

Some results of mesh convergence estimation for the spectral element method of different orders in FIDESYS industrial package

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Abstract. The article describes the Mesh convergence estimation for the method of spectral elements of different orders in FIDESYS industrial package. Estimation is done using the automated testing system developed for FIDESYS package. The tasks with known exact solutions were used for estimation. By changing the spectral element order the exponential decrease computational errors is shown against the degree of approximating polynomials.

1. Introduction

Using the method of spectral elements for sufficiently smooth functions one can show exponential decrease of computational error against the degree of the approximating polynomials.

In this paper, this feature is checked using the FIDESYS package with the help of automated testing system allowing to carry out a repeated calculation of the same problem while modifying some parameters (in this case, the degree of the approximating polynomials) for problems with known analytical solutions: Kirsch problem and Lamé problem.

2. Two-dimensional case (Kirsch problem)

Let's examine a tension with tensile stress $1e6$ N/m of square plate made of isotropic material with a Young's modulus of $2e11$ Pa and a Poisson's ratio of 0.3, with a side length of 10 m and a hole with a diameter of 0.5 m in the center.

The main advantage of the method of spectral elements is rapid convergence of solutions to the exact solution with a fewer number of nodes if compared with the finite element analysis method.

In connection with this, for more clear demonstration of exponential convergence, a rather coarse mesh was built:



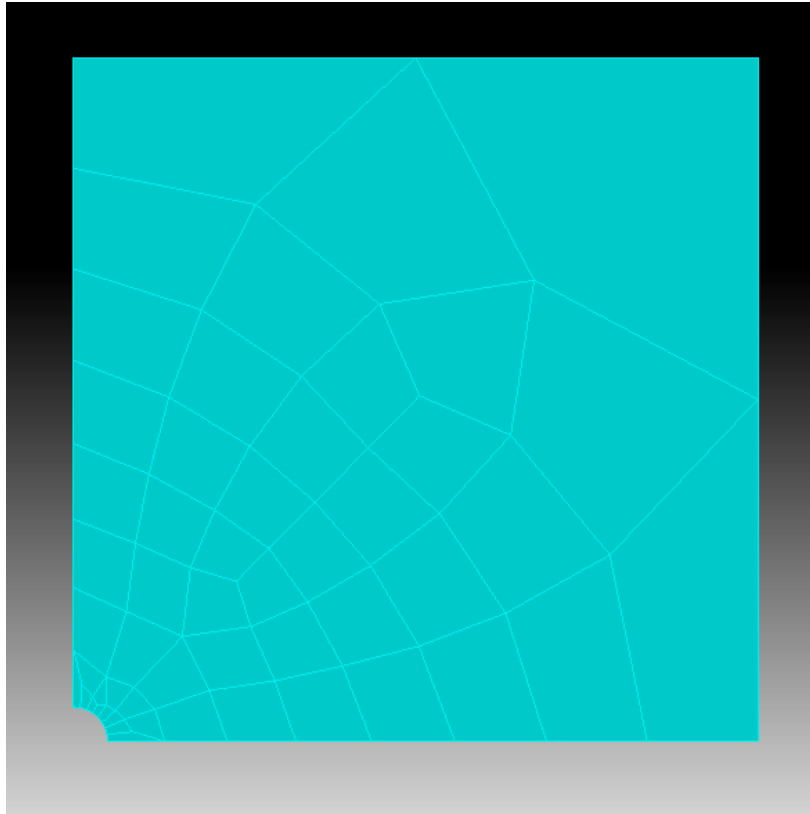


Figure 1. Mesh for Kirsch problem.

This problem is simulated using the FIDESYS package with the help of the following log file:

```
create surface rectangle width 10 zplane
create surface circle radius 0.25 zplane
subtract volume 2 from volume 1
webcut volume 1 with plane xplane preview
webcut volume 1 with plane xplane imprint nomerge
webcut volume 1 with plane yplane preview
webcut volume 1 with plane yplane imprint nomerge
delete volume 1
delete volume 3
curve 20 scheme bias fine size 0.4 factor 1.1 start vertex 18
curve 7 scheme bias fine size 0.4 factor 1.1 start vertex 8
curve 22 interval 8
surface 7 size auto factor 9
surface 7 scheme Auto
mesh surface 7
create displacement on curve 7 dof 1 fix 0

create displacement on curve 20 dof 2 fix 0
create pressure on curve 11 magnitude -1e6
create material elastic_modulus 2e11 poisson_ratio 0.3
set duplicate block elements off
block 1 surface 7
block 1 material 'Material 1'
```

block 1 element type QUAD8
 analysis type static dim2

As it is known, in the polar co-ordinates a tension on the hole border should be three times greater than the tensile stress, i.e., should be equal to 3σ .

With the help of automated testing system the analysis of this problem was carried out with the help of finite element analysis method, and using the method of spectral elements with approximating polynomials degrees from 2 to 9.

