

Aircraft nonlinear optimal control using fuzzy gain scheduling

I F Nusyirwan* and Z Y Kung

Department of Aeronautics, Automotive & Ocean Engineering, Universiti Teknologi Malaysia, Malaysia

*istaz@utm.my

Abstract. Fuzzy gain scheduling is a common solution for nonlinear flight control. The highly nonlinear region of flight dynamics is determined throughout the examination of eigenvalues and the irregular pattern of root locus plots that show the nonlinear characteristic. By using the optimal control for command tracking, the pitch rate stability augmented system is constructed and the longitudinal flight control system is established. The outputs of optimal control for 21 linear systems are fed into the fuzzy gain scheduler. This research explores the capability in using both optimal control and fuzzy gain scheduling to improve the efficiency in finding the optimal control gains and to achieve Level 1 flying qualities. The numerical simulation work is carried out to determine the effectiveness and performance of the entire flight control system. The simulation results show that the fuzzy gain scheduling technique is able to perform in real time to find near optimal control law in various flying conditions.

1. Introduction

Flight dynamics is nonlinear with the motion of an airplane subjected to inherent stability and external disturbance [1]. Hence a robust flight control system is important to stabilize and ensure favourable flight performance. Present generation aircraft has been augmented with sophisticated flight control system. This is especially true for fighter aircraft, which is designed to be inherently unstable to allow it to perform extreme manoeuvres. In view of this, optimal control and gain scheduling method are implemented to achieve the flight stability and controllability.

Linear quadratic (LQ) optimization is commonly utilised in flight control system. The general idea of LQ control is to minimize the performance index parameterized by the weighting matrices [2]. In performing stability augmented system (SAS) in flight control, the LQ optimization for tracking a command system is employed [3]. In addition, publications by [4, 5] shared a similar methodology in using LQ optimization to perform nonlinear control and they showed excellent controllability. Besides, gain scheduling method is commonly used to deal with nonlinear system and this approach reduces computational burden associated with the local linear system [6]. In agreement with research done by [7], the gain scheduling composes three important aspects: computation of linear parameter-varying models, design of linear control system and interpolation of controller gains.

In consonance with the second aspect of gain scheduling, similarity was found in using H-infinity controller together with gain scheduling method to improve the global stability [8, 9]. Meanwhile, the research study presented in [10] has highlighted the third aspect of gain scheduling, in which a global polynomial variable scheduling was examined.



In contrast, it is also stated that the conventional gain scheduling is time-consuming due to the ad-hoc nature implies more iterative work and requires more operating points [11]. Taking advantages of the research done in [12], the fuzzy logic is implemented in line with the gain scheduling. This is due to the fact that fuzzy inference system is prone to human understandable logic. Besides, fuzzy logic is widely used because of its easy realization and robustness [13]. Credible researches based on fuzzy gain scheduling could be found since the 1990's [14, 15] and similar approach was revised a decade later in [5, 11]. As presented by reference publications [5, 11, 14 and 15], they showed the capability of fuzzy gain scheduling in performing stable flight control. In addition, the stability based on fuzzy system with optimal feedback could also be found in [5, 16].

To date and to the best of author's knowledge, the work on the fuzzy gain scheduling on highly nonlinear flight dynamic is still insufficient. Previous publications showed that the application of fuzzy gain scheduling on a wide range of operating region. In fact, the most unstable performance could be observed at low speed manoeuvring flight. Therefore, the aim of this article is to perform fuzzy gain scheduling over highly nonlinear flight dynamics through examination of the eigenvalues. LQ Optimal control is applied on the particular operating point in order to search for the most suitable controller gains. The output is interfaced with the fuzzy gain scheduler to perform global flight control system. In this paper, full six-degree-of-freedom equations of motions are involved. Hence, for better illustration and understanding, Table 1 is provided to show all related abbreviations and symbols.

