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The mathematics of Soft logic

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Abstract. We strive to understand, by using mathematical tools, the phenomena where the observer can interpret simultaneously opposite situations, such as the distinct interpretations of the Necker cube. In this study we present a new coordinate system that draws a distinction between -0 and +0. The central theorem presented here postulates that when divisions and multiplications of 0 are combined with real numbers, the Mobius strip provides a model for non-standard analysis. We also suggest that the axis 0 is the fifth dimension extension of the quaternions. We also propose to add the soft logic capability to humanoid type robots, as a way to overcome the Turing test.

1. Introduction

The mathematician Gödel proved in 1931 that there are sentences in number theory, for which it is impossible to determine whether they are true or false, such as the so-called "liar paradox" (when a person says he is a liar). This surprising result presented by Gödel is true in all mathematical theories that contain number theory. It breaks with the prevalent deductive view, which has been developing since Euclid's *Elements.*

Following this result of Gödel, Wittgenstein delivered a seminar in 1939, at Cambridge University, about the foundation of mathematics. In this seminar, he suggested re-examining the basic assumptions of mathematicians. According to Wittgenstein, the mathematician is an inventor rather than a discoverer. Therefore, there is always a subjective dimension to the process of creating mathematics.

Wittgenstein hinted the idea that mathematical statements always have a dual meaning, both empirical and mathematical. Accordingly, there are two ways of examining them, an empirical way and a mathematical way. For example, there are two ways to validate that $\boldsymbol{625 = 25 \times 25}$. The first way is to test if the weight of 25 objects, each of which weighs 25, is in fact 625. Any other result would suggest that the statement is false. The second way is to obtain the result by the derivation of certain mathematical rules. An empirical meaning involves experiments and calculations, and a mathematical meaning involves mathematical logic and proofs.

In 1969, George Spencer-Brown published his book *"Laws of Form*". In the book, he developed a new mathematical operator, which he called a “distinction”. Distinction is the operation of pointing to an object, and by doing so, separating it from its environment. This operation have a new symbol ┐, which enables us to develop a new mathematical language with only one symbol.

This language is a primal language – since it contains the most fundamental that we can do - observe an object. Spencer-Brown produced this "primitive language" from the most abstract building blocks of language- that can be the basis for any language. Through the sign, the observer and the phenomena are united. In other words, the sign marks the active attention, which is a creative activity.

The distinction is a single operation, which agrees with the fact that we can really pay attention to only one thing. The act of separating a figure from its background always creates a single image (even if this figure is multiple).

The Boolean logic (the logic of the truth-values 0 and 1) is a higher level of logic than the one set by the Spencer-Brown language, which is a finer and more primitive one. Zero, unlike the empty place, is already creating. The operation of distinction is an act of marking a circle, with the intention of saying "it". According to Spencer-Brown it does not matter what you are looking at - what matters is the act of observation.

The basic axioms of Spencer-Brown are:

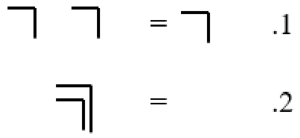

Figure 1. The laws of form

The first law says that if you go back to that which you have already noticed, and notice it again, it will not change anything. The second law interprets as follows: the creation of something from nothing. To distinguish inside the distinction is like nothing. Once more, we must remember that Spencer-Brown is looking for abstraction and for the most basic extension – in this primitive world there is only one object and one observer who is drawing a distinction.

According to studies of Marcelo Dascal, the mathematician and philosopher Leibniz, as a young researcher, aspired to develop a universal language with a single signal. As suggested by Dascal, Leibniz converted this first vision into a new one: to discover and develop a mathematical language that will demonstrate a softer logic than that of truth and falsehood. Leibniz had an ambitious plan to construct a universal language, which will prevent misunderstandings between people as well as serve as a scientific language that reflects thought. According to Leibniz, language is also a tool for thinking and it influences thinking. Therefore, precise formal language, such as expression and precise thinking, is necessary to reduce errors and increase certainty, thus allowing for the resolution of disputes. Dascal argues that Leibniz knew that no rational thinking, not even a “soft rationality”, is included in the formal computational model of rationality on computational language. In fact, Leibniz wrote in many occasions that the logic of two states is insufficient.

