.

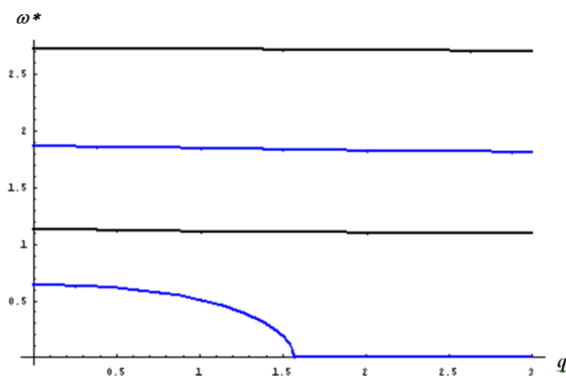


Figure 4. Graphics frequency–viscosity, $M^* = 1, \beta^* = 0$.

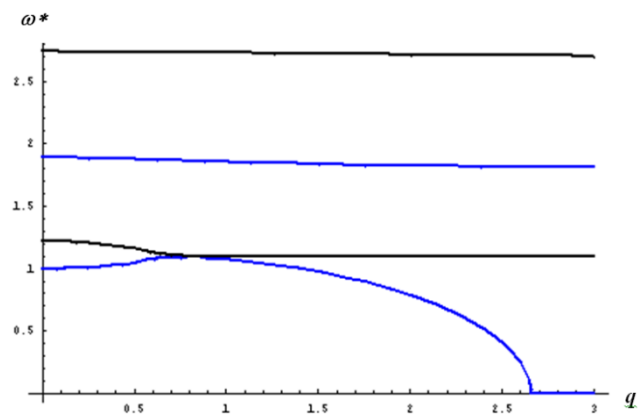


Figure 5. Graphics frequency–viscosity, $M^* = 1, \beta^* = 1$.

The graphs show that for all cases of MSD systems there are some values of the parameter $q = q^*(M^*, \beta^*)$ when $q \geq q^*$ the plate oscillates as if the attachment point of the MSD systems are fixed. The mass M^* and stiffness β^* affect the character of the frequency ω^* when $q \leq q^*$.

4. Conclusion

The paper is dedicated to a solution of the problem of oscillations of elastic system with joined mass, spring and damper. The mathematical model is obtained. The problem of vibration of a plate with attached MSD system is considered. The frequency range for different values of masses and spring stiffness is received.

Acknowledgements

This work was supported by the Russian Science Foundation (project 16-11-10299).

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