Table 1. List of abbreviations and symbols

Abbreviations	Definition
6DOF	six degrees of freedom
FGS	fuzzy gain scheduler
SAS	stability augmented system
LQ	linear quadratic
Symbols	Definition
$\dot{U}, \dot{V}, \dot{W}$	velocity derivatives in body frame with components in body system
U, V, W	body axis wind velocity components
$\dot{V}_t, \dot{\alpha}, \dot{\beta}$	wind axis velocity derivatives in body frame with components in body system
ϕ, θ, ψ	Euler angles
$\dot{\phi}, \dot{\theta}, \dot{\psi}$	Euler angles derivatives
P, Q, R	body axis angular velocity components
$\dot{P}, \dot{Q}, \dot{R}$	angular velocity derivatives in body frame with components in body system
X, Y, Z	body axis x, y and z component forces
l, m, n	body axis rolling, pitching and yawing moments
V_t	absolute wind
M	aircraft mass
g_d	gravity
$J_{x,y,z,xz}$	second moment of inertia corresponding to its axis
a_x, a_y, a_z	body axis x, y, and z component accelerations
$\delta_e, \delta_a, \delta_r, \delta_T$	elevator, aileron, rudder and throttle
\dot{x}	state derivative variables: $\dot{x} = [\dot{V}_t, \dot{\alpha}, \dot{\beta}, \dot{\phi}, \dot{\theta}, \dot{\psi}, \dot{P}, \dot{Q}, \dot{R}]^T$
x	states variables: $x = [V_t, \alpha, \beta, \phi, \theta, \psi, P, Q, R]^T$
y	output state variables: $y = [a_x, a_y, a_z]^T$
u	control state variables: $u = [\delta_e, \delta_a, \delta_r, \delta_T]^T$
r, z	reference input state variables performance output state variables
K	controller gains: $K = [K_\alpha, K_q, K_i]^T$
Q_m, R_m	state weighting matrix and control weighting matrix
ω_n, ξ	natural frequency, damping ratio

2. Nonlinear flight dynamics

Based on [3], the F-16 jet fighter aerodynamics data is employed and the 6DOF nonlinear equations of motion are derived based on first-order differential equation. The equations are as the following.

2.1. Force equations

$$\dot{U} = RV - QW - g_d \sin \theta + X/M \quad (1)$$

$$\dot{V} = -RU + PW + g_d \sin \phi \cos \theta + Y/M \quad (2)$$

$$\dot{W} = QU - PV + g_d \cos \phi \cos \theta + Z/M \quad (3)$$

written in wind axis,

$$\dot{V}_t = (U\dot{U} + V\dot{V} + W\dot{W})/V_t \quad (4)$$

$$\dot{\alpha} = (U\dot{W} - W\dot{U})/(U^2 + W^2) \quad (5)$$

$$\dot{\beta} = (\dot{V}_t - V\dot{V}_t)/(V_t \sqrt{U^2 + W^2}) \quad (6)$$

2.2. Euler kinematic equations

$$\dot{\phi} = P + \tan \theta (Q \sin \phi + R \cos \phi) \quad (7)$$

$$\dot{\theta} = Q \cos \phi - R \sin \phi \quad (8)$$

$$\dot{\psi} = (Q \sin \phi + R \cos \phi)/\cos \theta \quad (9)$$

2.3. Moment equations

$$\Gamma \dot{P} = J_{xz} [J_x - J_y + J_z] PQ - [J_z (J_z - J_y) + J_{xz}^2] QR + J_z \ell + J_{xz} n \quad (10)$$

$$J_y \dot{Q} = (J_z - J_x) PR - J_{xz} (P^2 - R^2) + m \quad (11)$$

$$\Gamma \dot{R} = [(J_x - J_y) J_x + J_{xz}^2] PQ - J_{xz} [J_x - J_y + J_z] QR + J_{xz} \ell + J_x n \quad (12)$$

where,

$$\Gamma = J_x J_z - J_{xz}^2$$

3. Linearization

As mentioned in the second aspect of gain scheduling, the flight control is performed under linear operating system or steady state condition, hence linearization is necessary. At equilibrium points, the trim function is constructed to determine the specific state variables and the control variables, which converge the state derivatives into zero. The constraint equations describe the relationship of 6DOF nonlinear equations and it is modelled based on analysis by [17]. To analyse with the nine equations, the numerical linearization method is utilized [3, 18]. A total number of 21 linear systems are selected according to various airspeed and altitude. The outcome of the linearized equations of motion are described in Equation 13.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (13a)$$

$$y = Cx + Du \tag{13b}$$

Taking steady state condition at sea level for airspeed of 300 ft/s, the linearized matrices are determined and further decomposed into longitudinal state dependent parameters, the linearized state space matrices are as presented in Equation 14.