Deliverables:

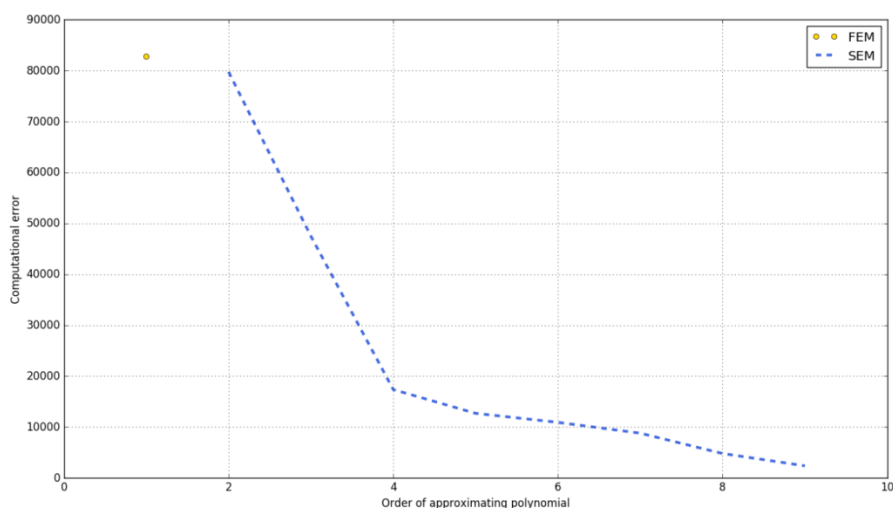


Figure 2. Computational errors against the approximating polynomials degree for Kirsch problem.

Yellow dot indicate the result for the finite element analysis method, the blue dotted line represents the results for the different degrees of the approximating polynomial for spectral elements method.

In order to verify that the computer error decreases exponentially, let's display the same results on the logarithmic axis.

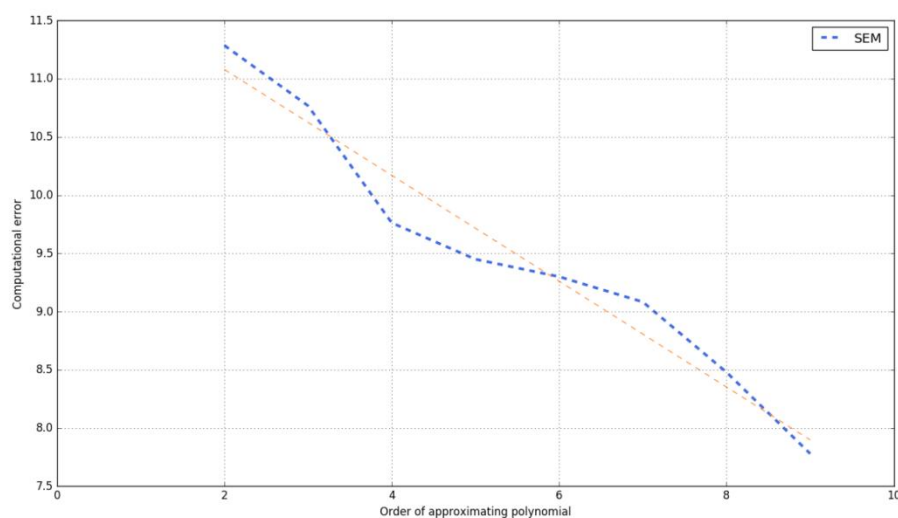


Figure 3. Dependence of computational errors on the degree of approximating polynomials for Kirsch problem; yellow dotted line represents a linear interpolation obtained by the method of least squares (logarithmic scale).

The resulting broken line, as expected, is quite close to straight line.

3. Three-dimensional case (Lame problem)

Let's examine a thick-walled cylinder with a height of 0.5 m and a radius of 2 m, a wall thickness of 1 m, made of isotropic material with a Young's modulus 2×10^{11} Pa and a Poisson's ratio of 0.3, loaded with internal and external pressures of 1×10^6 and 0.5×10^6 , respectively.

In this case, the generated mesh was even more coarse in order to demonstrate the exponential convergence.

This problem is simulated using the FIDESYS package with the help of the following log file:

```
create Cylinder height 0.5 radius 2
create Cylinder height 0.5 radius 1
subtract volume 2 from volume 1
webcut volume 1 with plane xplane offset 0 noimprint nomerge
webcut volume 1 with plane yplane offset 0 noimprint nomerge
delete Volume 1 3
volume 4 size auto factor 8
mesh volume 4
create Displacement on surface 27 dof 2 fix
create Displacement on surface 11 dof 1 fix
create Displacement on surface 29 31 dof 3 fix
create pressure on surface 30 magnitude 1e6
create pressure on surface 28 magnitude 0.5e6
create material elastic_modulus 200e9 poisson_ratio 0.3
set duplicate block elements off
block 1 volume 4
block 1 material 'Material 1'
block 1 element type HEX20
analysis type static dim3
```

One can check, e.g. stress σ_{zz} at the inner boundary of the wall (in cylindrical coordinates). In accordance with known analytical solution, it must be equal to $\sigma_{zz} = 2v \frac{p_a a^2 - p_b b^2}{b^2 - a^2} = -200,000$, Where a - inner radius, b - outer radius, p_a and p_b - internal and external pressures, respectively, and v - Poisson's ratio.

