Polar-symmetric problem of elastic diffusion for isotropic multi-component plane

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Abstract. The paper considers a polar-symmetric problem of finding a stress strain condition of a plane influenced by non-stationary volume elastic diffusion disturbances. The mathematical model is based on a connected system of equations of elastic diffusion in a polar coordinate system.
The solution of the problem is sought in an integral for and presented in the form of convolutions of Green's function with the right side of equation of motion and mass transfer. Laplace time and Hankel's radial coordinate transformations are used to find the Green's functions. The inverse Laplace transform is done analytically by residue. The inverse Hankel's transform is done numerically by quadrature formulas.

1. Introduction

Depending on the geometry of the area, sometimes, it is practical to consider mechanics problem in curvilinear coordinates. For instance, for plane polar-symmetric problems it is convenient to construct a mathematical model in polar coordinates, if the problem solution area is a circle, ring or spherical cavity. Besides, the same case is with a polar-symmetric problem for a plane as an extreme case for a circle problem when its radius tends to infinity. All these problems in a polar coordinate system will be one-dimensional which may simplify their solution.

On other hand, for solution of such problems one can use an approach close to an approach to the solutions of similar problems in a Cartesian coordinate system. Specifically, it is convenient to search for a solution of an initially boundary value problem by making use of the Laplace transformation and expansions by eigenfunctions. This will allow for reducing the initial system of equations of motion and mass transfer to a system of linear algebraic equations. Correspondingly, the solution in the Laplace transforms will be a rational function which allows us to come to originals by residues.
Here the main problem is finding a system of proper functions which are solutions of the corresponding Sturm-Liouville problem. The simplest solution of this problem is found within a Cartesian coordinate system. With certain boundary conditions such functions can be represented by sinus and cosine [1]. For curvilinear coordinates there are also specific solutions [2-4] the form of which will depend on both selected boundary conditions and the geometry of the area.

Further on, a polar-symmetrical problem for an isotropic plane will be considered where a Bessel function of the first kind can be used as a proper function. The medium is an N-component solid solution where non-stationary volumetric disturbance cause coupled elastic and diffusion fields.

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2. Statement of the problem

The equations describing coupled elastic and diffusion processes in an isotropic N-component solid solution in an arbitrary curvilinear coordinate system will be written as follows [5]:

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2}=(\lambda+\mu)\operatorname{graddiv}\mathbf{u}+\mu\Delta\mathbf{u}-\sum_{j=1}^{n}\alpha^{(j)}\operatorname{grad}\eta^{(j)}+\mathbf{F},$$

$$\frac{\partial\eta^{(q)}}{\partial t}=D^{(q)}\Delta\eta^{(q)}-\Lambda^{(q)}\Delta(\operatorname{div}\mathbf{u})+F^{(q)},\quad q=\overline{1,N}.$$

where t is time; $\mathbf{u}=\{u_1,u_2,u_3\}$ is displacement vector; $\eta^{(j)}=n^{(j)}-n_0^{(j)}$ is molar concentration increment of a j diffusant $n^{(j)}$ with reference to its initial concentration $n_0^{(j)}$; λ, and μ is elastic Lame constant; ρ is density of the environment; $\alpha^{(j)}$ is a coefficient of volume expansion due to mass transfer; $D^{(j)}$ is a coefficient of self-diffusion; $\Lambda^{(j)}=n_0D^{(j)}\alpha^{(j)}/(RT_0)$; R is absolute gas constant; T_0 - absolute ambient temperature, $\mathbf{F}$ is mass forces vector, $F^{(q)}$ is density of internal sources of mass transfer.

In case of polar symmetry the physical fields and mass forces vector will be written as follows:

$$\mathbf{u}=\{u_r(r),0,0\},\quad \eta^{(j)}=\eta^{(j)}(r),\quad \mathbf{F}=\{F_r(r),0,0\},$$

where r is a radial coordinate.

