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A mathematical model of the inline CMOS matrix sensor for investigation of particles in hydraulic liquids

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Abstract. One of the most effective ways to diagnose the state of hydraulic system is an investigation of the particles in their liquids. The sizes of such particles range from 2 to 200 µm and their concentration and shape reveal important information about the current state of equipment and the necessity of maintenance. In-line automatic particle counters (APC), which are built into hydraulic system, are widely used for determination of particle size and concentration. These counters are based on a single photodiode and a light emitting diode (LED); however, samples of liquid are needed for analysis using microscope or industrial video camera in order to get information about particle shapes. The act of obtaining the sample leads to contamination by other particles from the air or from the sample tube, meaning that the results are usually corrupted. Using the CMOS or CCD matrix sensor without any lens for in-line APC is the solution proposed by authors. In this case the matrix sensors are put into the liquid channel of the hydraulic system and illuminated by LED. This system could be stable in arduous conditions like high pressure and the vibration of the hydraulic system; however, the image or signal from that matrix sensor needs to be processed differently in comparison with the signal from microscope or industrial video camera because of relatively short distance between LED and sensor. This paper introduces mathematical model of a sensor with CMOS and LED, which can be built into hydraulic system. It is also provided a computational algorithm and results, which can be useful for calculation of particle sizes and shapes using the signal from the CMOS matrix sensor.

1. Introduction

Most automatic particle counters that are built into hydraulic systems (in-line) are based on a single photodiode as a photo-sensor. This approach has advantages: the relatively small size of the sensor, low cost and reliability; however, it is not possible to distinguish particle shapes, which can be considered as important information about processes in entire hydraulic systems [1, 2]. In addition, photodiodes confine the sensitivity and maximum concentration limit. The present authors propose the use of a matrix sensor in order to deal with these problems. A matrix sensor consists of several relatively small photodiodes with integration scheme, digitizer and interface on the crystal. CMOS and CCD matrix sensors are widely used for obtaining pictures in different applications.

There are several mathematical models of the data provided by photoelectric particle sensors, which are based on the effect of illumination of the photodiode by LED [3–5]. All of these models utilize the light blocking method [6, 7]. In this method the estimation of particle sizes is calculated by relating the change of light intensity to the illuminated cross-sectional area of the particle in the sensor volume. This change in light intensity is estimated using a photodiode and some dedicated electronics. Figure 1 reflects the typical composition of such sensors. They consist of glass tube coated by nontransparent material with a small aperture in its centre. Light from the LED goes through the aperture to the photo-sensor.

