Additive formulation elastoplasticity at finite strain

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Abstract. In the article a method of investigation of elastic-plastic deformation for solids under complex loading is presents. Resolving equation based on the principle of virtual power. For simulation of plastic deformation taken the associated flow law with updating the stressed state. Numerical implementation is based on using 8-node finite element discretization. Numerical calculations show the suitability of the proposed methodology.

1. Introduction

There are many publications, where solutions of nonlinear problems of a solid mechanic are discussed, for example [10-13].In this paper a numerical algorithm of the investigation of stress-strain state of the elastic–plastic solids with large deformations is described.

2. Kinematics

The deformation gradient tensor **F**, the left Cauchy–Green tensor $\mathbf{B}=\mathbf{F}\cdot\mathbf{F}^{\mathrm{T}}$, the corresponding spatial gradient of velocity $\mathbf{h}=\dot{\mathbf{F}}\cdot\mathbf{F}^{-1}$ and the rate of deformation $\mathbf{d}=\mathrm{sym}(\mathbf{h})$ are used for describing of kinematics of a continuum. The basic relationships are described in [1, 6-9].

3. Constitutive equations

The constitutive equations are obtained using the free energy function ψ that plays the role of an elastic potential and yield function. The Cauchy stress tensor is defined as [1, 5]

$$\mathbf{\Sigma}=\frac{2}{J}\mathbf{B}\cdot\frac{\partial\psi}{\partial\mathbf{B}}, \tag{1}$$

where $J=\det(\mathbf{F})$ is a changing of volume. For isotropic material the free energy ψ is defined as $\psi=\psi\left(I_{1\mathbf{B}},I_{2\mathbf{B}},I_{3\mathbf{B}}\right)$, where $I_{i\mathbf{B}}$ is the corresponding invariants of tensor $\mathbf{B}$.

After linearization (1) the rate of Cauchy stress is defined as

$$\dot{\mathbf{\Sigma}}=2\left\{\frac{1}{J}\dot{\mathbf{B}}\cdot\frac{\partial\psi}{\partial\mathbf{B}}+\frac{1}{J}\left[\mathbf{B}\cdot\frac{\partial^2\psi}{\partial\mathbf{B}^2}\right]\cdot\cdot\dot{\mathbf{B}}-\frac{1}{J}\mathbf{B}\cdot\frac{\partial\psi}{\partial\mathbf{B}}I_{1\mathbf{d}}\right\}=\mathbf{\Lambda}_{\Sigma}\cdot\cdot\mathbf{d}+\mathbf{h}\cdot\mathbf{\Sigma}+\mathbf{\Sigma}\cdot\mathbf{h}^{\mathrm{T}}-\mathbf{\Sigma}I_{1\mathbf{d}},$$

where $\mathbf{\Lambda}_{\Sigma}=\frac{4}{J}\mathbf{B}\cdot\frac{\partial^2 W}{\partial\mathbf{B}\partial\mathbf{B}}\cdot\mathbf{B}$ – fourth order constant tensor.

Or

$$\mathbf{\Sigma}^{Tr}=\mathbf{\Lambda}_{\Sigma}\cdot\cdot\mathbf{d}, \tag{2}$$

where $\mathbf{\Sigma}^{Tr} = \dot{\mathbf{\Sigma}} + \mathbf{h} \cdot \mathbf{\Sigma} + \mathbf{\Sigma} \cdot \mathbf{h}^{\mathbf{T}} - I_{\mathbf{1d}}\mathbf{\Sigma}$ is the Truesdell stress rate. Therefore, constitutive model can be defined as linear relation between an objective derivative of Cauchy stress tensor and rate of deformation.

The theory of flow is used for describing plastic deformation [2-5]. The total deformation rate is represented as a sum of elastic and plastic parts: $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p$ [2-5]. For the plastic deformation rate must hold association flow rule:

$$\mathbf{d}^p = \dot{\gamma} \frac{\partial \Phi}{\partial \mathbf{\Sigma}}, \tag{3}$$

where $\dot{\gamma}$ is the consistency parameter, Φ is a yield function.