Correspondingly, the equations of motion and mass transfer for an isotropic plane will take the following forms:

$$\rho\frac{\partial^2u_r}{\partial t^2}=(\lambda+2\mu)\left(\frac{\partial^2u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}\right)-\sum_{j=1}^{N}\alpha^{(j)}\frac{\partial\eta^{(j)}}{\partial r}+F_r,$$

$$\frac{\partial\eta^{(q)}}{\partial t}=-\Lambda^{(q)}\left(\frac{\partial^3u_r}{\partial r}+\frac{2}{r}\frac{\partial^2u_r}{\partial r^2}-\frac{1}{r^2}\frac{\partial u_r}{\partial r}+\frac{u_r}{r^3}\right)+D^{(q)}\left(\frac{\partial^2\eta^{(q)}}{\partial r^2}+\frac{1}{r}\frac{\partial\eta^{(q)}}{\partial r}\right)+F^{(q)}.$$

Further on it is convenient to switch to non-dimensional numbers (with the same symbols they are marked with "*" which is omitted later on) as follows:

$$r^*=\frac{r}{L},\quad u=\frac{u_r}{L},\quad \tau=\frac{Ct}{L},\quad \eta_q=\frac{\eta^{(q)}}{n_0^{(q)}},\quad C^2=\frac{\lambda+2\mu}{\rho},$$

$$\alpha_q=\frac{n_0^{(q)}\alpha^{(q)}}{\lambda+2\mu},\quad D_q=\frac{D^{(q)}}{CL},\quad \Lambda_q=\frac{\Lambda^{(q)}}{n_0^{(q)}CL},\quad F_1=\frac{F_rL}{\lambda+2\mu},\quad F_{q+1}=\frac{F^{(q)}L}{Cn_0^{(q)}},$$

As a result, we will obtain (prime mark means a derivative with respect to r and period means a derivative with respect to non-dimensional time τ)

$$\begin{aligned}\ddot{u}&=u''+\frac{u'}{r}-\frac{u}{r^2}-\sum_{q=1}^{N}\alpha_q\eta_q'+F_1,\\ \dot{\eta}_q&=-\Lambda_q\left(u'''+\frac{2u''}{r}-\frac{u'}{r^2}+\frac{u}{r^3}\right)+D_q\left(\eta_q''+\frac{\eta_q'}{r}\right)+F_{q+1}.\end{aligned}\tag{1}$$

The formulation of the problem is concluded by the initial conditions which will be assumed to be null.

3. Solution method

The solution of the problem is sought in an integrated form. Assume that $G_{nm}(r,\tau),\ n,m=\overline{1,N+1}$ - are Green's function of the subject problem, i.e. the solution of the following Cauchy problems:

$$\begin{gathered}\left(G''_{1m}+\frac{G'_{1m}}{r}-\frac{G_{1m}}{r^2}\right)-\sum_{j=1}^{N}\alpha_j G'_{j+1,m}+\delta_{1m}\delta(r-\xi)\delta(\tau)=\ddot{G}_{1m},\\ -\Lambda_q\left(G'''_{1m}+\frac{2G''_{1m}}{r}-\frac{G'_{1m}}{r^2}+\frac{G_{1m}}{r^3}\right)+D_q\left(G''_{q+1,m}+\frac{G'_{q+1,m}}{r}\right)+\delta_{q+1,m}\delta(r-\xi)\delta(\tau)=\dot{G}_{q+1,m},\\ G_{1m}\big|_{\tau=0}=\dot{G}_{1m}\big|_{\tau=0}=G_{q+1,m}\big|_{\tau=0}=0.\end{gathered} \tag{2}$$

Thus, the solution of the problem (1) will take the form (asterisks will mean convolution by time τ and radius r):

$$u=\sum_{m=1}^{N+1}G_{1m}(r,\tau)**f_m(r,\tau),\quad \eta_q=\sum_{m=1}^{N+1}G_{q+1,m}(r,\tau)**f_m(r,\tau), \tag{3}$$