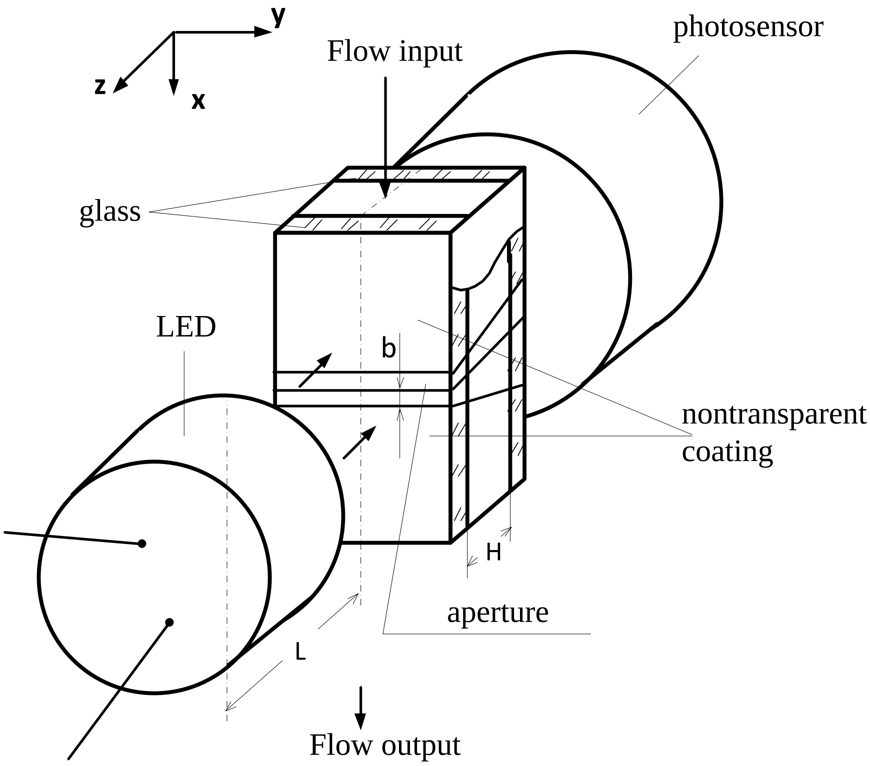

Figure 1. Typical composition of the optoelectronic part of the inline sensor.

In [5], the model based on geometrical optics has been described. This model can be used for the determination of sensitivity, and estimation of the accuracy influenced by particles position inside the volume. The sensor proposed by the authors is based on the matrix CMOS photo-sensor, instead of using a single photodiode, and it has quite a similar composition but with the exception of the necessity of an aperture. In this case, the mathematical model should be different in comparison with model in [5], because of the signal that is obtained from CMOS matrix and the absence of the aperture.

2. The mathematical model of the inline CMOS matrix sensor

The mathematical model described in this paper is based on the assumption that the LED and CMOS matrix sensor can be represented as equivalent planes that consist of several unit elements as the relatively small pixels, as illustrated in figure 2. It is certainly appropriate for the CMOS and can be applicable for the LED. The LED consists of a lens and a relatively small crystal in its focus. This composition can be considered optically as a significantly larger area at the higher distance, emitting through the lens aperture [5].

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The LED is introduced as the plane with dimensions of A_S and B_S. The Cartesian coordinates of lower left corner are denoted as (x_{S0}, y_{S0}, H). The LED unit element has dimensions dx and dy with coordinates depending on its position that can be described as (x_S, y_S, H). The size of matrix sensor is represented by A_R and B_R. The lower left matrix corner has Cartesian coordinates (0, 0, 0). Each pixel has dimensions Δx and Δy with coordinates (x_m, y_m, 0). A particle that is placed between the LED and the matrix has coordinates (x_p, y_p, z_p) of its mass centre.

The output signal from the matrix sensor can be evaluated as two dimensional signal obtained from each pixel U_{mm}. In the case of particle absence it can be determined using following relation [8]:

$$U_{mm} = k\int_{0}^{t_{\exp}}\int_{x_m}^{x_m+\Delta x}\int_{y_m}^{y_m+\Delta y} E(x_1,y_1,t)P(x_1,y_1)dx_1dy_1dt\,, \tag{2.1}$$

where $t_{\exp}$ stands for exposure time, $E(x_1, y_1, t)$ denotes irradiance of point with coordinates (x_1, y_1), $P(x_1, y_1)$ stands for special conversion function that is defined by the quantum efficiency of the photo-sensor and other characteristics, settings of the matrix etc., and k denotes the coefficient of proportionality.