4. Integration algorithm of the flow rules

For the solution problem of a plastic flow the general return method is used [5, 14-16]. The solution at time ${}^k t$ is known. The stress at time ${}^{k+1}t = {}^k t + \Delta^{k+1} t$ is defined from the initial conditions at time ${}^k t$. The trial stress is defined as

$$ {}^{k+1}\tilde{\mathbf{\Sigma}} = {}^k\mathbf{\Sigma} + {}^{k+1}\Delta\mathbf{\Sigma}. \tag{4}$$

The plastic flow $\Delta^{k+1}\gamma$ can be computed from equation

$$ {}^{k+1}\mathbf{\Sigma} + \Delta^{k+1}\gamma \frac{\partial^{k+1}\Phi}{\partial^{k+1}\Sigma} \cdot\cdot {}^{k+1}\mathbf{\Sigma} = {}^{k+1}\tilde{\mathbf{\Sigma}}. \tag{5}$$

The solution is obtained by solving a nonlinear system of equations (5) with linearized Newton scheme using (2)–(4).

5. Variation formulation

The research algorithm is based on an Update Lagrange formulation. The principle of virtual work in terms of the virtual velocity is used [1-5]:

$$\int_{\Omega} \mathbf{\Sigma} : \boldsymbol{\delta}\mathbf{d} d\Omega = \int_{\Omega} \mathbf{f} \cdot \boldsymbol{\delta}\mathbf{v} d\Omega + \int_{S^\sigma} \mathbf{p} \cdot \boldsymbol{\delta}\mathbf{v} dS,$$

where Ω is the current volume, S^σ is the surface on which the force $\mathbf{p}$ is applied, $\mathbf{f}$ is the body force vector, $\mathbf{v}$ is a velocity vector. After linearization the system of linear equations is obtained, where the unknown is the increment of displacement in the current state $\boldsymbol{\Delta}^{k+1}\mathbf{u}$. For solving general system of equations the arc-length method is applied [5]. The current state is defined as ${}^{k+1}\mathbf{R} = {}^k\mathbf{R} + \Delta^{k+1}\mathbf{u}$. Then the trial stress is calculated by (4). And if $\Phi({}^{\mathbf{k+1}}\tilde{\mathbf{\Sigma}}) \le 0$ then the Cauchy stress ${}^{k+1}\mathbf{\Sigma} = {}^{k+1}\tilde{\mathbf{\Sigma}}$, else the radial return method with an iterative refinement of the current mode of deformation is applied [5].

6. Numerical example

As an example the potential of elastic deformation is considered:

$$\psi = \frac{\lambda + 2\mu}{8}\left(I_{\mathbf{1B}} - 3\right)^2 + \mu\left(I_{\mathbf{1B}} - 3\right) - \frac{\mu}{2}\left(I_{\mathbf{2B}} - 3\right),$$

here λ, μ are Lame parameters. The von Mises yield criterion with isotropic hardening is used: $\Phi = \sigma_i - \xi(\chi) \le 0$, where $\sigma_i = \sqrt{1.5\, dev\mathbf{\Sigma} : dev\mathbf{\Sigma}}$, $\xi(\chi) = \sigma_T + h\chi + (\sigma_\infty - \sigma_T)(1 - e^{-\delta\chi})$ is the hardening function. The material data for isotropic elasticity and the von Mises yield condition are given as follows: E=206.9 GPa, $\nu = 0.29$, $\sigma_\infty = 0.715$ GPa, $\sigma_T = 0.450$ GPa, $h = 0.129$, $\delta = 16.93$.

The numerical implementation is based on the finite element method. An 8-node brick element is used [2-5, 9].

6.1. Necking of a circular bar. The necking of a circular bar is an example widely investigated in the literature; see e.g. [14] or [15]. To initialize the necking process radius in the center is reduced by 1.8 %. Figure 1 displays the final deformed structure and the equivalent plastic strain, which concentrates in the necking zone. The results are in very good agreement with the computational reference solutions of [14] and [15].