To find Greens functions let us employ the Laplace and Hankel transformation. The literature [6, 7] offer the following formulas for Hankel transformation from derivatives

$$H_k[f']=-pH_{k-1}[f]-(1-k)H_k\left[\frac{f}{r}\right],\quad H_k\left[f'+(1-k)\frac{f}{r}\right]=-pH_{k-1}[f],$$

$$H_k\left[f''+\frac{f'}{r}-\frac{k^2}{r^2}f\right]=-p^2H_k[f],\quad H_k\left[f''+\frac{f'}{r}\right]=-p^2H_k[f]+k^2H_k\left[\frac{f}{r^2}\right]$$

Here operator H_k (and the corresponding index further on) means Hankel of k order; p means a parameter of Hankel transformation; $f=f(r)$.

Let us present that

$$H_0\left[f'''+\frac{2f''}{r}-\frac{f'}{r^2}+\frac{f}{r^3}\right]=-p^3H_1[f].$$

As a matter of fact

$$H_0\left[f'''+\frac{2f''}{r}-\frac{f'}{r^2}+\frac{f}{r^3}\right]=H_0\left[\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)'\right]+H_0\left[\frac{1}{r}\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)\right].$$

Consider the augend:

$$\begin{gathered}H_0\left[\left(f''+\frac{f'}{r}-\frac{1}{r^2}f\right)'\right]=\int_0^\infty\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)'J_0(pr)rdr=\int_0^\infty J_0(pr)rd\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)=\\ =J_0(pr)r\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)\Big|_0^\infty-\int_0^\infty\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)\frac{d}{dr}[J_0(pr)r]dr=\\ =-\int_0^\infty\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)[J_0(pr)+prJ'_0(pr)]dr=-H_0\left[\frac{1}{r}\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)\right]+\\ +p\int_0^\infty\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)rJ_1(pr)dr=-H_0\left[\frac{1}{r}\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)\right]-p^3H_1[f].\end{gathered}$$

In this case $J_0(x)$, $J_1(x)$ are Bessel functions of the first kind, zero and first order correspondingly. Thus,

$$\begin{gathered}H_0\left[f'''+\frac{2f''}{r}-\frac{f'}{r^2}+\frac{f}{r^3}\right]=-H_0\left[\frac{1}{r}\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)\right]-p^3H_1[f]+\\ +H_0\left[\frac{1}{r}\left(f''+\frac{f'}{r}-\frac{f}{r^2}\right)\right]=-p^3H_1[f].\end{gathered}$$

Which was to be proved. Now we apply the Laplace transformation to the equations (2), then, a first order Hankel transformation to the first equation (2), and a zero order Hankel transformation, to the second equation, which results into (s is a parameter of Laplace transformation, index L is a Laplace transform)

$$\begin{aligned}&\left(p^2+s^2\right)G_{1m}^{H_1L}-\sum_{j=1}^{N}\alpha_j pG_{j+1,m}^{H_0L}=f_{1m}^{H_1L},\\&-\Lambda_q p^3G_{1m}^{H_1L}+\left(D_qp^2+s\right)G_{q+1,m}^{H_0L}=f_{q+1,m}^{H_0L}.\end{aligned}\tag{4}$$

Where

$$G_{1m}^{H_1L}=\int_0^\infty rJ_1(pr)dr\int_0^\infty G_{1m}(r,\tau)e^{-s\tau}d\tau,$$

$$G_{q+1,m}^{H_0L}=\int_0^\infty rJ_0(pr)dr\int_0^\infty G_{q+1,m}(r,\tau)e^{-s\tau}d\tau,$$

$$f_{1m}^{H_1L}=\delta_{1m}\int_0^\infty rJ_1(pr)dr\int_0^\infty\delta(r-\xi)\delta(\tau)e^{-s\tau}d\tau=\delta_{1m}\xi J_1(p\xi),$$

$$f_{q+1}^{H_0L}=\delta_{q+1,m}\int_0^\infty rJ_0(pr)dr\int_0^\infty\delta(r-\xi)\delta(\tau)e^{-s\tau}d\tau=\delta_{q+1,m}\xi J_0(p\xi).$$

