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Synthesis of hybrid systems of pattern recognition on the basis of procedure of consecutive correction of decision functions

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Abstract. Hybrid systems of pattern recognition in the conditions of large volumes of the training selections and not stationarity of classification objects are offered. Asymptotic properties of their decision function are investigated.

1. Introduction

Perspective nature of sharing of parametrical and local methods of approximation was for the first time noted in work [1]. Its idea was used at restitution of stochastic dependences in the conditions of existence of partial data on their look [2, 3]. The developed models were called hybrid. This concept is natural and is based on procedure of correction of initial parametrical approximation. The next analogs of the used approach are semi-parametrical models of stochastic dependences [4].

The offered technique was developed at the solution of the two-alternate problem of pattern recognition in the conditions of existence of partial data on a type of the dividing surface equation [5, 6].

In this work the technique of synthesis of hybrid systems of pattern recognition in the conditions of large volumes of the training selections and not stationarity of objects of classification is offered. The basis of a technique is made by serial procedure of correction of decision functions.

2. Synthesis of hybrid system of pattern recognition

Let there is a training selection $V = \left(x^i, \sigma\left(x^i\right), i = \overline{1,n}\right)$ of large volume n, where $\sigma(x^i)$ - instructions on situation x^i belonging to one of two classes Ω_1, Ω_2. The objects of classification which are characterized by a feature set $x = \left(x_v, v = \overline{1,k}\right)$, can be nonstationary. These conditions are characteristic, for example, at data processing of remote sensing. Let's apply the principles of hybrid model operation at synthesis of system of pattern recognition.

The idea of the offered approach consists in realization of the following actions:

1. Let's carry out decomposition of the initial training selection V on N parts $V_j = \left(x^i, \sigma\left(x^i\right), i \in I_j\right)$, $j = \overline{1,N}$. The basis for realization of this operation is formation of selections V_j according to their accessory to various time frames of overseeing by signs of the classified objects.

2. By data V_1 construct the decisive rule

$$\overline{m}_{12}^{1}(x): \begin{cases} x \in \mathbf{\Omega}_1, \ if \quad \bar{f}_{12}^{1}(x) \geq 0, \\ x \in \mathbf{\Omega}_2, \ if \quad \bar{f}_{12}^{1}(x) < 0, \end{cases} \tag{1}$$

where the nonparametric assessment of the equation of the dividing surface is

$$\bar{f}_{12}^{1}(x) = \frac{1}{n_1 \prod\limits_{v=1}^{k} c_v(n_1)} \sum_{i \in I_1} \mathbf{\sigma}\left(x^i\right) \prod_{\mathbf{v}=1}^{k} \mathbf{\Phi}\left(\frac{x_v - x_v{}^i}{c_v(n_1)}\right);$$

n_1 - quantity of elements of set I_1.

Here values are $\mathbf{\sigma}\left(x^i\right)$=1, when $x^i \in \mathbf{\Omega}_1$ and $\mathbf{\sigma}\left(x^i\right)$=-1, when $x^i \in \mathbf{\Omega}_2$. Nuclear functions $\mathbf{\Phi}(u_v)$ in nonparametric assessment $\bar{f}_{12}^{1}(x)$ obey H: positive, the symmetric and the normalized [7]. Diffuseness coefficients c_v of nuclear functions decrease with body height n_1.

In this case the decisive rule (1) corresponds to criterion of a maximum of a posterior probability.

3. By results of computing experiment to create data $V_2 = \left(x^i, q_1\left(x^i\right), i \in I_2\right)$. They reflect difference of "decisions" $\overline{\mathbf{\sigma}}_1\left(x^i\right)$ of rule $\overline{m}_{12}^{1}(x)$ (1) from "instructions" $\mathbf{\sigma}\left(x^i\right)$ from the training selection V_1. Here values of the correcting function are

$$q_1\left(x^i\right) = \begin{cases} 0 \ \forall \ \mathbf{\sigma}\left(x^i\right) = \overline{\mathbf{\sigma}}_1\left(x^i\right), \\ \left(\bar{f}_{12}^{1}\left(x^i\right) + \mathbf{\Delta}\right) \ \forall \ \overline{\mathbf{\sigma}}_1\left(x^i\right) = -1 \ \ and \ \ \mathbf{\sigma}\left(x^i\right) = 1, \\ \text{-}\left(\bar{f}_{12}^{1}\left(x^i\right) + \mathbf{\Delta}\right) \ \forall \ \overline{\mathbf{\sigma}}_1\left(x^i\right) = 1 \ \ and \ \ \mathbf{\sigma}\left(x^i\right) = -1. \end{cases}$$