$$\begin{bmatrix} \dot{Vt} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -0.028 & -6.2682 & -32.17 & -2.6422 \\ -0.0007 & -0.6099 & 0 & 0.9012 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0710 & 0 & -0.7192 \end{bmatrix} \begin{bmatrix} Vt \\ \alpha \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} 0.0112 \\ -0.0013 \\ 0 \\ -0.0628 \end{bmatrix} [\delta_e] \tag{14a}$$

$$[a_z] = [0.0066 \quad 5.6540 \quad 0 \quad 0.9238] \begin{bmatrix} Vt \\ \alpha \\ \theta \\ q \end{bmatrix} + [0.0119][\delta_e] \tag{14b}$$

4. Linear quadratic optimal control

Linearized equations of motion is constructed based on Equation 13 with respect to specific airspeed and altitude. The linear LQ optimal controller is designed based on the highly nonlinear region and this can be done by examining the root locus plot for the longitudinal mode of motion [1]. Based on the selected flying conditions, the root locus plots behave irregular pattern as shown in Figure 1 and this implies the nonlinearity of flight dynamics.

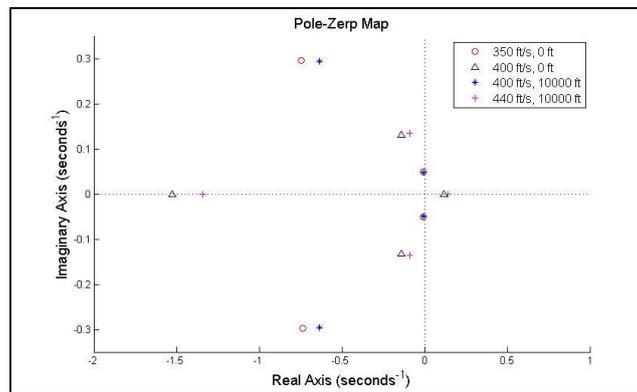


Figure 1. Root locus plots for several operating points

Taking [2, 3] as references, the optimal control for pitch rate SAS can be formulated based on Figure 2. The longitudinal SAS is performed by controlling the pitch rate. Pilot input or the reference pitch rate command is given to manoeuvre the jet fighter. In addition, the characteristics of actuator and alpha sensor are modelled in term of transfer function. This is to obtain more realistic results in performing the stability augmentation.

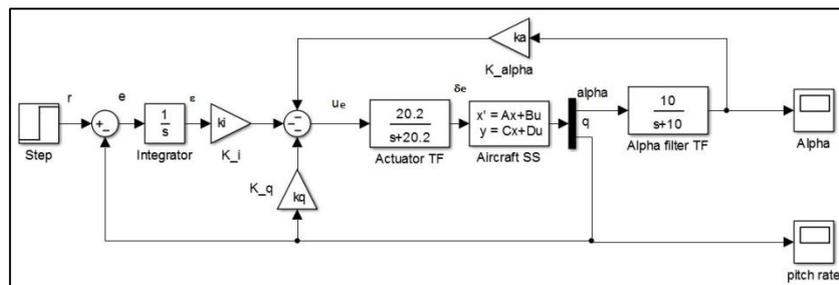


Figure 2. Pitch stability augmented system block diagram

Based on Figure 2, the LQ optimal control for command tracking is designed and derived. The mathematical formulation can be simplified in linear state space model as shown in Equation 15 and the control is static output feedback as listed in Equation 16.

$$\dot{x} = Ax + Bu + Gr \quad (15a)$$

$$y = Cx + Du + Fr \quad (15b)$$

$$z = Hx \quad (15c)$$

$$u = -Ky \quad (16)$$

where,

$$x = \begin{bmatrix} \alpha \\ q \\ \delta_e \\ \alpha_F \\ \epsilon \end{bmatrix}, \quad u = [u_e], \quad r = [q_c], \quad z = [q], \quad y = \begin{bmatrix} \alpha_F \\ q \\ \epsilon \end{bmatrix}, \quad K = \begin{bmatrix} K_\alpha \\ K_i \\ K_q \end{bmatrix}$$