The solution of the system (4) takes the following form:

$$\begin{aligned}&G_{11}^{H_1L}(p,\xi,s)=\frac{P_{11}(p,s)}{P(p,s)}\xi J_1(p\xi),\quad G_{1,q+1}^{H_1L}(p,\xi,s)=\frac{P_{1,q+1}(p,s)}{P(p,s)}\xi J_0(p\xi)\\&G_{q+1,1}^{H_0L}(p,\xi,s)=\frac{P_{q+1,1}(p,s)}{P(p,s)}\xi J_1(p\xi),\\&G_{q+1,q+1}^{H_0L}(p,\xi,s)=\left[\frac{1}{s+D_qp^2}+\frac{P_{q+1,q+1}(p,s)}{Q_q(p,s)}\right]\xi J_0(p\xi),\\&G_{q+1,p+1}^{H_0L}(p,\xi,s)=\frac{P_{q+1,p+1}(p,s)}{P(p,s)}\xi J_0(p\xi),\quad p=\overline{1,N},\quad p\neq q.\end{aligned}\tag{5}$$

where

$$P(p,s)=\left(s^2+p^2\right)\prod_{j=1}^{N}\left(s+D_jp^2\right)-\sum_{j=1}^{N}\alpha_j\Lambda_jp^4\prod_{r=1,r\neq j}^{N}\left(s+D_rp^2\right),\tag{6}$$

$$Q_q(p,s)=P(p,s)\left(s+D_qp^2\right);$$

$$P_{11}(p,s)=\prod_{j=1}^{N}\left(s+D_jp^2\right),\quad P_{1,q+1}(p,s)=\alpha_qp\prod_{j\neq q}^{N}\left(s+D_jp^2\right);\tag{7}$$

$$\begin{aligned}&P_{q+1,1}(p,s)=\Lambda_qp^3\prod_{j=1,\,j\neq q}^{N}\left(s+D_jp^2\right),\\&P_{q+1,p+1}(p,s)=\alpha_p\Lambda_qp^4\prod_{j=1,\,j\neq\{p,q\}}^{N}\left(s+D_jp^2\right),\quad p=\overline{1,N},\ p\neq q,\\&P_{q+1,q+1}(p,s)=\alpha_q\Lambda_qp^4\prod_{j=1,\,j\neq q}^{N}\left(s+D_jp^2\right).\end{aligned}\tag{8}$$

Note. The formulas (6) - (8) must assume:

- if $N=1$

$$\prod_{j=1,\, j\neq q}^{1}\left(s+D_j\lambda_n^2\right)=1,$$

- if $N=2$

$$\prod_{j=1,\, j\neq p,q}^{2}\left(s+D_j\lambda_n^2\right)=1.$$

The polynomials $P(p,s)$ and $Q_q(p,s)$ have the same structure as those of the similar polynomials in [1]. Therefore, the originals of Laplace transformed Green functions (5) will be written as follows:

$$G_{11}^{H_1}(p,\xi,\tau)=\tilde{G}_{11}(p,\tau)\xi J_1(p\xi),\quad G_{1,q+1}^{H_1}(p,\xi,\tau)=\tilde{G}_{1,q+1}(p,\tau)\xi J_0(p\xi),$$