In this study, we propose to see the Necker cube phenomenon as a basis for the development of a mathematical language in accordance with Leibniz’s vision of soft logic. By using the new coordinate system, that was developed by Yale Landsberg, we make a distinction between -0 and +0. By observing the Necker cube phenomenon with the Quaternions number system as was done by Martin Hay, we discover the existence of a fifth dimension which is the 0 axis. All this allows us to prove that the Mobius strip can be a model for non-standard analysis.

2. The Necker cube and ± 0

If we draw a hexagon **Figure 2** and its three principal diagonals, we obtain a flat figure that can be interpreted as a three-dimensional cube.

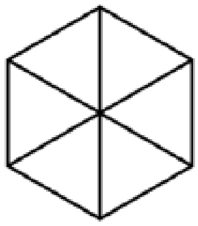

Figure 2. An hexagon with 3 main diagonal

This form is called a Necker cube and It can be interpreted in two different ways, as follows:

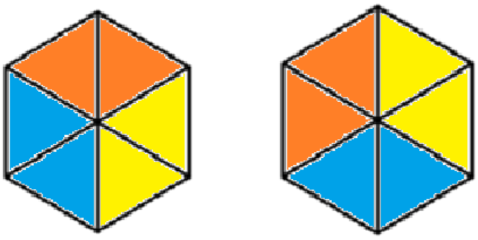

Figure 3. Two interpretations of the Necker cube

A cube has 6 faces, 8 vertices and 12 edges. The flat figure has 6 vertices and one point,at the center of the hexagon, that can appear as either of two more vertices.

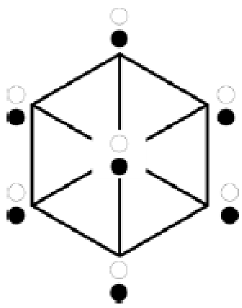

Figure 4. Polarity in hexagon

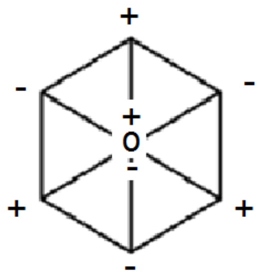

Figure 5. A bipartite graph

We shall use the signs of - and + to create a Bipartite graph **Figure 5**. with the edges of the hexagon, by denoting the vertex of each diagonal with one of these signes. The center of the hexagona shall be marked by the letter O.

Each interpretation of the Necker cube brings to mind opposite signs ± of O, either + or -. Therefore, we can conclude that: -O=+O . The center depends on the point of view of the observer. If the vertices are to attain some numerical values, the only conclusion will be as follows:

$$O = \emptyset$$

The Hexagon's vertices will be $\pm\emptyset$ Thus, we discover a distinction between - $\emptyset$ and + $\emptyset$. A distinction between $\pm\emptyset$ also exists in the theory of calculus. The two marks of $\emptyset$ appear while looking at the limit of a function's derivative from left or right. In the following section, we shall extend the concept of $\pm\mathbf{0}$ by building a new coordinate system.

3. The new coordinate system

The ability to interpret a drawing in two different ways also exists in **Figure 6.** The diagonal can be interpreted as moving upwards or as moving away from us, sideways. This property may be more easily observed in a simple corner **Figure 7.** Now, we will create a coordinate system. The first number that was invented was 1 so we shall locate it in place of B **Figure 8.** Then, the number 2 was invented, so we shall locate it in place of C: **Figure 9.**

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Figure 6.

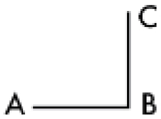

Figure 7.

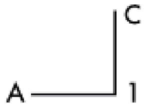

Figure 8.

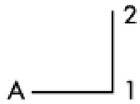

Figure 9.