Optimal control using linear quadratic for command tracking simplify the tuning parameters. By adding the time-dependent weighting component [2, 3], the general performance index or cost function is described in Equation 17.

$$J = \frac{1}{2} \int_0^{\infty} (\tilde{x}^k \tilde{x}^T P_m \tilde{x} + \tilde{x}^T (Q_m + C^T K^T R_m K C) \tilde{x}) dt \quad \text{Or } J = \frac{1}{2} \text{tr}(P_k X) \quad (17)$$

where,

$$X = \bar{x} \bar{x}^T = A_c^{-1} B_c r_0 r_0^T B_c^T A_c^{-T}, \quad A_c = A - BKC, \quad B_c = G - BKF$$

P_m is the solution for Lyapunov equation for optimal control shown in Equation 18.

$$\begin{aligned} 0 &\equiv A_c^T P_0 + P_0 A_c + P_m \\ 0 &\equiv A_c^T P_1 + P_1 A_c + P_0 \\ &\vdots \\ 0 &\equiv A_c^T P_{k-1} + P_{k-1} A_c + P_{k-2} \\ 0 &\equiv A_c^T P_k + P_k A_c + k! P_{k-1} + Q_m + C^T K^T R_m K C \end{aligned} \quad (18)$$

By observing the SAS presented in Figure 2, the integrator makes the whole system become type 1 control system. The natural performance index indicates the performance requirement that minimize the tracking error without huge amount of control effort [5]. Hence, the whole tuning parameter could now be simplified into single weighting value [2, 3]. This could be done by assigning the parameters into $P_m = H^T H$, $Q_m = 0$, $R_m = \rho$. Whereas the ρ term is the only tuning parameter left.

The LQ optimal control is performed with an initial guess on the feedback matrix, K and after few iterations, the cost function value, J converges into a solution that indicates the minimum performance index and results in the most appropriate controller gains. Apparently, following the LQ optimal design procedure and suitable tuning parameter, ρ , the response for pitch rate can be simulated using step input function. The result is presented in Figure 3. By checking the eigenvalues, it is found out that with the aid of the LQ optimal control, the short period poles shift to $-1.833 \pm 1.7398i$ and this gives a response of $\omega_n = 2.523$, $\zeta = 0.725$ that satisfies the Level 1 flying quality [19]. It is essential to highlight the advantages of using LQ optimal control. The search for the most suitable controller gains is all done by computer, which highly reduced human work load. Besides, this method enables to cope with the ad-hoc nature of fundamental trial and error gain tuning method. According to Figure 3, the LQ optimal control showed outstanding response, which avoided overshoot and settled at time less than 3 seconds.

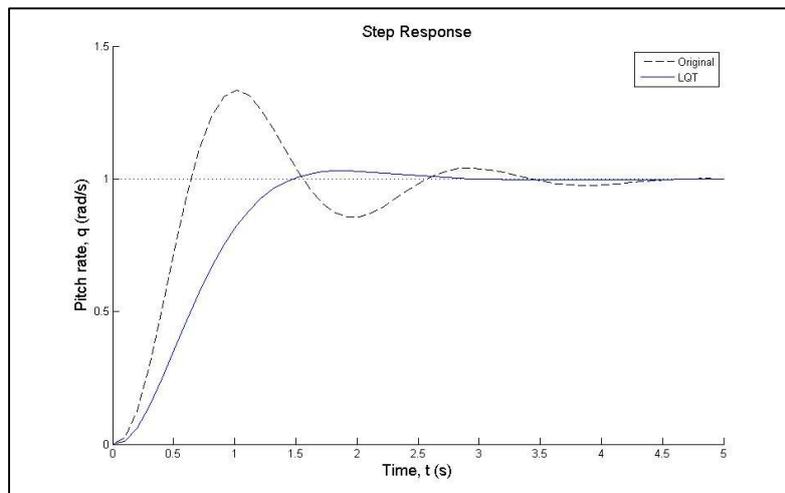


Figure 3. Pitch rate step response at airspeed of 300 ft/s at sea level.

