Flow past a porous cylinder in a rectangular periodic cell: Brinkman and Darcy models comparison

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Abstract. The problem of the gas suspension flow around a porous cylinder in a periodic rectangular cell within the models of the Stokes – Darcy and the Navier–Stokes – Brinkman using the boundary element and finite volumes method was solved. Streamlines of the carrier phase flow and the air and inertia-less particles capture coefficients of at varying porosities of cylinder medium and periodic cylinder packing were compared.

1. Introduction

A flow around a porous body is widely encountered in the field of gas suspension filtration. Fully or partially porous bodies can act as an aerosol filter element. To evaluate the efficiency of filters with porous elements, the flow of gas suspension in an ordered or random packing should be calculated. The flow field of the carrier phase can be found with a good accuracy in the approximation of a circular or rectangular periodic cell model [1-3]. To the best of our knowledge, there are not enough comparisons of accuracy of the predictions of various flow models in the region with mixed homogeneous and porous areas. The study of the air flow past a porous cylinder in a rectangular periodic cell on the base of two flow models is the aim of the present work.

2. The problem statement

The problem of incompressible gas flow around a porous cylinder in a periodic rectangular cell using two mathematical models is solved. The first model describes the fluid flow in a homogeneous field in the Stokes approximation and in the porous cylinder domain on the base of the Darcy law. The obtained boundary value problem is solved by the boundary element method (BEM). Within the second model a combination of the Navier–Stokes and the extended Brinkman equations for uniform and porous areas, respectively, is implemented. The solution of the model equations is obtained using the CFD code ANSYS/FLUENT by the finite volume method (FVM). Let us describe problem for both models.

2.1. The Stokes – Darcy model

Let us consider a two-dimensional flow of an incompressible viscous fluid with suspended particles of a porous cylinder with the radius R_c in the periodic rectangular cell. As the scale of a linear size and speed select the radius of the cylinder R_c and the average speed U on the left side of the cell,

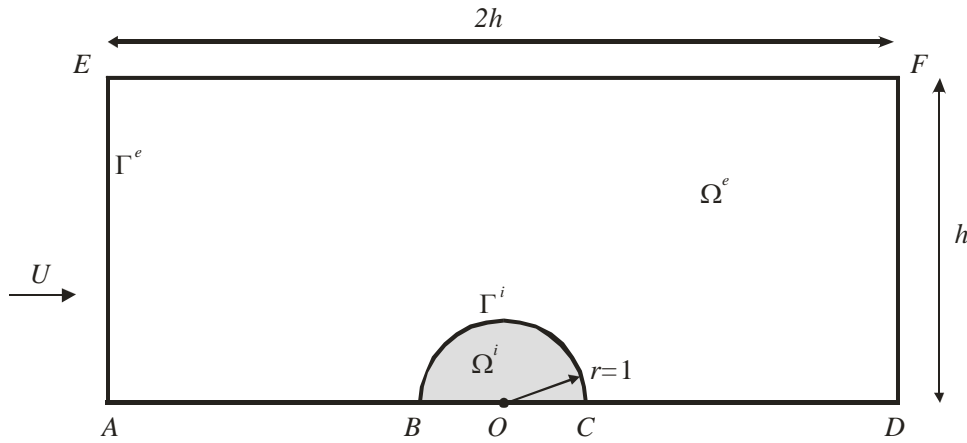respectively. In dimensionless coordinates xy the rectangular periodic cell of the porosity ε has the

Figure 1. The geometry of the computational domain.

height $h=\sqrt{\pi/(1-\varepsilon)}/2$ and width $2h$. The computational domain Ω consists of a homogeneous area Ω^e of the external flow and a porous medium Ω^i of the internal flow inside the cylinder with the radius $r=1$: $\Omega=\Omega^e\cup\Omega^i$ (figure 1).

The external flow in the area Ω^e is described in the Stokes approximation. In this case, the stream function $\psi^e(x,y)$ of the external flow satisfies the biharmonic equation

$$\Delta^2\psi^e=0 \qquad (1)$$

and the conditions on the boundaries Γ^e and Γ^i. On the lines AE and DF the periodic conditions are set

$$\psi^e(-h,y)=\psi^e(h,y),\quad \psi^{e'}(-h,y)=-\psi^{e'}(h,y),\quad \omega^e(-h,y)=\omega^e(h,y),\quad \omega^{e'}(-h,y)=-\omega^{e'}(h,y),\quad (2)$$

where $\omega^e=-\Delta\psi^e$ is the vorticity (a prime denotes differentiation with respect to the outward normal to the boundary). On the top line EF

$$\psi^e=h,\quad \omega^e=0. \qquad (3)$$

On the axis-lines AB and CD − the symmetry conditions hold

$$\psi^e=0,\quad \omega^e=0. \qquad (4)$$

In the porous domain Ω^i the flow is described within the Darcy model. The stream function $\psi^i(x,y)$ of the fluid flow inside the cylinder satisfies the Laplace equation