In the presence of a pattern recognition error the correcting function accepts value inverse on a sign of a nonparametric assessment of the equation of the dividing surface $\bar{f}_{12}^{1}(x)$ also exceeds it on a small $\mathbf{\Delta}$.

4. To estimate the correcting function $q_1(x)$ on selection V_2 on the basis of nonparametric regression

$$\bar{q}_1(x) = \frac{\sum\limits_{i \in I_2} q_1\left(x^i\right) \mathbf{\beta}_i(x)}{\sum\limits_{i \in I_2} \mathbf{\beta}_i(x)}, \ \mathbf{\beta}_i(x) = \prod_{\mathbf{v}=1}^{k} \mathbf{\Phi}\left(\frac{x_v - x_v{}^i}{c_v(n_2)}\right).$$

5. Then the intermediate hybrid algorithm of pattern recognition will register as

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$$\overline{m}_{12}^{2}(x): \begin{cases} x \in \mathbf{\Omega}_1, \; if \quad \bar{f}_{12}^{2}(x) \geq 0, \\ x \in \mathbf{\Omega}_2, \; if \quad \bar{f}_{12}^{2}(x) < 0, \end{cases}$$

where

$$\bar{f}_{12}^{2}(x) = \bar{f}_{12}^{1}(x) + \overline{q}_1(x).$$

The choice of best values $c_v(n_2), v = \overline{1,k}$ is carried out from a condition of a minimum of a statistical assessment of probability of an error of pattern recognition in the mode of "the sliding examination".

6. Generally serial correction procedure of the dividing surface equation has an appearance as

$$\bar{f}_{12}^{t}(x) = \bar{f}_{12}^{t-1}(x) + \overline{q}_{t-1}(x), \; t = \overline{2,N}\,. \tag{2}$$

Nonparametric assessment of the correcting function

$$\overline{q}_{t-1}(x) = \frac{\sum\limits_{i \in I_t} q_{t-1}\left(x^i\right)\boldsymbol{\beta}_i(x)}{\sum\limits_{i \in I_t} \boldsymbol{\beta}_i(x)}, \; \boldsymbol{\beta}_i(x) = \prod_{\nu=1}^{k} \mathbf{\Phi}\left(\frac{x_v - x_v{}^i}{c_v(n_t)}\right)$$

is formed according to computing experiment

$$q_{t-1}\left(x^i\right) = \begin{cases} 0 \; \forall \; \boldsymbol{\sigma}\left(x^i\right) = \overline{\boldsymbol{\sigma}}_{t-1}\left(x^i\right), \\ \left(\bar{f}_{12}^{t-1}\left(x^i\right) + \Delta\right) \; \forall \; \overline{\boldsymbol{\sigma}}_{t-1}\left(x^i\right) = -1 \; \; and \; \; \boldsymbol{\sigma}\left(x^i\right) = 1, \\ -\left(\bar{f}_{12}^{t-1}\left(x^i\right) + \Delta\right) \; \forall \; \overline{\boldsymbol{\sigma}}_{t-1}\left(x^i\right) = 1 \; \; and \; \; \boldsymbol{\sigma}\left(x^i\right) = -1. \end{cases}$$

Here $\overline{\boldsymbol{\sigma}}_{t-1}\left(x^i\right)$ - is "solution" of algorithm $\overline{m}_{12}^{t-1}(x)$ in conditions x^i, and $\boldsymbol{\sigma}\left(x^i\right)$ - are "instructions" from selection $V_t = \left(x^i, \boldsymbol{\sigma}\left(x^i\right), i \in I_t\right)$.