$$G_{q+1,1}^{H_0}(p,\xi,\tau)=\tilde{G}_{q+1,1}(p,\tau)\xi J_1(p\xi),$$

$$G_{q+1,\,p+1}^{H_0}(p,\xi,\tau)=\tilde{G}_{q+1,\,p+1}(p,\tau)\xi J_0(p\xi),$$

$$\tilde{G}_{1m}(p,\tau)=e^{\gamma\tau}\left[A_{1m}^{(1)}\cos\beta\tau-A_{1m}^{(2)}\sin\beta\tau\right]+\sum_{j=1}^{N}A_{1m}^{(j+2)}e^{s_{j+2}\tau},\quad m=\overline{1,N+1},$$

$$\tilde{G}_{q+1,p}(p,\tau)=e^{\gamma\tau}\left[A_{q+1,p}^{(1)}\cos\beta\tau-A_{q+1,p}^{(2)}\sin\beta\tau\right]+\sum_{j=1}^{N}A_{q+1,p}^{(j+2)}e^{s_{j+2}\tau},\ p=\overline{1,N},$$

$$\tilde{G}_{q+1,q+1}(p,\tau)=e^{\gamma\tau}\left[A_{q+1,q+1}^{(1)}\cos\beta\tau-A_{q+1,q+1}^{(2)}\sin\beta\tau\right]+\sum_{j=1}^{N}A_{q+1,q+1}^{(j+2)}e^{s_{j+2}\tau}+A_{q+1,q+1}^{(N+3)}e^{-D_qp^2\tau},$$

where s_1 and $s_2=\bar{s}_1$ are complex conjugate values, and $s_{j+2},\ \ j=\overline{1,N}$ are real zeros of a polynomial $P(p,s)$. Meanwhile, $\beta=\operatorname{Re}s_1<0$, $\gamma=\operatorname{Im}s_1$, $s_{j+2}<0,\ \ j=\overline{1,N}$. The coefficients $A_{nm}^{(l)},\ l=\overline{1,N+2}$ are found as follows:

$$A_{1m}^{(1)}=2\operatorname{Re}\frac{P_{1m}(\lambda,s_1)}{P'(\lambda,s_1)},\ A_{1m}^{(2)}=2\operatorname{Im}\frac{P_{1m}(\lambda,s_1)}{P'(\lambda,s_1)},\ A_{1m}^{(j+2)}=\frac{P_{1m}(\lambda,s_{j+2})}{P'(\lambda,s_{j+2})},$$

$$A_{q+1,p}^{(1)}=2\operatorname{Re}\frac{P_{q+1,p}(\lambda,s_1)}{P'(\lambda,s_1)},\ A_{q+1,p}^{(2)}=2\operatorname{Im}\frac{P_{q+1,p}(\lambda,s_1)}{P'(\lambda,s_1)},\ A_{q+1,p}^{(j+2)}=\frac{P_{q+1,p}(\lambda,s_{j+2})}{P'(\lambda,s_{j+2})},$$

$$A_{q+1,q+1}^{(1)}=2\operatorname{Re}\frac{P_{q+1,q+1}(\lambda,s_1)}{Q_q'(\lambda,s_1)},\ A_{q+1,q+1}^{(2)}=2\operatorname{Im}\frac{P_{q+1,q+1}(\lambda,s_1)}{Q_q'(\lambda,s_1)},$$

$$A_{q+1,q+1}^{(j+2)}=\frac{P_{q+1,q+1}(\lambda,s_{j+2})}{Q_q'(\lambda,s_{j+2})},\ A_{q+1,q+1}^{(N+3)}=\frac{P_{q+1,q+1}(\lambda,-D_q\lambda^2)}{Q_q'(\lambda,-D_q\lambda^2)}.$$