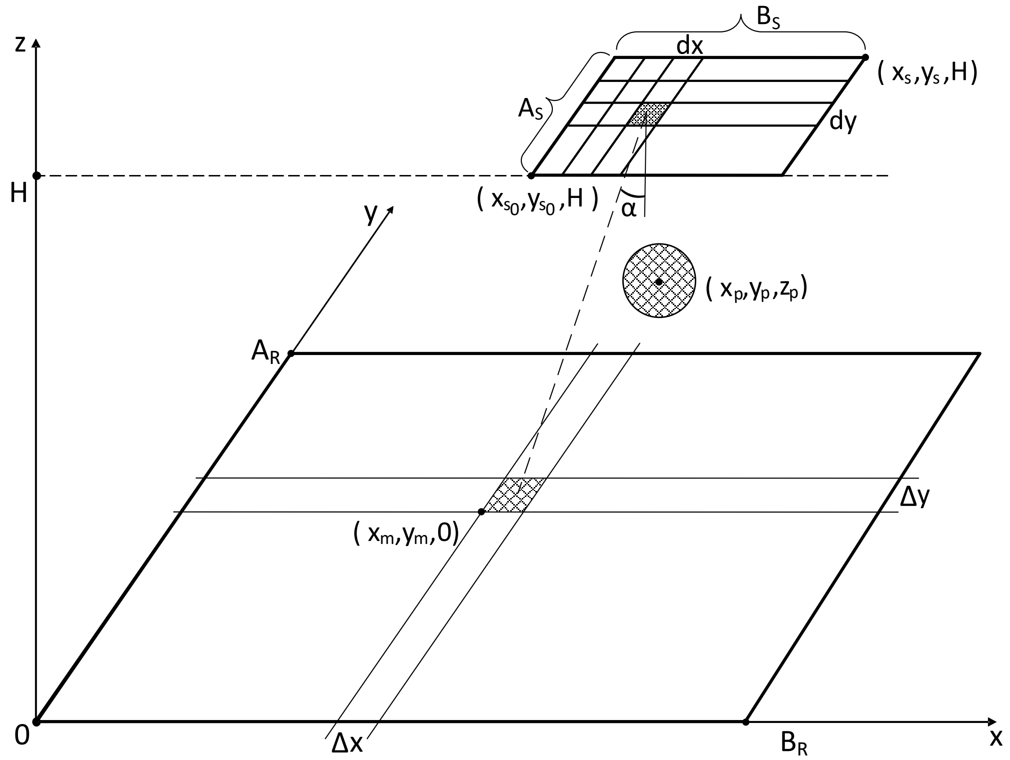

Figure 2. The geometry of the inline CMOS matrix sensor.

The total irradiance of the pixel can be determined in case of constant radiant exitance of light source [8] and absence of particle using following equation [9]:

$$E(x_1,y_1,t)=\int_{x_{s0}}^{x_{s0}+B_s}\int_{y_{s0}}^{y_{s0}+A_s}\frac{M\cdot cos^2(\alpha)}{l^2}dydx\,, \tag{2.2}$$

where M stands for radiant exitance; α denotes angle between perpendicular line drawn to the surface of the photo-sensor and line that connects the light source unit and the unit of the photo-sensor; l stands for distance between these units. As follows from figure 2, the α angle can be determined geometrically using following equation:

$$\alpha=\arccos\frac{H}{\left[(x-x_1)^2+(y-y_1)^2+H^2\right]^{\frac{1}{2}}}\,, \tag{2.3}$$

If the light unit source illuminates according to the Lambert law [10, 11], the irradiance $E(x_1,y_1,t)$ made by that unit source with radiant exitance M_1 can be estimated as [12]:

$$E_1=\frac{M_1}{2}\left(\frac{dx_s}{\sqrt{H^2+dx_s{}^2}}arctg\frac{dy_s}{\sqrt{H^2+dx_s{}^2}}+\frac{dy_s}{\sqrt{H^2+dy_s{}^2}}arctg\frac{dx_s}{\sqrt{H^2+dy_s{}^2}}\right), \tag{2.4}$$

The total irradiance of the entire matrix sensor can be calculated using integration according to equation (2.2).

If there is a particle between light source and matrix, the relation (2.1) should be rewritten. In order to do so, let us propose that particle absorbs light totally, which means that if the light ray crosses the particle shape, the irradiance made by that light ray is null. In case of not crossing, the irradiance can be estimated using (2.4), therefore, (2.1) can be transformed as follows:

$$U_{mm}=k\int_0^{t_{\exp}}\int_{x_m}^{x_m+\Delta x}\int_{y_m}^{y_m+\Delta y}E(x_1,y_1,t)P(x_1,y_1)H(\xi)dx_1dy_1dt\,, \tag{2.5}$$

where $H(\xi)$ is the unit step function [13], ξ denotes the fact of whether the light ray crosses the particles or not. ξ depends on $x,y,x_1,y_1,x_p(t),y_p(t),z_p(t)$. The geometrical task of determining ξ can be considered as the task of crossing between the line and surface (of particle) [14].