The numbers 3,4,5,6, etc., which were invented later, shall be placed along the vertical axis, as follows:

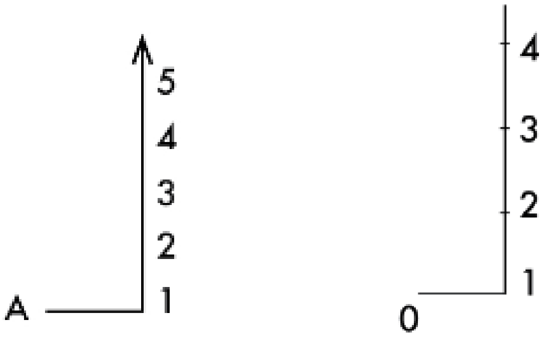

Figure 10. New location for 0

The invention of the number 0 revolutionized mathematics. Thus, as it is befitting, we propose to position the number 0 perpendicularly, in place of A. Let us keep in mind the following statements:

- The left end of the number's coordinate is marked with 0.
- It is important to note: We measure from 0, but count from 1.
- It is possible to build a basis that will be a different length than 1.

Having established the foundation for this number coordinate, we can begin the construction of our argument.

1st lemma: The horizontal segment (0, 1] is dual to the vertical segment (1, ∞]

Proof:

Let us consider the following function **Figure 11.**: $f(x) = \frac{1}{x}$

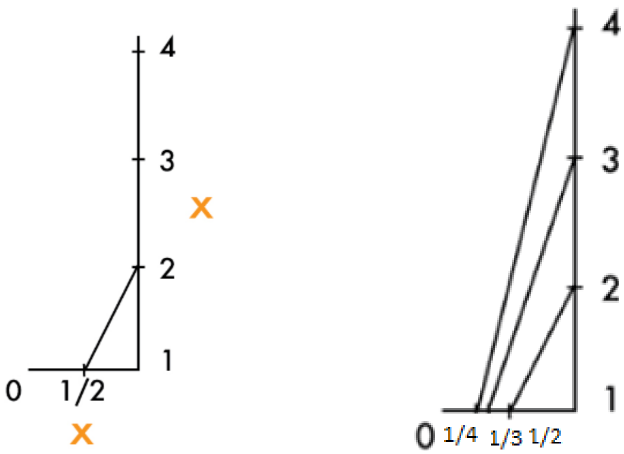

Figure 11. The connection between x to 1/x

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This function creates a one-to-one correspondence between the segment (0, 1] and the segment$(\mathbf{1}, \infty]$.

Q.E.D

We would like to stress that the line we drew connects two different points on the same coordinate.

2nd lemma: The new number's coordinate connects 0 to infinity.

Proof:
Let us consider again the function $f(x) = \frac{1}{x}$
When x tends to 0, 1/x tends to infinity and vice versa.

Q.E.D

Thus, we hope, and as it is demonstrated below, the link between 0 and infinity becomes more intuitive.

3rd Lemma: All lines between opposite points intersect at one point.

Proof:
Let us observe the following drawing **Figure 12.** connecting x to 1/x:

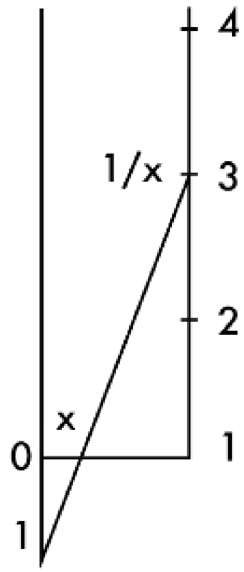

Figure 12. A proof of lemma 3

Let us apply the properties of similar triangles:

$$\frac{x}{1} = \frac{1-x}{\frac{1}{x}-1}$$

All the lines intersect at one point, which is located one unit below the 0.
This same point is the beginning of the Cartesian coordinate.
Conclusion:

4th Lemma: The height of 0 from the beginning of the Cartesian coordinate is 1.

5th lemma: A distinction between -0 and +0 can be made by applying the new coordinate system.

Proof:
As we have already seen, the height of 0 is 1, in relation to the Cartesian coordinate's origin point.
We would also like to suggest extending this new coordinate system to the negative numbers.
Let us consider the next drawing:

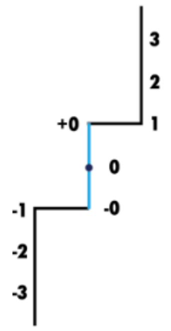

Figure 13. A distinction between -0 and +0

The number 0 is also opposite to the number -1, but it is not identical to the 0 opposite to the number 1.
Hence, we suggest to denote the two different "zeros" as -0 and +0.

Q.E.D

6th lemma: There is a line of 0.

Proof:
As we have seen, the new coordinate system draws a distinction between -0 and +0.