The inverse Hankel transformation is found as follows:

$$G_{11}(r,\xi,\tau)=\int_0^\infty G_{11}^{H_1}(p,\xi,\tau)pJ_1(pr)dp=\int_0^\infty\tilde{G}_{11}(p,\tau)\xi J_1(p\xi)pJ_1(pr)dp,$$

$$G_{1,q+1}(r,\xi,\tau)=\int_0^\infty G_{1,q+1}^{H_1}(p,\xi,\tau)pJ_1(pr)dp=\int_0^\infty\tilde{G}_{1,q+1}(p,\tau)\xi J_0(p\xi)pJ_1(pr)dp,$$

$$G_{q+1,1}(r,\xi,\tau)=\int_0^\infty G_{q+1,1}^{H_0}(p,\xi,\tau)pJ_0(pr)dp=\int_0^\infty\tilde{G}_{q+1,1}(p,\tau)\xi J_1(p\xi)pJ_0(pr)dp,$$

$$G_{q+1,k+1}(r,\xi,\tau)=\int_0^\infty G_{q+1,k+1}^{H_0}(p,\xi,\tau)pJ_0(pr)dp=\int_0^\infty\tilde{G}_{q+1,k+1}(p,\tau)\xi J_0(p\xi)pJ_0(pr)dp.$$

To find solutions u and η_q, first, let us find convolution by time in (3). In this case

$$G_{11} ** f_1 = \int_0^\infty d\xi \int_0^\tau dt \int_0^\infty \tilde{G}_{11}(p,t)\xi J_1(p\xi) p J_1(pr) f_1(\xi, \tau - t) dp = \int_0^\infty S_{11}(p,\tau) p J_1(pr) dp, \qquad (9)$$

and in the same way

$$G_{1,q+1} ** f_{q+1} = \int_0^\infty S_{1,q+1}(p,\tau) p J_1(pr) dp,$$

$$G_{q+1,m} ** f_1 = \int_0^\infty S_{q+1,m}(p,\tau) p J_0(pr) dp, \quad m = \overline{1, N+1},$$

where

$$\begin{aligned}
S_{11}(p,\tau) &= \int_0^\infty \xi J_1(p\xi) d\xi \int_0^\tau \tilde{G}_{11}(p,t) f_1(\xi, \tau - t) dt, \\
S_{1,q+1}(p,\tau) &= \int_0^\infty \xi J_0(p\xi) d\xi \int_0^\tau \tilde{G}_{1,q+1}(p,t) f_{q+1}(\xi, \tau - t) dt, \\
S_{q+1,1}(p,\tau) &= \int_0^\infty \xi J_1(p\xi) d\xi \int_0^\tau \tilde{G}_{q+1,1}(p,t) f_1(\xi, \tau - t) dt, \\
S_{q+1,k+1}(p,\tau) &= \int_0^\infty \xi J_0(p\xi) d\xi \int_0^\tau \tilde{G}_{q+1,k+1}(p,t) f_{k+1}(\xi, \tau - t) dt,
\end{aligned} \qquad (10)$$

Here the functions $S_{mn}(p,\tau)$ can be found analytically with integration table [8]. The remaining integrals in (9) are found numerically.

4. Example

As an example let us consider a single component plane made of aluminum with the following mechanical properties [9]

$$\lambda + 2\mu = 1.26 \cdot 10^{11} \frac{H}{м^2}, \quad T_0 = 800\,K, \quad \rho = 2700 \frac{кг}{м^3}, \quad D = 7.73 \cdot 10^{-14} \frac{м^2}{c}, \quad L = 1\,м.$$

Set

$$f_1(r,\tau) = e^{-\varepsilon r} H(\tau), \quad f_2(r,\tau) = 0, \quad \operatorname{Re}\varepsilon > 0.$$