5. Fuzzy gain scheduling

A series of linear systems are taken into examination and LQ optimal control is performed. Each linear controller design results in three controller gains, K_α , K_q and K_i . The results for each controller gains are fed into the FGS. Fundamental fuzzy controller is used to schedule the controller gains and the fuzzy controller design is well discussed in [20]. Design of fuzzy system is based on Model Rules, R^i .

IF $x_i(t)$ is $M_k(x_i)$ **AND** $x_j(t)$ is $M_l(x_j)$, **THEN** $K = \sum \alpha_i k_i$

where R^i is the i^{th} rule of the fuzzy system, $x_i(t)$ and $x_j(t)$ are the current inputs or states fed into the fuzzy system, M_k and M_l are the k^{th} and l^{th} membership function subjected to inputs $x_i(t)$ and $x_j(t)$ respectively, k_i is the controller gain with respect to the input membership function, α_i is the weighting function.

The fuzzy system consists of four main components [20] and each component is described in detailed. First, the “rule-base” definition is determined by subjecting reference inputs to the specific outputs. Second, the inference mechanism acts as the decision maker based on the inputs parameter. Third, the fuzzification interface processes the given inputs into information for inference mechanism. The last components describe the defuzzification interface, which gives the conclusions based on the results of inference mechanism. The fuzzification inference utilises triangular membership functions for both inputs and it is shown in Figure 4. There will be only one rule involves in the whole process, which indicates four input membership functions and concludes using the **OR** defuzzification method. The actual corresponding output is weighted using the area rule given in Equation 19.

$$K = \frac{\sum_{i=1}^4 Ar_i c_i}{\sum_{i=1}^4 Ar_i} \quad (19)$$

where K is the output controller gain, Ar_i is the i^{th} triangular area of output membership function and c_i is the centre of i^{th} output membership function.

21 linear systems are taken into consideration and these cover airspeeds for 300, 250, 400, 440, 500, 540 and 600 ft/s, and altitude for 0, 10000 and 20000 ft. The outputs from the fuzzy system are tabulated graphically and displayed as shown in Figure 5.