The line that connects light source unit (x_S, y_S, H) with matrix unit (x_m, y_m, 0) can be described using following relation [14]:

$$\frac{x-x_s}{x_m-x_s}=\frac{y-y_s}{y_m-y_s}=\frac{z-z_s}{z_m-z_s} \tag{2.6}$$

The (2.6) can be represented in a parametric form with w parameter:

$$\begin{cases} x = (x_m - x_s)w + x_0, \\ y = (y_m - y_s)w + y_0, \\ z = (z_m - z_s)w + z_0, \end{cases} \tag{2.7}$$

In case of crossing between the line and the surface, the following equation should have a real solution:

$$\xi\left[(x_m - x_s)w + x_0 - x_p, (y_m - y_s)w + y_0 - y_p), (z_m - z_s)w + z_0 - z_p\right] = 0 \tag{2.8}$$

The implemented algorithm was based on equations (2.1) – (2.8) and the method of ray tracing. The result is illustrated as a picture, showing the shadow from the particle. The signal without particle (calculated using (2.2) – (2.4)) was also used as a reference in order to determine the equivalent diameter D_e of the particle [3], [15]. The equivalent diameter means the diameter of the sphere with the same volume as a real particle has. The difference between signal with and without the particle can be used for particle size estimation.

The proposed simulation is aimed to determine relation between equivalent diameter of the particle and inline sensor (figure 1) parameters. These parameters are sizes of the light plane A_S and B_S, the distance between this plane and matrix sensor H, the distance between particle and matrix sensor z_p.

3. The results of mathematical modelling

The real sensor dimensions are following: the minimum distance between the particle and matrix sensor is determined by the thickness of matrix's glass (roughly 1 mm) and the thickness of glass plate (roughly 1 mm). The maximum distance measured from the particle to the matrix depends on dimension of the channel (roughly 1 mm), therefore, the particle z_p coordinate varies from 2 to 3 mm. We can present H and z_p in the one figure because they are measured along the same z axis.

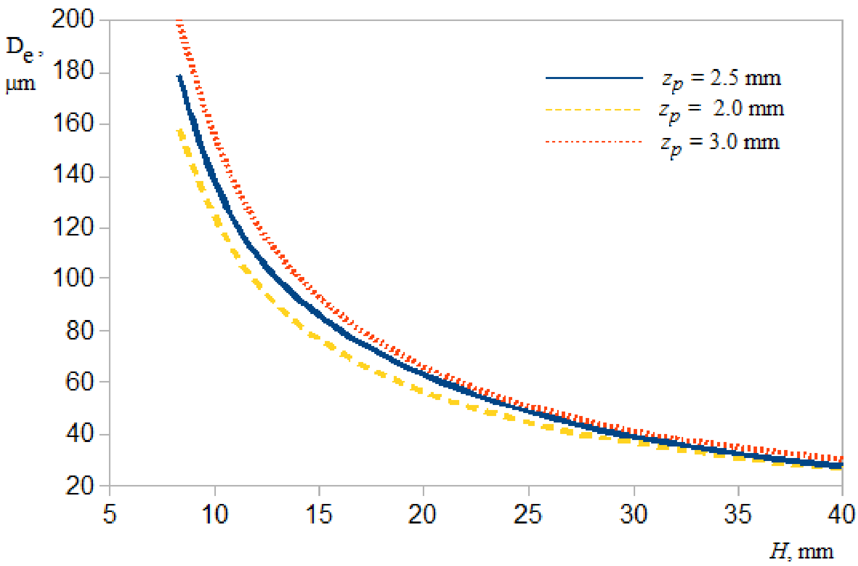

Figure 3. The relationship between equivalent diameter of particle and the distance between matrix sensor and source of light.

Figure 3 presents the relation between equivalent diameter D_e and distance between the matrix and the source of light in case of using 10 µm equivalent diameter of the particle and fixed z_p-coordinates of the particle. The model was calibrated at the H distance of 100 mm.

As follows from figure 3, the equivalent diameter increases significantly depending on the contracted distance between matrix sensor and source of light H from initial size of 10 µm at 100 mm (not shown on figure) to approximately 160-200 µm at the distance 10 mm. The increasing dependence of the z_p-coordinate should also be mentioned.

Figure 4 illustrates the dependence between the equivalent and the real diameter D_r of the particle at the distance 10 mm and 30 mm. The sensor was calibrated for $z_p = 2$ mm. Figure 4 shows that the particle with diameter 20 µm can be incorrectly measured as particle with diameter ~25 µm in case of $H = 10$ mm and $z_p = 3$ mm and roughly the same size if $H = 30$ mm.