$$\Delta\psi^i=0 \qquad (5)$$

and the symmetry conditions on the line BOC

$$\psi^i=0,\quad \omega^i=0. \qquad (6)$$

On the line BC between the free space and the porous medium (the boundary of the cylinder) the conditions are

$$u_r^e=u_r^i,\quad p^e=p^i,\quad \frac{\partial u_\theta^e}{\partial r}=-\alpha_s S\left(u_\theta^i-u_\theta^e\right), \qquad (7)$$

where u_r, u_θ are the radial and tangential components of the fluid flow velocity, indexes e and i correspond to the external and internal flow, p is the pressure, $S=R_c/\sqrt{k}$, k is the permeability of the porous medium. The dimensionless slip coefficient α_s is determined by the viscosity of the carrier medium and geometric parameters of the porous medium. The last condition in (7) is a widely known Beavers condition [4].

2.2. The Navier–Stokes – Brinkman model
The flow in the rectangular periodic cell in the approximation of a laminar flow of a viscous incompressible fluid is described by the extended Brinkman equations in the porous region and the Navier–Stokes equations in the homogeneous region [3, 6, 7]:

$$\nabla \cdot \bar{U} = 0, \tag{8}$$

$$\varepsilon_f^{-2} \rho \bar{U} \cdot \nabla \bar{U} = -\nabla P + \frac{\mu}{\varepsilon_f} \Delta \bar{U} - b \frac{\mu}{k} \bar{U}, \tag{9}$$

where ε_f is the porosity of the cylinder medium. The quantity b equals zero and unity outside and inside the cylinder, respectively. The conditions at external boundaries of the rectangular cell correspond to the conditions accepted in the previous model: the periodic conditions on the vertical boundaries of the cell and the symmetry conditions on the top and bottom boundaries.

3. Calculation results
The boundary value problems (1) – (7) are solved together by BEM [5]. The equations (8) – (9) are numerically integrated by FVM using ANSYS/FLUENT code. The numerical studies using the two models described were conducted for the same values of the parameters.

Figure 2 shows the flow streamlines in the periodic cell of $\varepsilon = 0.96$, calculated by two models for the two values of the parameter *S*. The solid and dashed lines correspond to the streamlines obtained by BEM and FVM respectively. In the Stokes – Darcy model α_S is assumed to be $\alpha_S = 1$ in the Beavers condition (7). To better visualize the flow in the porous cylinder more fluid flow streamlines near the symmetry axis are given. It is seen that the streamlines obtained by two models agree well away from the porous cylinder and the agreement is better for larger S. Near and on the porous cylinder there is a significant difference between two models.
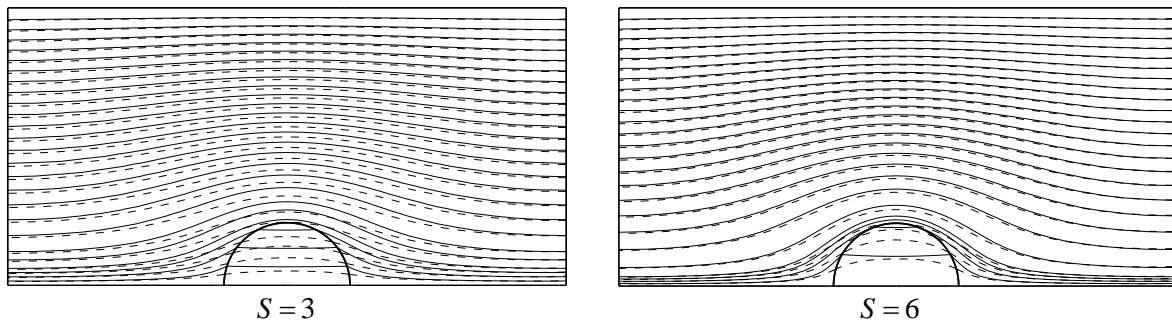

$S = 3$ $S = 6$

Figure 2. The fluid flow streamlines in the periodic cell.