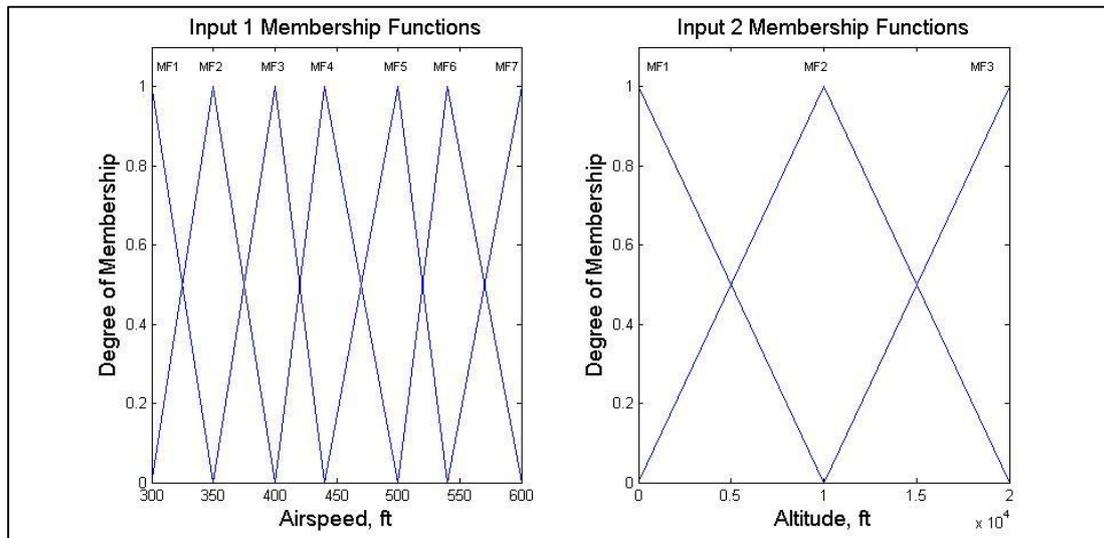


Figure 4. Airspeed (left) and altitude (right) membership function

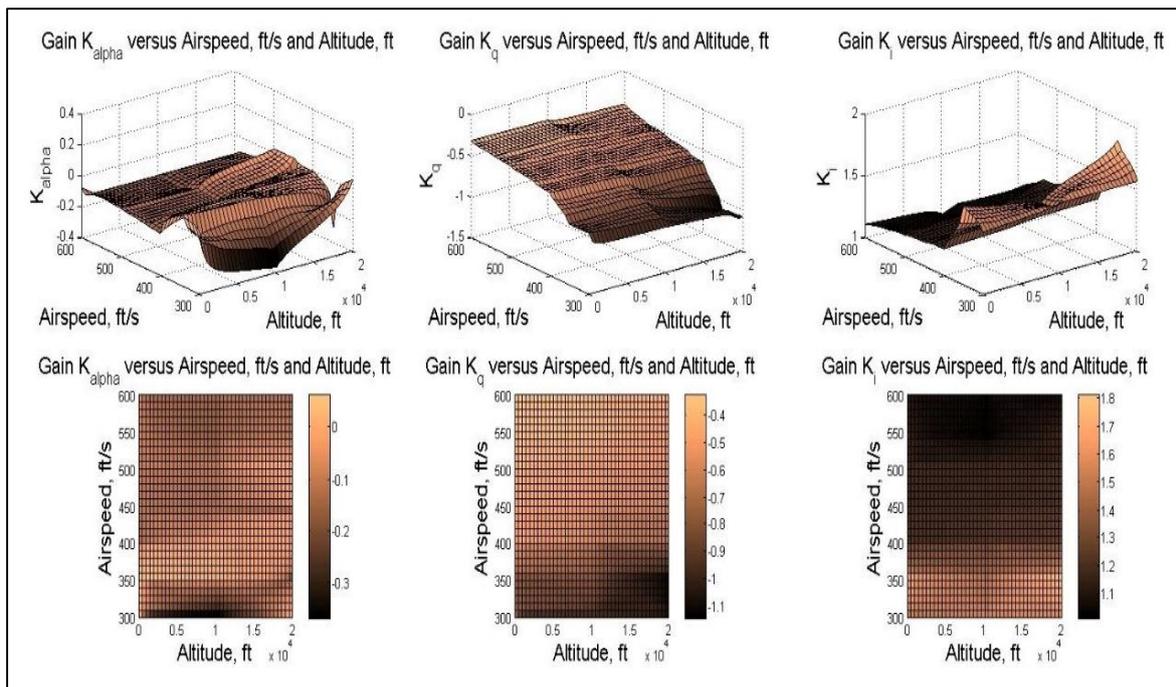


Figure 5. Controller gains K_{α} , K_q and K_i versus airspeed and altitude

A simple simulation on pitch rate stability augmented system is conducted by randomly selecting the flying condition. Two sets of controller gain (output from FGS and pre-set controller gain) are applied and compared. Based on the simulation work on flying condition at airspeed of 375 ft/s and altitude at 5000 ft above sea level, the step response is presented in Figure 6. The response shows non-oscillating and non-overshooting results as compared to pre-set controller gain. Besides, by checking on the eigenvalues, the short period poles are located at $-2.115 \pm 1.913i$ and this gives a response of $\omega_n = 2.852, \zeta = 0.742$ that also satisfies Level 1 flying quality. It is vital to emphasize that the LQ optimal control serve as the effective controller gain finder at a particular operating point whereas the fuzzy gain scheduling serve as the global control system for a wide range of operating range.