Figure 5 represents the two dimensional discrete signal obtained from the matrix sensor as a result of the simulation using the same conditions as in figure 4.

According to the figure 5 it should be mentioned that the shape of the particles was more easily recognized in the case of 30 mm distance between the matrix and the source of light. The dimensions of the central spot between (c) and (d) vary more than between (a) and (b).

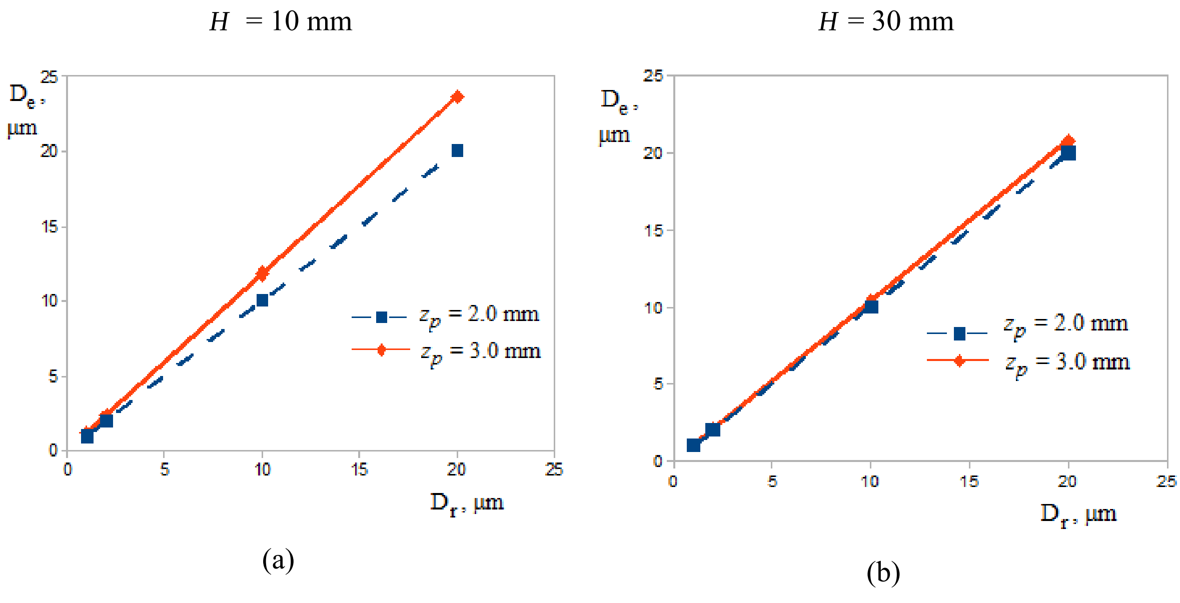

Figure 4. The relationship between the equivalent diameter of the particle and the real diameter. These graphs were obtained for a different distance H between matrix sensor and light source.