Then time convolutions (3) will give the following expressions for functions $S_{nm}(p,\tau)$ in (10):

$$S_{11}(p,\tau) = \frac{p}{\left(\varepsilon^2 + p^2\right)^{3/2}} \left[A_{11}^{(1)} I_1(\gamma,\beta,\tau) - A_{11}^{(2)} I_2(\gamma,\beta,\tau) + A_{11}^{(3)} I_3(s_3,\tau) \right],$$

$$S_{21}(p,\tau) = \frac{\varepsilon}{\left(\varepsilon^2 + p^2\right)^{3/2}} \left[A_{21}^{(1)} I_1(\gamma,\beta,\tau) - A_{21}^{(2)} I_2(\gamma,\beta,\tau) + A_{21}^{(3)} I_3(s_3,\tau) \right],$$

where I_m, $m = \overline{1,3}$ are found by as follows [1,8]:

$$I_1(\gamma,\beta,\tau) = \frac{1}{\gamma^2 + \beta^2} \left[e^{\gamma\tau} (\beta \sin\beta\tau + \gamma \cos\beta\tau) - \gamma \right],$$

$$I_2(\gamma,\beta,\tau) = \frac{1}{\gamma^2 + \beta^2} \left[e^{\gamma\tau} (\gamma \sin\beta\tau - \beta \cos\beta\tau) + \beta \right], \quad I_3(s,\tau) = \frac{e^{s\tau} - 1}{s}.$$

The inversed Hankel transforms are calculated numerically. The results of calculations are presented in Figures below.

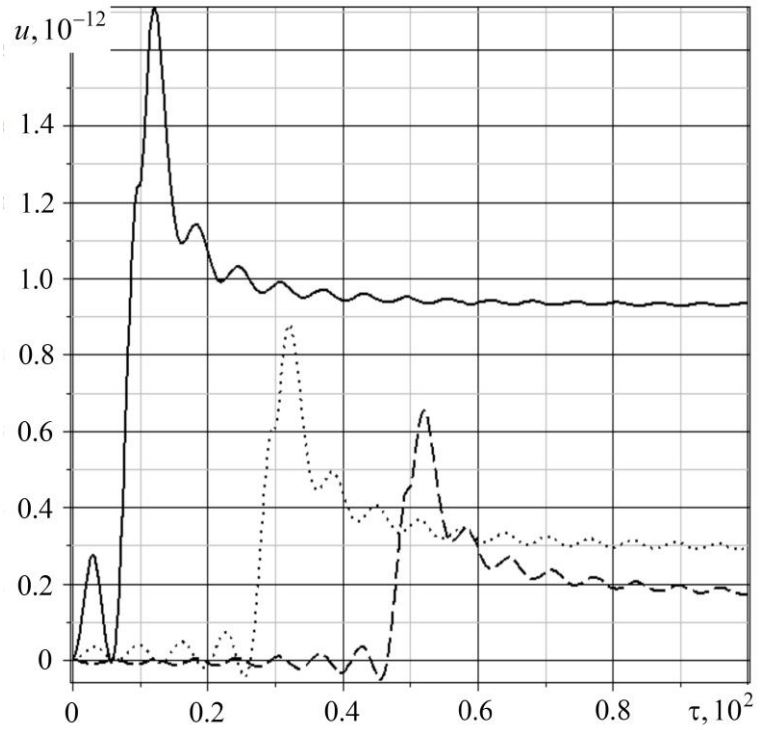


Figure 1. Time-displacement relation $u(x,\tau)$ where $r=10$ (full line), $r=30$ (dotted line), $r=50$ (dash line)