On the basis of decision functions $\bar{f}_{12}^{t}(x), \; t = \overline{2,N}$ serial synthesis of algorithms of pattern recognition is carried out

$$\overline{m}_{12}^{t}(x): \begin{cases} x \in \mathbf{\Omega}_1, \; if \quad \bar{f}_{12}^{t}(x) \geq 0, \\ x \in \mathbf{\Omega}_2, \; if \quad \bar{f}_{12}^{t}(x) < 0, \; t = \overline{2,N}. \end{cases} \tag{3}$$

Expression (2) is a nonparametric assessment of decision function

$$f_{12}^{t}(x) = f_{12}^{t-1}(x) + q_{t-1}^{0}(x). \tag{4}$$

For simplification of the subsequent transformations we will consider that

$$f_{12}^{t-1}(x) = p_1^{t-1}(x) - p_2^{t-1}(x).$$

In expression (4)

$$q_{t-1}^0(x) = \int q_{t-1}^0 \; p\left(q_{t-1}^0 \big/ x \right) d\, q_{t-1}^0 .$$

Hereinafter the infinite limits are passed.

3. Asymptotic properties of hybrid decision function

Let's consider asymptotic properties of one-dimensional hybrid model (2) type at a known collateral probability density $p(x)$ of distribution x in classes $\mathbf{\Omega}_1,\ \mathbf{\Omega}_2$. Let the algorithm of classification (3) correspond to criterion of maximum likelihood. The correcting function is optimum in sense of a minimum of a mean squared deviation between $q_{t-1}^0(x)$ and $\bar{q}_{t-1}(x)$. In these conditions nonparametric estimates of the equation of the dividing surface $f_{12}^{t-1}(x)$ and the correcting function $q_{t-1}^0(x)$ in (4) are presented in the form

$$\bar{f}_{12}^{t-1}(x) = \bar{p}_1^{t-1}(x) - \bar{p}_2^{t-1}(x),$$
$$\bar{q}_{t-1}(x) = \left(n_t c_t \; p_t(x)\right)^{-1} \sum_{i \in I_t} q_{t-1}\left(x^i\right) \mathbf{\Phi}\left(\frac{x - x^i}{c_t} \right). \tag{5}$$

Here $\bar{p}_j^{t-1}(x)$ - is nonparametric assessment of a probability density of distribution x in class $\mathbf{\Omega}_j$.

Let $f_{12}^{t-1}(x),\ p_t(x)$ are limited and continuous with all the derivatives to the second order inclusive. We will designate these conditions through G_{t-1}.

Theorem. Let: 1) the equation of the dividing surface $f_{12}^{t-1}(x)$ and a collateral probability density $p_t(x)$ of distribution x in classes obey G_{t-1}; 2) nuclear functions $\mathbf{\Phi}(u) \geq 0$ in nonparametric statistics $\bar{q}_{t-1}(x)$, $\bar{f}_{12}^{t-1}(x)$ obey H; 3) sequences $c_t = c(n_t) \to 0$, $\mathbf{\Delta} = \Delta(n_t) \to 0$, $\dfrac{\left(n_{1,t-1} + n_{2,t-1}\right)}{n_{1,t-1}\, n_{2,t-1}\, c_{t-1}} \to 0$, $n_t c_t \to \infty$ at $n_t \to \infty$, $n_{1,t-1} \to \infty$, $n_{2,t-1} \to \infty$.

Then the hybrid model $\bar{f}_{12}^{t}(x)$ possesses properties of an asymptotic unbiasedness and solvency of rather decision function $f_{12}^{t}(x)$.

Proof.

1) Let's consider expression
2)

$$M\left(\bar{f}_{12}^{t}(x)\right) = M\left(\bar{f}_{12}^{t-1}(x) + \bar{q}_{t-1}(x)\right) = M\left(\bar{f}_{12}^{t-1}(x)\right) + M\left(\bar{q}_{t-1}(x)\right), \tag{6}$$

where M - is the sign of expected value.