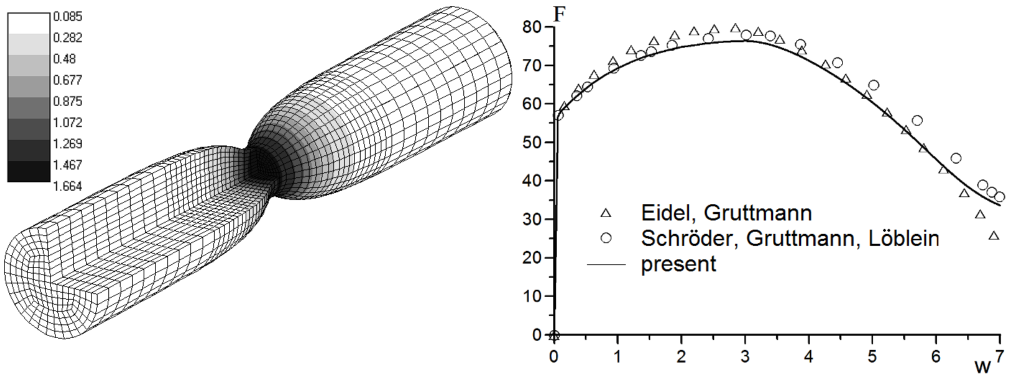

Figure 1. Left: Equivalent plastic strain at final structure.
Right: Computational results of applied force F [kN] versus axial elongation w [mm].

6.2. Conical shell. In the second example of isotropic elastoplasticity a conical shell subjecting to a constant ring load is considered [9–10]. Figure 2 displays equivalent plastic strain for the several deformed structure.

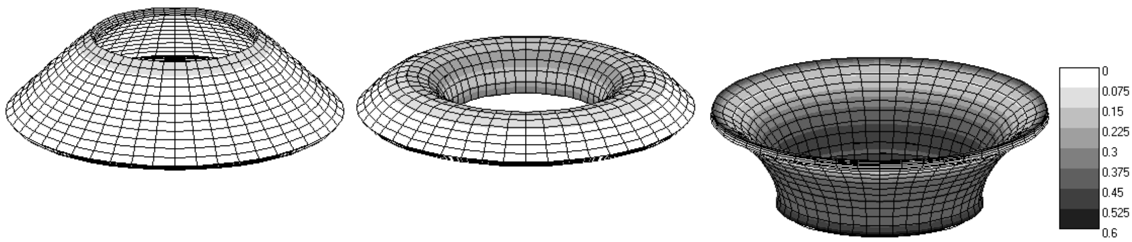

Figure 2. Equivalent plastic strain on deformed structures at different stages of the punching process.

6.3. Drawing of a Circular Blank. Two different materials are considered for orthotropic yielding. For material A the shear stresses dominate in the yield criterion, for material B the normal stresses are predominant in yielding. As expected the plastic strains concentrate for material A at a 45º angle in the (x, y)-plane and for material B along the x-and y-axes, see Figure 3.

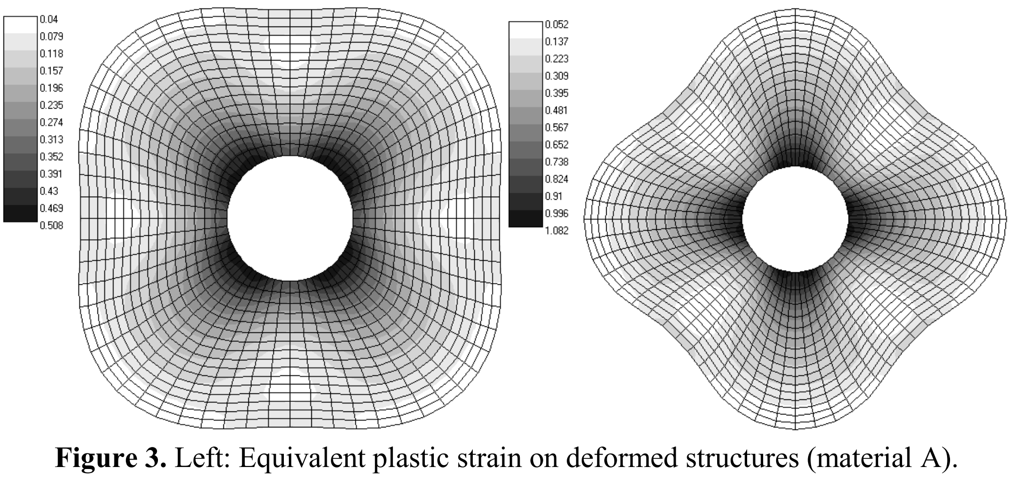

Figure 3. Left: Equivalent plastic strain on deformed structures (material A).
Right: Equivalent plastic strain on deformed structures (material B).

7. Conclusions

In the paper an additive formulation of the elastic-plastic deformation presents and finite element implementation is considered. The physical conditions obtained using the free energy function. The linearized equations derived from the principle of virtual work for the current state. Mises yield criterion with isotropic hardening is used. The effectiveness of present research algorithm of large elastic-plastic deformation demonstrate resolved problems, results are compared with the results of other authors.

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