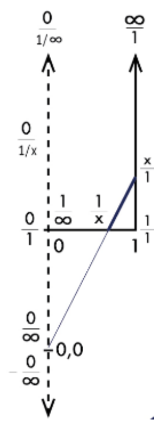

Figure 14. The line of 0

As seen in the drawing above, a line can be drawn to pass through them.
This line will be comprised of all of 0's multiplications and divisions.

Q.E.D

Non-standard analysis is a mathematical theory that extends the real number line by adding infinite and infinitesimal numbers (for more details see Appendix A).
Conclusion:
7th lemma: A model for nonstandard analysis can be constructed.

4. The Mobius strip model

The Möbius strip, having only one side, can serve as a model for the understanding of the relationship between mathematics and physics. This is due to its ability to represent the fact that the viewer is not merely a watcher, but that he is also able to see himself as an observer in the world.

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Theorem 1: The Mobius strip is a model of a non-standard analysis

Figure 15. A Mobius strip

Proof:

As we have observed, while presented on a plane, the new numbers coordinate creates a non-standard analysis model. To demonstrate that the Mobius strip includes the new coordinate system in it, let us observe the next drawing:

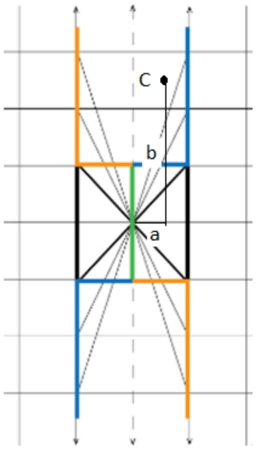

Figure 16. A model for the Mobius strip

Let us take a straight piece of paper and on it, draw the new coordinate system in the plane. Draw a square in the middle and fold the piece of paper twice, along the diagonal line. The resulting form will be a Mobius strip. The coordination of the point C on the Mobius strip are:

$$\boldsymbol{a + b\bar{0}}$$

Q.E.D

5. Quaternions and the fifth dimension

Hamilton invented the field of quaternions in 1843. It is a four-dimensional extension of the complex numbers. Each element in the field written by using the following expression**:**

$$a + bi + cj + dk$$

For more details, see appendix B.

Theorem 2: We can extend the quaternions to the 5th dimension by the 0 axis.

Proof:

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Chirality is a fundamental symmetry in nature. It occurs at all scales, from the chiral molecules in the living body and up to spiral galaxies. As demonstrated below, chirality can be manifested as in these four different objects (red, yellow, green, blue), arranged at the corners of a tetrahedron.
Such a tetrahedron has two formations which are related, as (+) to (–) or as clockwise to anticlockwise. (Note the order of green, blue and yellow in the photographs below).

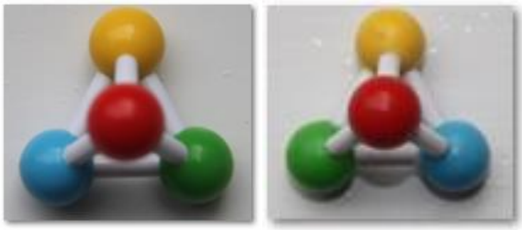

Figure 17. Chirality of the tetrahedron

In one formation, all the corners can be denoted (+) (or "is"), while in the other formation all the corners can be denoted (-) (or "not").

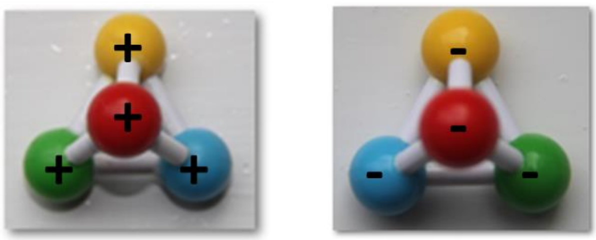

Figure 18. Two opposite chirality's

The two formations can be nested in a cube, so that opposite corners of the cube are labelled as (+) and (–) opposites. Each face of the cube then has two (+) corners and two (-) corners.