By definition of expected value, if $q_{t-1}\left(x^i\right) = q_{t-1}^0\left(x^i\right) + \Delta$, we have

$$M\left(\bar{q}_{t-1}(x)\right) = \left(c_t\ p_t(x)\right)^{-1} \int q_{t-1}^0(\tau)\, \Phi\left(\frac{x-\tau}{c_t}\right) p_t(\tau)\, d\tau +$$
$$+ \Delta \left(c_t\ p_t(x)\right)^{-1} \int \Phi\left(\frac{x-\tau}{c_t}\right) p_t(\tau)\, d\tau .$$

Let's carry out replacement of variables in integrals of the received expression $(x-\tau)c_t^{-1} = u$. Let's spread out functions $q_{t-1}^0(x - c_t u)$, $p_t(x - c_t u)$ in Taylor series in a point x. Taking into account properties of nuclear functions at rather great values n_t we will receive

$$M\left(\bar{q}_{t-1}(x)\right) = q_{t-1}^0(x) + \Delta + \frac{c_t^2}{2 p_t(x)}\left(\left(q_{t-1}^0(x)\ p_t(x)\right)^{(2)} + \Delta\ p_t^{(2)}(x)\right) + 0\left(c_t^4\right). \tag{7}$$

Here $\left(q_{t-1}^0(x)\ p_t(x)\right)^{(2)}$, $p_t^{(2)}(x)$ - are flexons of the corresponding functions. By symbol $0\left(c_t^4\right)$ are designated the composed order trifles c_t^4.

From the analysis of expression (7) when performing conditions $c_t \to 0$, $\Delta \to 0$ $\forall$ $n_t \to \infty$ property of asymptotic not shift of the correcting function $\bar{q}_{t-1}(x)$ follows.

Asymptotic expression first composed in (6) is determined by analogy

$$M\left(\bar{f}_{12}^{t-1}(x)\right) = M\left(\bar{p}_{1,t-1}(x) - \bar{p}_{2,t-1}(x)\right) =$$
$$= \frac{1}{c_{t-1}} \int \Phi\left(\frac{x-\tau}{c_{t-1}}\right) p_{1,t-1}(\tau)\, d\tau - \frac{1}{c_{t-1}} \int \Phi\left(\frac{x-\tau}{c_{t-1}}\right) p_{2,t-1}(\tau)\, d\tau .$$

After simple transformations we will receive

$$M\left(\bar{f}_{12}^{t-1}(x) - f_{12}^{t-1}(x)\right) \sim \frac{c_{t-1}^2}{2}\left(p_{1,t-1}^{(2)}(x) - p_{2,t-1}^{(2)}(x)\right). \tag{8}$$

In this expression $p_{j,t-1}^{(2)}(x)$ - density flexon of probability $p_{j,t-1}(x)$ on x, $j = 1, 2$.

From condition $c_{t-1} \to 0$ at $n_{1,t-1} \to \infty$, $n_{2,t-1} \to \infty$, property of asymptotic not shift of nonparametric statistics $\bar{f}_{12}^{t-1}(x)$ follows.

Taking into account expression (6) and the received results (7), (8) asymptotic not shift of hybrid decision function $\bar{f}_{12}^t(x)$ follows.

2) Let's investigate asymptotic properties of a mean square deviation

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$$M\left(f_{12}^{t}(x)-\bar{f}_{12}^{t}(x)\right)^2 = M\left(f_{12}^{t-1}(x)+q_{t-1}^{0}(x)-\bar{f}_{12}^{t-1}(x)-\bar{q}_{t-1}(x)\right)^2 =$$
$$= M\left(f_{12}^{t-1}(x)-\bar{f}_{12}^{t-1}(x)\right)^2 + M\left(q_{t-1}^{0}(x)-\bar{q}_{t-1}(x)\right)^2 +$$
$$+2M\left(\left(f_{12}^{t-1}(x)-\bar{f}_{12}^{t-1}(x)\right)\left(q_{t-1}^{0}(x)-\bar{q}_{t-1}(x)\right)\right). \quad (9)$$