In this case, the results are as follows:

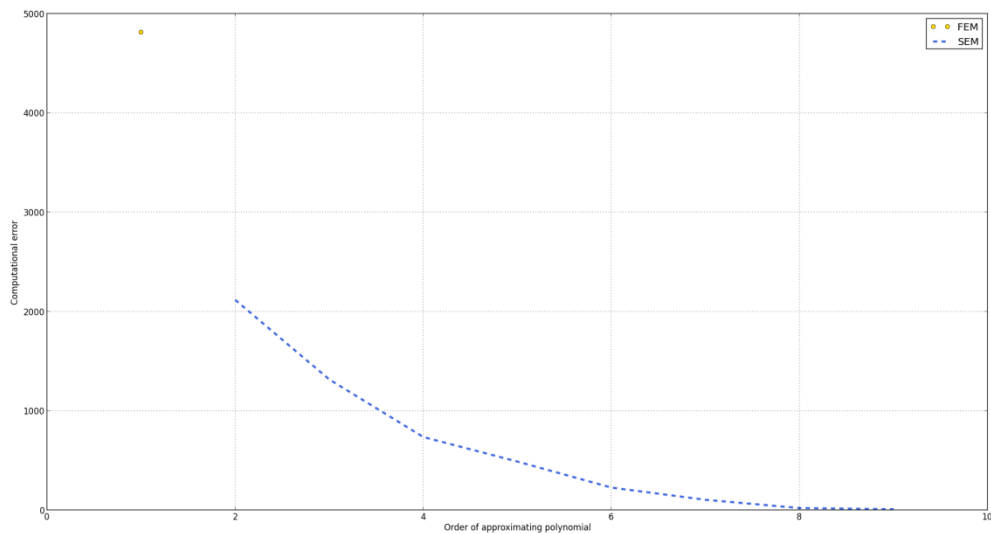


Figure 4. Computational errors against the approximating polynomials degree for Lamé problem.

Now let's just use a logarithmic scale:

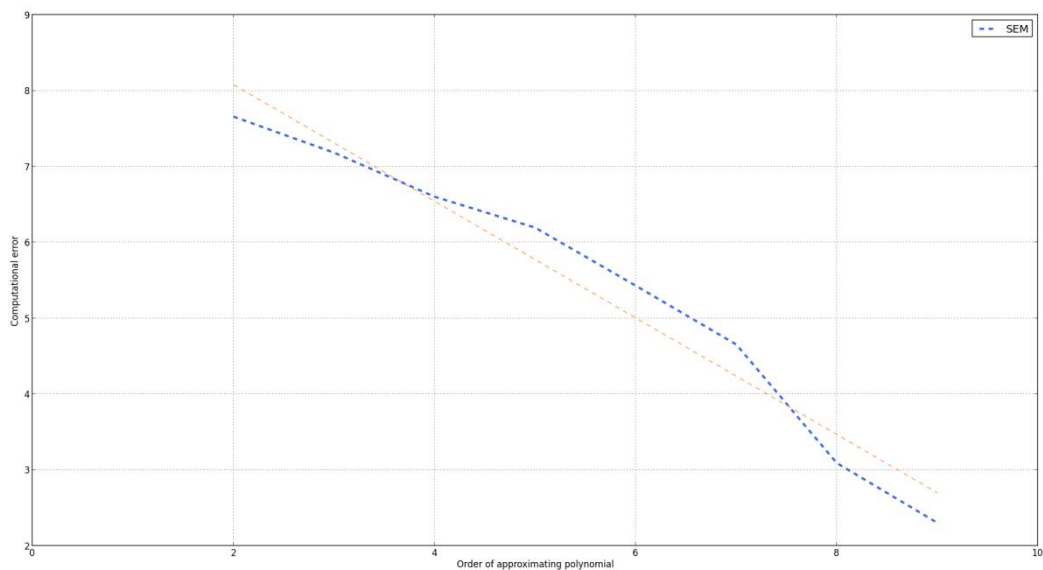


Figure 5. Dependence of computational errors on the degree of approximating polynomials for Lamé problem; yellow dotted line represents a linear interpolation obtained by the method of least squares (logarithmic scale)

In this case, the proximity of the resulting broken line to straight line is even more obvious than it is for the two-dimensional case.

4. Conclusion

The article describes the checking of exponentially fast decrease of computational errors for the spectral element method depending on the degree of the approximating polynomials using the FIDESYS package for problems with known analytical solutions: the Kirsch problem and the Lamé problem.

The obtained results confirm the exponential rate of convergence to the exact solution.

Acknowledgments

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References

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