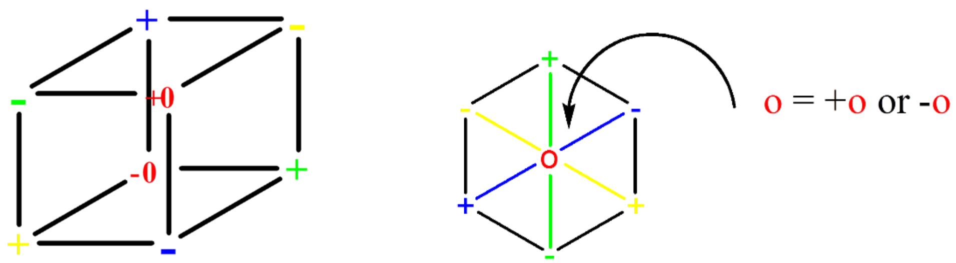

Figure 19. The Necker cube and ± 0

Now the Necker cube effect can be viewed in terms of the nested tetrahedrons. The brain exploits the difference between what a chiral tetrahedron is (+) and what it is not (-) to draw distinctions. It is a chiral system. The universe's chirality provides the basis for consciousness. It is possible to relate the red, yellow, green and blue vertexes to quaternions, conventionally represented as ± 1, ± i, ±j and ±k.

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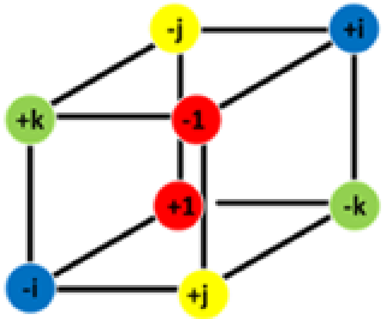

×	1	*i*	*j*	*k*
1	1	*i*	*j*	*k*
i	*i*	−1	*k*	−*j*
j	*j*	−*k*	−1	*i*
k	*k*	*j*	−*i*	−1

Figure 20. The Necker cube and quaternions

Each face of the cube can be represented by 4 units of the vertices of the face. We use the multiplication Table and check which unit multiplication will make all the results become negative.

face	i	J	k	-i	-j	-k
[(-1)(-i)(j)(k)]	[(1)(-i)((j)(-k)]	**[(-1)(-i)(-j)(-k)]**	[(-1)(i)(j)(-k)]	[(-1)(i)((- j)(k)]	[(1)(i)(j)(k)]	[(1)(-i)(-j)(k)]
[(-1)(i)(j)(-k)]	**[(-1)(-i)(-k)(-j)]**	[(-1)(-i)(-j)(k)]	[(1)(i)(-j)(-k)]	[(1)(i)(k)(j)]	[(1)(i)(j)(-k)]	[(-1)(-i)(j)(k)]
[(-1)(i)(-j)(k)]	[(-1)(-i)(j)(k)]	[(1)(-i)(-j)(k)]	**[(-1)(-i)(-j)(-k)]**	[(1)(i)(-j)(-k)]	[(-1)(i)(j)(-k)]	[(1)(i)(j)(k)]
[(1)(-i)(j)(-k)	[(1)(i)(-j)(-k)]	[(-1)(i)(j)(-k)]	[(1)(i)(j)(k)]	[(-1)(-i)(j)(k)]	[(1)(-i)(-j)(k)]	**[(-1)(-i)(-j)(-k)]**
[(1)(-i)(-j)(k)]	[(1)(i)(j)(k)]	[(1)(-i)(-j)(-k)]	[(-1)(-i)(j)(k)]	**[(-1)(-i)(-j)(-k)]**	[(-1)(i)(j)(k)]	{(1)(i)(-j)(-k)]
[(1)((i)(-j)(-k)]	[(-1)(i)(-j)(k)]	[(1)(i)((j)(k)]	[(1)(-i)(-j)(k)]	[(1)(-i)(j)(-k)]	**[(-1)(-i)((-j)(-k)]**	[(-1)(i)(j)(-k)]

For every face there is only one such unit and this is the way we assign to each face of the cube a unit element. The 4 numbers coding a face behave under multiplication as a unitary quaternion. The two interpretations of the Necker cube are like jumping from the positive faces to the negative faces.

We can add a fifth dimension:

$$\boldsymbol{a + bi + cj + dk + e\bar{0}}$$

Q.E.D

6. Conclusion and Application

The possibility to see the Necker cube in two different ways is analogous to the measurement process in quantum theory and the influence the observer has on the measurement's result. We also note here a possible application in the study of quantum mechanics - more specifically, on the zero point energy.