Let's consider expression

$$M\left(q_{t-1}^{0}(x)-\bar{q}_{t-1}(x)\right)^2 = \left(q_{t-1}^{0}(x)\right)^2 - 2\, q_{t-1}^{0}(x)\,\bar{q}_{t-1}(x) + M\left(\bar{q}_{t-1}(x)\right)^2. \quad (10)$$

Carrying out similar transformations, we will calculate

$$M\left(\bar{q}_{t-1}(x)\right)^2 = \left(n_t c_t\; p_t(x)\right)^{-2}\left[\sum_{i\in I_t} M\left(\left(q_{t-1}^{0}(i)+\Delta\right)^2 \Phi^2\left(\frac{x-x^i}{c_t}\right)\right)+\right.$$
$$\left.+\sum_{i\in I_t}\sum_{\substack{j\in I_t\\ j\neq i}} M\left(\left(q_{t-1}^{0}(i)+\Delta\right)\Phi\left(\frac{x-x^i}{c_t}\right)\right) M\left(\left(q_{t-1}^{0}(j)+\Delta\right)\Phi\left(\frac{x-x^j}{c}\right)\right)\right]=$$
$$=\left(n_t c_t\; p_t(x)\right)^{-2}\left[n_t\left(\int\left(q_{t-1}^{0}(\tau)\right)^2 \Phi^2\left(\frac{x-\tau}{c_t}\right) p_t(\tau)\,d\tau +\right.\right.$$
$$\left.+2\Delta\int q_{t-1}^{0}(\tau)\,\Phi^2\left(\frac{x-\tau}{c_t}\right) p_t(\tau)\,d\tau + \Delta^2\int\Phi^2\left(\frac{x-\tau}{c_t}\right) p_t(\tau)\,d\tau\right)+$$
$$\left.+n_t\left(n_t-1\right)\left(\int\left(q_{t-1}^{0}(\tau)+\Delta\right)\Phi\left(\frac{x-\tau}{c_t}\right) p_t(\tau)\,d\tau\right)^2\right]\sim$$
$$\sim\frac{\left(q_{t-1}^{0}(x)\right)^2\int\Phi^2(u)\,du}{n_t c_t\; p_t(x)}+\left(q_{t-1}^{0}(x)+\Delta\right)^2+\frac{q_{t-1}^{0}(x)\,\Delta\, c_t^2}{p_t(x)}\left(\left(q_{t-1}^{0}(x)\; p_t(x)\right)^{(2)}+\Delta\; p_t^{(2)}(x)\right). \quad (11)$$

Substituting expressions (7), (11) in (10), at $n_t \to \infty$ we will receive

$$M\left(q_{t-1}^{0}(x)-\bar{q}_{t-1}(x)\right)^2 \sim \frac{\left(q_{t-1}^{0}(x)\right)^2\int\Phi^2(u)\,du}{n_t c_t\; p_t(x)}+$$
$$+\frac{q_{t-1}^{0}(x)\,\Delta\, c_t^2}{p_t(x)}\left(\left(q_{t-1}^{0}(x)\; p_t(x)\right)^{(2)}+\Delta\; p_t^{(2)}(x)\right)(1-\Delta)+\Delta^2.$$

When performing these transformations composed trifles $0(1/n_t)$, $0(c_t/n_t)$, $0(\Delta/(n_t c_t))$, $0\left(c_t^4\right)$ weren't considered.

Let's define asymptotic expression for a mean square deviation

$$M\left(f_{12}^{t-1}(x)-\bar{f}_{12}^{t-1}(x)\right)^2=M\left(p_{1,t-1}(x)-\bar{p}_{1,t-1}(x)\right)^2-$$
$$-2M\left(\left(p_{1,t-1}(x)-\bar{p}_{1,t-1}(x)\right)\left(p_{2,t-1}(x)-\bar{p}_{2,t-1}(x)\right)\right)+M\left(p_{2,t-1}(x)-\bar{p}_{2,t-1}(x)\right)^2. \quad (12)$$