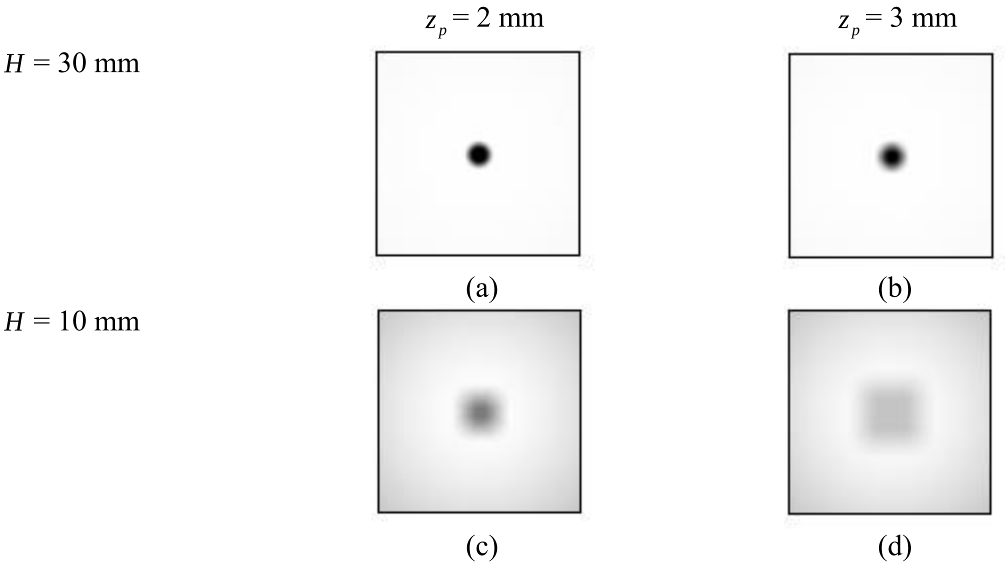

Figure 5. The two dimensional discrete signal obtained from the matrix as a result of simulation.

Figure 6 illustrates the relationship between the equivalent diameter and the area of the light source, S. This area can be determined by the multiplication of A_S and B_S . As follows from figure 6, the equivalent diameter increases simultaneously with increasing light source area, and can be approximated using the square root function. As follows from Figure 7, the increased area of the light source influences the particle shape.

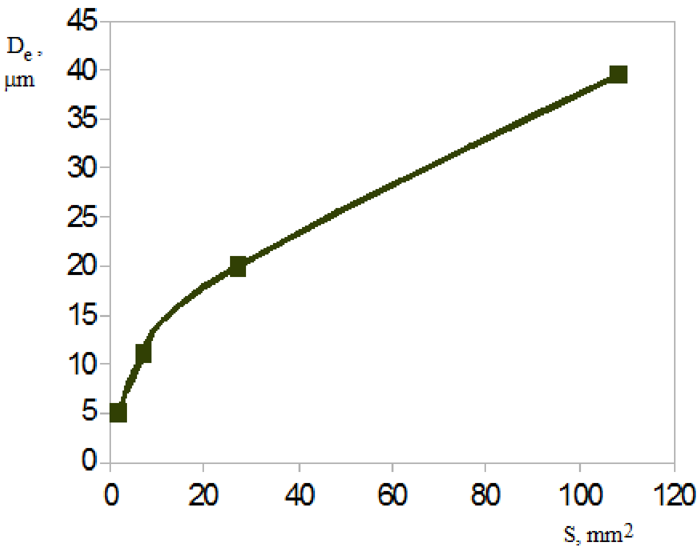

Figure 6. The relationship between the equivalent diameter of particle and the area of the light source.

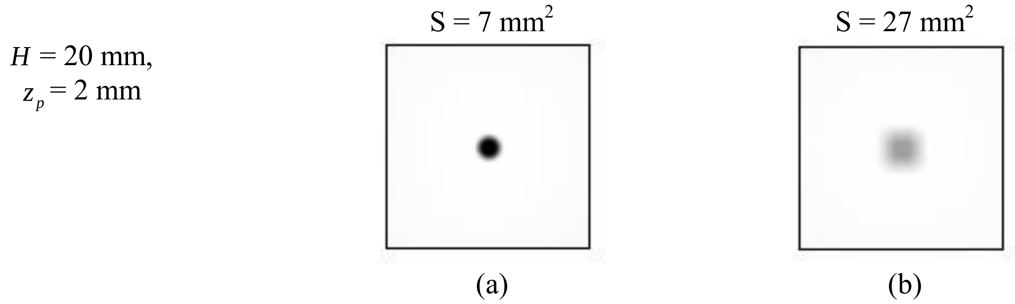

Figure 7. The two dimensional discrete signal obtained from the matrix as a result of simulation.