The physics world today holds a conflict between the idea of locality (the speed of light limit) and the idea of nonlocality. In our study, we propose a new coordinate system which may unify these two concepts under one framework.

In the development of special relativity, Einstein developed the idea of measuring the length of a body by measuring the time movement of light. Later, Minkowsky developed the geometry of the fourth dimension by using the Lorentz transformation. The new coordinate system, presented here, is based on a method, developed by Minkowsky, of measuring a distance by Manhattan metrics (Taxicab geometry).

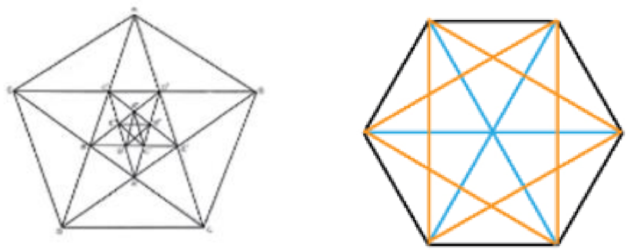

Figure 21. Two polygons and Paradigm shifts

In Pythagoras's time, the discovery of irrational numbers such as the golden ratio in the pentagon gave rise to a paradigm shift. We believe that a similar paradigm shift is possible today, through the discovery of non-local numbers, which appear on the 0 axis. This axis arises from a deep observation of the hexagon and its diagonals.

7. Acknowledgment
The writers of this article would like to thank Yale Landsberg, who developed the Beth-El Non-Standard Geometry of the Real Number Line as well as Martin Hay, the inventor of Chiralkine systems http://chiralkine.com .

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9. Appendix

A. Non-standard analysis
The earliest development of the infinitesimal calculus by Newton and Leibniz was formulated using expressions such as infinitesimal number. These formulations were criticized by George Berkley and others. There was a challenge to develop a consistent theory on analysis using infinitesimal and the first one who did that was Abraham Robinson.

Non-standard analysis is a mathematical theory that extends the real line number by adding to them infinite numbers and infinitesimal numbers. This new set of numbers called hyperreal number. Is marked as R *.

Robinson wrote: "The idea of infinitely small or *infinitesimal* quantities seems to appeal naturally to our intuition. At any rate, the use of infinitesimals was widespread during the formative stages of the Differential and Integral Calculus. As for the objection [...] that the distance between two distinct real numbers cannot be infinitely small, Gottfried Wilhelm Leibniz argued that the theory of infinitesimals implies the introduction of ideal numbers which might be infinitely small or infinitely large compared with the real numbers but which were *to possess the same properties as the latter*."

By looking on the set of all infinite sequences of real numbers named M, we define addition and multiplication operations between the sequences by action on each component. All fixed sequences (c,c,c,…) represent the real number c. The infinitesimal numbers are those sequences that are 0 from a certain place.

By using the axioms of set theory (Zorn's Lemma) we show the existence of a maximum set D while group R * is defined by the quotient of their division.

$$R^* = M / D.$$

It is important to note here the non-constructive character of this building.

B. Quaternion

Quaternions are a number system that extends the complex numbers This system invented by William Hamilton in 1843, and applied to mechanics in three-dimensional space. Hamilton knew that the complex numbers could be interpreted as a point in a plane. He was looking for a way to do the same for points in three-dimensional space. One day he discovered that he can solve the problem in 4-dimeson space.

The Quaternions have 4 unit elements 1,I,j,k with the following rules of multiplication.

×	**1**	***i***	***j***	***k***
1	1	*i*	*j*	*k*
i	*i*	−1	*k*	−*j*
j	*j*	−*k*	−1	*i*
k	*k*	*j*	−*i*	−1

A feature of quaternions is that the multiplication of two quaternions is noncommutative Hamilton defined a quaternion as the quotient of two directed lines in a three-dimensional space or equivalently as the quotient of two vectors.
Each element in the field can be written by using the following expression:
$a + bi + cj + dk$

Quaternions have many applications in theoretical and applied mathematics. For example, calculation involving three-dimensional rotation. Is also has many application in in physics and computer graphics.