Asymptotic expression for a mean square deviation $\bar{p}_{j,t-1}(x)$ from $p_{j,t-1}(x)$, $j=1,2$ is received in [8]

$$M\left(p_{j,t-1}(x)-\bar{p}_{j,t-1}(x)\right)^2\sim\frac{\int\mathbf{\Phi}^2(u)du}{n_{j,t-1}\,c_{t-1}}+\frac{c_{t-1}^4\int\left(p_{j,t-1}^{(2)}(x)\right)^2dx}{4}.$$

It is easy to show that

$$M\left(\left(p_{1,t-1}(x)-\bar{p}_{1,t-1}(x)\right)\left(p_{2,t-1}(x)-\bar{p}_{2,t-1}(x)\right)\right)\sim c_{t-1}^4\;p_{1,t-1}^{(2)}(x)\,p_{2,t-1}^{(2)}(x).$$

Then at rather great values of $n_{j,t-1}$, $j=1,2$ asymptotic expression for a mean square deviation (12) is presented in the form

$$M\left(f_{12}^{t-1}(x)-\bar{f}_{12}^{t-1}(x)\right)^2\sim\frac{\int\mathbf{\Phi}^2(u)du\left(n_{1,t-1}+n_{2,t-1}\right)}{n_{1,t-1}\,n_{2,t-1}\,c_{t-1}}. \quad (13)$$

The composed trifles $0\left(c_{t-1}^4\right)$ are excluded. Substituting expression (11), (13) in (9) at $n_{1,t-1}\to\infty$, $n_{2,t-1}\to\infty$, $n_t\to\infty$, we will receive

$$M\left(f_{12}^{t}(x)-\bar{f}_{12}^{t}(x)\right)^2\sim\frac{\int\mathbf{\Phi}^2(u)du\;\left(n_{1,t-1}+n_{2,t-1}\right)}{n_{1,t-1}\,n_{2,t-1}\,c_{t-1}}+$$
$$+\frac{\left(q_{t-1}^0(x)\right)^2\int\mathbf{\Phi}^2(u)\,du}{n_tc_t\;p_t(x)}+\left(q_{t-1}^0(x)+\Delta\right)^2+\frac{q_{t-1}^0(x)\,\Delta\,c_t^2}{p_t(x)}\left(q_{t-1}^0(x)\,p_t(x)\right)^{(2)}+\Delta^2.$$

In the course of transformations the composed trifles $0\left(c_{t-1}^4\right)$, $0\left(c_t^2\;\Delta\right)$ weren't considered. From conditions $c_t\to 0$, $\dfrac{\left(n_{1,t-1}+n_{2,t-1}\right)}{n_{1,t-1}\,n_{2,t-1}\,c_{t-1}}\to 0$, $n_t\,c_t\to\infty$, $\Delta\to 0$ at $n_t\to\infty$, $n_{1,t-1}\to\infty$, $n_{2,t-1}\to\infty$, convergence in mean square statistics $\bar{f}_{12}^{t}(x)$ follows. The last statement and asymptotic not shift of hybrid model $\bar{f}_{12}^{t}(x)$ defines property of its solvency.

4. Acknowledgment

Hybrid systems provide the effective solution of problems of pattern recognition in the conditions of non-uniform data of large volume. Sources of the non-uniform training selection are: not stationarity of objects of research, existence of the completed admissions of data and their polytypic character. The structure of hybrid system of pattern recognition in a two-alternative problem of classification is based on consecutive procedure of correction of the equation of the dividing surface. At each its stage the intermediate hybrid model of the equation of the dividing surface is specified by the correcting function. Estimation of the correcting function is carried out by results of computing experiment with the intermediate decisive rule and use of one of uniform parts of the training selection. As its model nonparametric regression is used. The hybrid equation of the dividing surface possesses properties of asymptotic convergence.

The type of the correcting function and feature of the training selection generate family of hybrid systems of pattern recognition . Use of consecutive procedure of formation of structure of hybrid system allows to organize the accounting of partial aprioristic data on a type of the equation of the dividing surface. Further development of a technique of synthesis of hybrid system of pattern recognition is connected with its generalization on a multialternative problem of classification and the analysis of its properties in a multidimensional case.

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