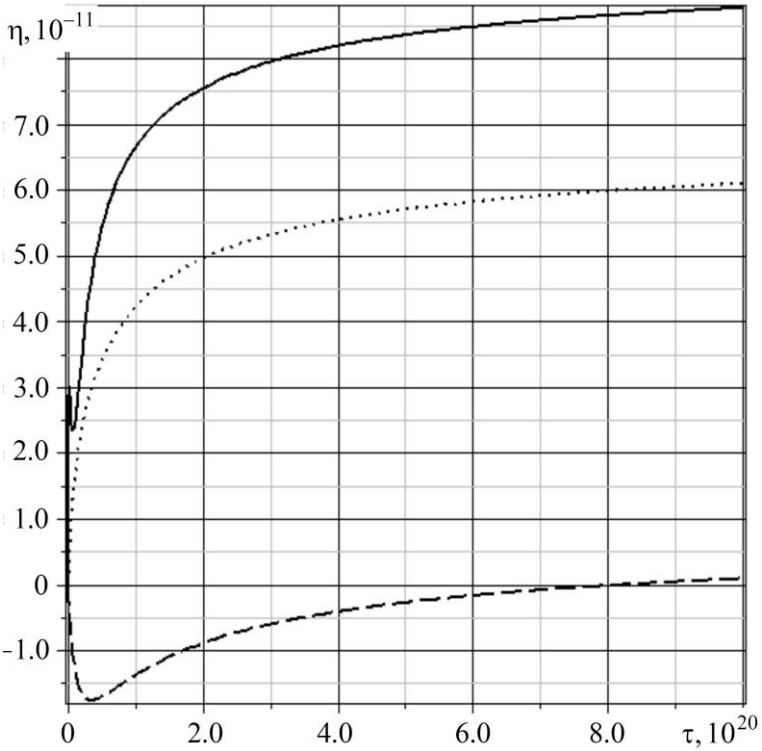


Figure 2. Relation of time and increment of concentration $\eta(x,\tau)$ where $r=1$ (solid line), $r=3$ (dotted line), $r=5$ (dash line)

The calculations demonstrate that volumetric disturbances set in the right side of the equation of motion induce both mechanical displacements (Fig. 1) and mass transfer (Fig.2).

Conclusion

This paper introduces an algorithm of solution of a polar-symmetric non-stationary problem of elastic diffusion for an isotropic plane. There are dominant functions found allowing for identifying displacement fields and increment of ambient component concentration by preset volumetric disturbances. The work of the algorithm is exemplified by an effect of link of mechanic and diffusion field. The results of calculations are portrayed in graphs of relations of the desired fields and time in various points of a plane.

References

[1] Davydov S A, Zemskov A V, Tarlakovskii D V 2015 *Lobachevskii Journal of Mathematics* **36**,4 503.
[2] Sneddon I 1951 *Fourier transforms* (New York –Toronto – London).
[3] *Slepyan L.I., Yakovlev Y.S. Integralniye preobrazovaniya v nestatsionarnykh zadachakh mekhaniki.- L.: Sudostroyeniye, 1980, -344 s.* (Slepyan L.I., Yakovlev Y.S., Integral transformations in non-stationary problems of mechanics, in Russian)
[4] Tranter C J 1966 *Integral transforms in mathematical physics* (London: Methuen & Co. Ltd. New York: John Wiley & Sons, Inc)
[5] Zemskov A V, Tarlakovskiy D V *2015 Journal of Applied Mechanics and Technical Physics*, **56**,6 1023
[6] *Gorshkov A.G., Medvedevsky A.L., Rabinsky L.N., Tarlakovsky D.V. Volny v sploshnykh sredakh. Uchebnoye posobiye*: dlya vuzov. – M.: Phizmatlit, 2004. – 472 s. (Gorshkov A.G., Medvedevsky A.L., Rabinsky L.N., Tarlakovsky D.V., Waves in continuous mediums, in Russian)
[7] Piessens R. 2000 *The Hankel Transform*: (Boca Raton: CRC Press LLC)
[8] *Prudnikov A.P., Brychkov Y.A., Marichev O.I., Integraly I ryady. Tom 1. Elementarniye funktsii. – M.: Nauka, 1981, 797 s.* (Prudnikov A.P., Brychkov Y.A., Marichev O.I., Integrals and serials. Volume 1. Elementary functions. In Russian).
[9] *Fizicheskiye velichiny: Sprovochnik / Babichev A.P., Babushkina N.A., Bratkovsky A.M., I dr. Pod obshchey redaktsiyey Grigorieva I.S., Meylikhova I.Z. – M.: Energoatomizdat, 1991.* (Physical quantities: reference manual, edited by Grigorieva I.S., Meylikhova I.Z. In Russian)