To estimate the inertialess particles coefficient for the flow around the porous cylinder the carrier phase capture as a function $Q(S)$ should be calculated. The quantity Q is defined as the ratio of the initial ordinate of the limiting streamline passing through the cylinder to its radius and is found as the value of stream function of the upper point of the cylinder with the coordinates $x = 0, y = 1$: $Q = \psi(0,1)$. The calculated functions $Q(S)$ for the various values ε and α_S are shown in figure 3. With the growth of S (decrease in the permeability of the cylinder) the value Q decreases monotonically. The parameter α_S in the Darcy model with the Beavers boundary condition (7) significantly affects the value Q and this effect is larger for less porosity ε. An approximate model of Darcy's flow in the porous cylinder yields the low values for the considered α_S due to neglecting the air tangential velocity at the cylinder boundary.
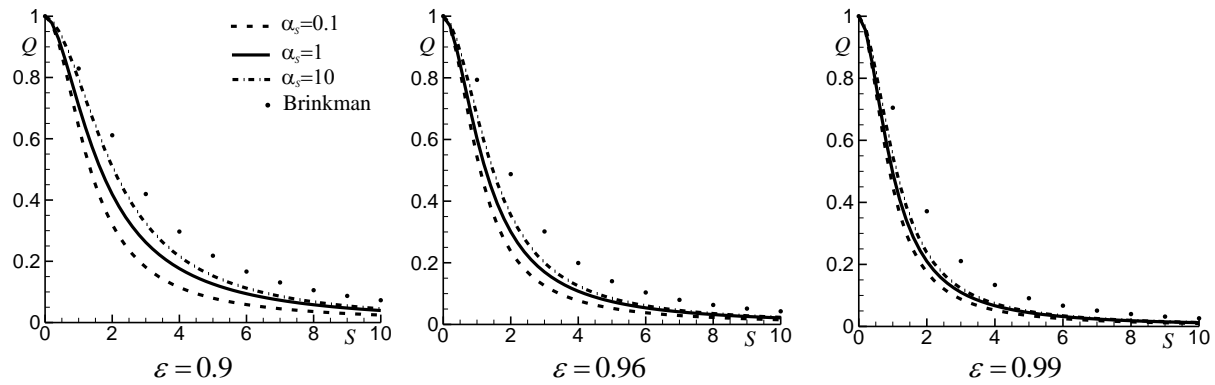
Figure 3. The coefficient of air capture $Q(S)$.

In the case of a porous fiber of an aerosol filter the air capture coefficient Q gives the efficiency of the capture of inertialess particles [3]. Using the numerically found streamfunction $\psi^e(x, y)$ we can calculate the particle deposition coefficient by the interception phenomena [8]. The contribution of the interception to the particles deposition becomes considerable for the particle sizes that are comparable with cylinder radius and depends on the ratio $\delta = r_p / R_c$, where r_p is the particle radius. The total efficiency E of capture of inertialess particles can be found as $E(\delta) = \psi(0, 1 + \delta)$. On the base of the analytical Kuwabara model the formula was obtained in [3]

$$E(\delta) = \psi(0, 1 + \delta) = (1 + \delta) + C\left(a_1(1 + \delta)^{-1} - a_2(1 + \delta) + a_3(1 + \delta)^3\right),$$
$$a_1 = 0.5 - 0.25\alpha, \ a_2 = \text{Ku} - 0.5\alpha + 0.5 - \ln(1 + \delta), \ a_3 = -0.25\alpha,$$
(10)

$$C = \left(\text{Ku} + 2(1 + \alpha)S^{-2}\right)^{-1}, \ \text{Ku} = -0.5\ln\alpha - 0.75 + \alpha - 0.25\alpha^2, \ \alpha = 1 - \varepsilon.$$

The dependencies $E(\delta)$ for the two values of S at $\varepsilon = 0.96$ and $\alpha_S = 1$ are given in figure 4. In [3] it was shown that the value of the parameter $S < 3$ for real porous bodies. The curve $E(\delta)$ is higher than the interception efficiency of a solid cylinder. The difference is the air capture coefficient Q. In general, the Darcy model gives low values of the particle capture rate as compared with the model of Brinkman. The curves $E(\delta)$ obtained by BEM are in a better agreement with an analytical formula (10).
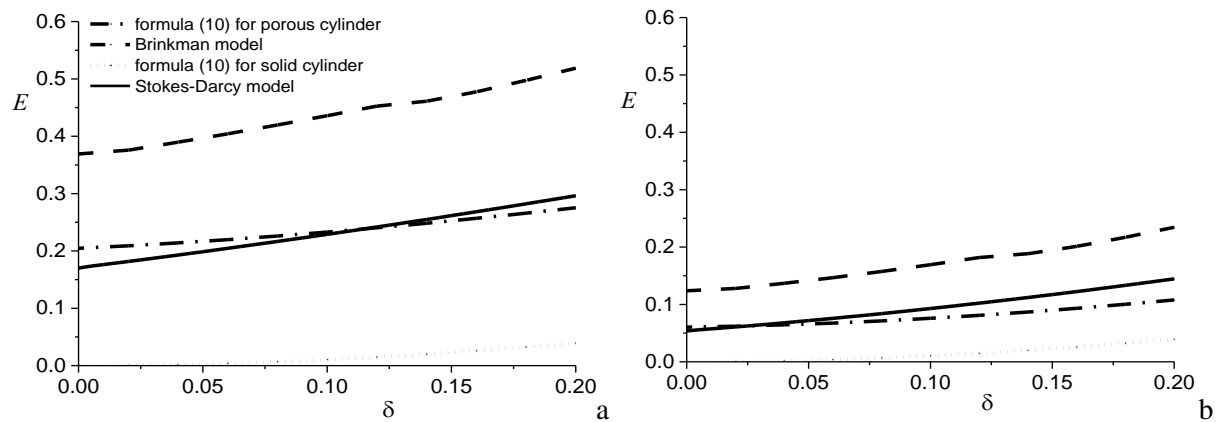

Figure. 4. The dependence $E(\delta)$ at $S=3$(a), 6(b).

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