4. Discussion

As follows from figure 3, the signal level measured as D_e increased significantly as a consequence of decreased distance between light source and the matrix. For example, it was only 10 µm of equivalent diameter at the distance of 100 mm and about 200 µm at the distance of 10 mm. We have here an increase more than 20 times. This dependence might be explained and approved by the inverse-square law. This might be used to improve the sensitivity of the sensor; however, the short distance between the light source and the matrix decreases the accuracy of the sensor. Generally, accuracy is influenced by several factors: one of the most important is the random position of the particle in the sensor channel caused by changing of the flow conditions. The distance between light source and matrix sensor influences the sensitivity of the sensor to the changing of particle z_p coordinate. The differences in z_p coordinate for particles in the sensor channel cannot be lower than approximately 1 mm. Figure 3 shows that particles with the same actual size will be determined as particles with different sizes depending on the distance between the matrix sensor and the source of light. The closer the light source to the matrix, the higher the error in particle size estimation. This is illustrated more clearly on figure 4, where the comparison was made between particle size estimation for two fixed distances 10 mm and 30 mm. The sensor was calibrated at z_p coordinate 2 mm. If the particle flows at the 3 mm z_p coordinate, the measured particle size will be roughly 25 µm instead of the actual size of 20 µm, for the distance H = 10 mm; however, at the distance 30 mm the measured particle size (approximately 20.5 µm) will be almost correct. The difference looks clearer for relatively big particles, therefore, it could be recommended to choose an appropriate distance between the light source and the matrix sensor in order to satisfy both sensitivity and accuracy of the sensor.

The distance H between the light source and the matrix influences the apparent particle shape. As follows from figure 6, for the H distance 10 mm, the shapes appear to be smoother in comparison with those for the distance 30 mm. For the case when z_p coordinate varies from 2 mm to 3 mm, the shadow size changes more in case of H = 10 mm.

An additional parameter that can be changed in order to improve the sensor sensitivity, is the light source area S. As shown on figure 6, the equivalent diameter depends as the square root of the area of the light source. This can also be predictable because the irradiance increases proportionally with the area of the light source, and the signal rises proportionally with the area of the particle. The equivalent diameter should be related to the light source area as the square root; however, the increased source area might lead to the same drawbacks as was shown in case of short H distance, for example, the smoothed particle shape. This effect is illustrated on figure 7. In case of S = 7 mm^2 the shape of the particle shadow is more accurate in comparison with S = 27 mm^2. The other parameters of the sensor was fixed H = 20 mm and z_p = 2 mm.

The results obtained during simulation had good correlation with those anticipated by the basic laws of optics. This can be considered as a positive result of testing the proposed model. The results that were obtained using the proposed mathematical model can advise the implementation during the development of a real inline CMOS matrix sensor by providing an estimate of its characteristics and supporting the choice of the right parameters. First of all, using a relatively short distance between light source and the matrix can be proposed, because this approach leads to the increased signal from a particle; however, using shorter distance might cause decrease of sensor accuracy influenced by increased sensitivity to the z_p coordinate of the particle. It also affects the prediction of the shape of the particle. In addition, the increased area of the light source can be suggested in order to improve the sensitivity of the sensor; however, as shown in figure 7, an increased light source area leads to decreased sensor accuracy similar to the case of contracting the distance between light source and matrix.

5. Conclusion

A mathematical model of the in-line sensor based on matrix photo-sensor has been proposed. The proposed model is based on the assumption that the LED and CMOS matrix sensor can be represented as equivalent planes, with the particle between them, without utilizing any optical elements. The simulation results obtained using ray tracing algorithm show that the optimal characteristic of the sensor can be found by choosing the appropriate distance between the LED and CMOS matrix, as well as the size of LED. It was shown that the signal level is controversial to the accuracy of the sensor. The concrete values of sensor parameters should be determined using proposed mathematical model and obtained results.

An additional investigation should be performed for different pixel sizes[16]. The state-of-the-art matrix sensors have the minimum pixel size lower than 1 μm. In this paper the authors simply intended to demonstrate the principles, but did not consider the effect of pixel size. Further research should take into account the limit of pixel size.

The proposed model should be implemented with more accuracy in case of particle sizes lower than several micrometers, because it might be influenced by interaction with the light. This interaction is described generally by Mie theory and can lead to a bigger shadow than predicted by the classical model. In this study we investigate particles bigger than 10 μm, which have insignificant interaction with light. Further research will be focused on investigation of the dependence between particle sizes in range 0.5 to 10 μm and the particle shadow dimension and its optical density when the light wavelength is fixed.

Another issue is the influence of the liquid flow. This study was aimed to investigate static images; however, there are several cases that should be considered for different particle velocities and exposure time. This might lead to the necessity of processing particle tracks.

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