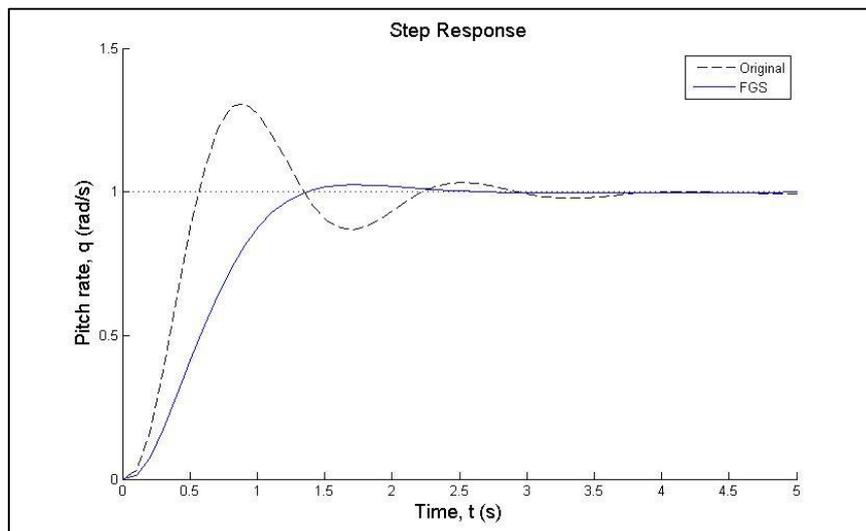


Figure 6. Pitch rate step response at airspeed of 375 ft/s and altitude of 5000 ft above sea level

6. Conclusion

In conclusion, the FGS is designed and established in longitudinal flight control system. The nonlinear control for longitudinal motion is approximated using 21 linearized models and the selection of the linearized system is based on the highly nonlinear region where the eigenvalues show irregular pattern. Using LQ optimal control, the tuning parameter is simplified into a single weighting value, ρ . Besides, this method presents the most suitable controller gain following the optimal cost function and in the meantime reduces the time required for controller gain tuning. The controller gains are determined and fed into FGS. Two inputs: airspeed and altitude are required to schedule the specific controller gains K_α , K_q and K_i . The computer simulation demonstrates the effectiveness in using FGS and the capability in achieving Level 1 flying qualities at all time.

Acknowledgments

The authors wish to acknowledge the Department of Aeronautics, Automotive and Ocean Engineering, Universiti Teknologi Malaysia for supporting in this research. In addition, the authors also would like to present their gratitude to Universiti Teknologi Malaysia.

References

- [1] Nelson R C 1998 *Flight Stability and Automatic Control* (New York: McGraw-Hill)
- [2] Lewis F L and Syrmos V L 1995 *Optimal Control* (New York: Wiley)
- [3] Steven B L and Lewis F L 1992 *Aircraft control and simulation* (New York: Wiley)
- [4] Almutairi S H and Aouf N 2014 *Proc. of the 11th IEEE International*
- [5] Bai Y, Zhuang H and Wang D 2006 *Advanced Fuzzy Logic Technologies in Industrial Applications* (London: Springer) 207-21
- [6] Rugh W J and Shamma J S 2000 *Automatica* **36** 1401-25
- [7] Leith D J and Leithead W E 2000 *Int. J. of control* **73** 1001-25
- [8] Nichols R A, Reichert R T and Rugh W J 1993 *IEEE Trans. on control systems technology* **1** 67-79
- [9] Ravanbod K M and Noll D 2012 *7th IFAC Symp. on Robust Control Design*
- [10] Choi T, Jimenez H and Mavris D N 2015 *J. Aerospace Science and Technology* **46** 493-505
- [11] Oosterom M and Babuska R 2006 *Control Engineering Practice* **14** 769-81
- [12] Takagi T and Sugeno M 1985 *IEEE Trans. on system, man and cybernetics* **15** 116-32
- [13] Takana K and Sugeno M 1992 *Fuzzy Sets System* **45** 135-56
- [14] Gonsalves P G and Zacharias G L 1994 *Proc. of the 3rd IEEE conf. on Fuzzy System*

- [15] Fujimori A, Wu Z, Nikiforuk P N and Gupta M M 1997 *Proc. of AIAA Guidance, Navigation and Control Conference*
- [16] Wu S J and Lin C T 2000 *IEEE Trans. on Fuzzy System* **8** 171-85
- [17] Marco A D, Duke E and Nerndt J 2007 *AIAA Modelling and Simulation Technologies Conf.*
- [18] Chapra S and Canale R 2005 *Numerical methods for engineers* (New York: McGraw-Hill)
- [19] MIL-F-8785C1980 Flying qualities of piloted airplanes *Technical report MIL-F-8785C*
- [20] Passino K M and Yurkovich S 1998 *Fuzzy Control* (Menlo Park: Addison Wesley Longman)