

Mixed convection aiding flow in a vertical porous annulus-two temperature model

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Abstract. The effect of convective heat transfer on mixed convection flow in a vertical porous annulus embedded with fluid saturated porous medium for aiding flow is studied. The inner surface of the annular cylinder is heated with constant temperature whereas the outer surface remains at ambient temperature. The governing partial differential equations are solved using Finite Element Method (FEM). It is assumed that the Darcy law is applicable and thermal non-equilibrium TNE exists between solid and fluid phases of porous medium. The aiding flow behavior of heat transfer with respect to Radius ratio R_r , Aspect ratio A_r and Radiation parameter R_d for different values of Peclet number Pe are investigated.

Key words: Mixed convection, Finite Element Method, Thermal non-equilibrium

1. Introduction

The heat transfer in porous medium has become an increasingly important issue during last few decades due to its significance in many engineering and industrial applications such as heat exchangers, thermal insulation of buildings, solar power generation. The insight of the convective heat transfer in porous medium is well described by Nield and Bejan [1], Vafai [2], Ingham and Pop [3]. The significance of the porous annular cylindrical geometry is well known due to its vast applications in industry as well as in the research field. The convective heat transfer in porous medium for different geometries including those of radiation effect, mass transfer is investigated by eminent researchers [4-



24]. Thermal non-equilibrium approach which comparatively yields more accurate result than the thermal equilibrium approach has been reported in many investigations so far [25-33]. Two-dimensional steady mixed convection in a vertical porous layer was investigated numerically by Saeid and Pop [34] using TNE model. Duwairi et al. [35] studied the effects of oscillating plate temperature on transient mixed convection heat transfer from a porous vertical surface embedded in a saturated porous medium with internal heat generation or absorption, using the Galerkin's method. Studies addressing the various aspects of the mixed convection analyses using TNE modelling have been reported by Bera et al. [36] and Manish et al. [37]. Kumari and. Pop [38] studied mixed convection boundary layer flow past a horizontal circular cylinder embedded in a bi-disperse porous medium. In our previous works, the TNE model was applied for mixed convection in vertical cylinder fixed with saturated porous medium [39]. In the present study, the effect of aspect ratio, radiation parameter and radius ratio on mixed convection aiding flow in a vertical porous annulus is considered.

2. Physical Model

Figure 1 depicts the schematic of the geometry under consideration with inner radius r_i and outer radius r_o where r and z represent the radial and vertical directions respectively, of the annulus. The inner wall of the annulus is heated to the constant temperature T_w whereas the outer wall is maintained at the constant temperature T_∞ , such that $T_w > T_\infty$.

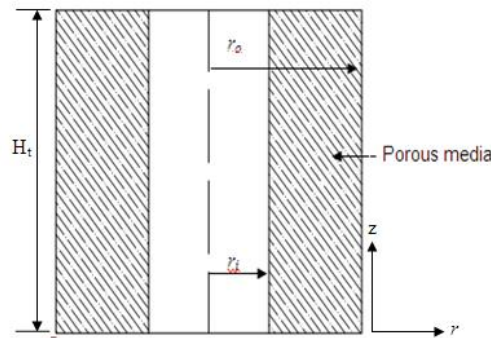


Figure 1. Vertical Annular Cylinder

Following assumptions are applicable [5,13]

- The fluid flow obeys Darcy law
- The fluid and solid matrix of porous medium are in thermal equilibrium
- There is no phase change
- The Fluid properties are constant except the variation of density with temperature.

The heat and fluid flow in porous medium are governed by following non-dimensional form of equations.

Momentum equation

$$\frac{\partial^2 \bar{\psi}}{\partial \bar{z}^2} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{\psi}}{\partial \bar{r}} \right) = \frac{\bar{r} Ra}{Pe} \frac{\partial \bar{T}_f}{\partial \bar{r}} \quad (1)$$

Energy equation of fluid

$$Pe \left[\frac{\partial \bar{\psi}}{\partial \bar{r}} \frac{\partial \bar{T}_f}{\partial \bar{z}} - \frac{\partial \bar{\psi}}{\partial \bar{z}} \frac{\partial \bar{T}_f}{\partial \bar{r}} \right] = \left[\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\left(1 + \frac{4R_d}{3} \right) \bar{r} \frac{\partial \bar{T}_f}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{T}_f}{\partial \bar{z}^2} \right] + H(\bar{T}_s - \bar{T}_f) \quad (2)$$

Energy equation for solid

$$\left(\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\left(1 + \frac{4R_d}{3} \right) \bar{r} \frac{\partial \bar{T}_s}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{T}_s}{\partial \bar{z}^2} \right) = H\gamma(\bar{T}_s - \bar{T}_f) \quad (3)$$

The non-dimensional boundary conditions are

$$\begin{aligned} \text{At } \bar{r} = \bar{r}_i, \quad \bar{\psi} = 1, \bar{T}_f = \bar{T}_s = -\frac{1}{2} \\ \text{At } \bar{r} = \bar{r}_o, \quad \bar{\psi} = 0, \bar{T}_f = \bar{T}_s = -\frac{1}{2} \\ \text{At } \bar{z} = 0, \bar{\psi} = 1 \end{aligned} \quad (4)$$

The heat transfer relations in terms of Nusselt number are

For fluid,

$$\overline{Nu}_f = \frac{1}{\bar{z}} \int_0^{\bar{z}} \frac{\partial \bar{T}_f}{\partial \bar{r}} \bigg|_{\bar{r}=\bar{r}_i, \bar{r}_o} \bar{d}\bar{z} \quad (5)$$

For solid,

$$\overline{Nu}_s = -\frac{1}{\bar{z}} \int_0^{\bar{z}} \left(1 + \frac{4}{3} R_d \right) \frac{\partial \bar{T}_s}{\partial \bar{r}} \bigg|_{\bar{r}=\bar{r}_i, \bar{r}_o} \bar{d}\bar{z} \quad (6)$$

The total Nusselt number is given by the relation.

$$\overline{Nu}_t = \left(\frac{-1}{\gamma + 1} \right) \frac{1}{\bar{z}} \times \int_0^{\bar{z}} \left\{ \gamma \frac{\partial \bar{T}_f}{\partial \bar{r}} \bigg|_{\bar{r}=\bar{r}_i, \bar{r}_o} \bar{d}\bar{z} + \left(1 + \frac{4}{3} R_d \right) \left(\frac{\partial \bar{T}_s}{\partial \bar{r}} \right)_{\bar{r}=\bar{r}_i, \bar{r}_o} \right\} \bar{d}\bar{z} \quad (7)$$

2. Results and Discussion

Finite element method is used to solve the governing partial differential equations 1-3, subjected to boundary conditions 4. The finite element equations are code in an in-house computer code to solve the problem under consideration. Results are presented in terms of Nusselt number that indicates the heat transfer at hot surface. Figure 2 shows the aiding flow variation of average Nusselt number with respect to Rr for different values of Pe with $Ra = 100$, $\gamma = 5$, $H = 10$, $Ar = 5$ and $Rd = 1$. In equation 2-3, the term γ indicates the thermal conductivity ratio between fluid and solid phases whereas H shows the inter-phase convective coupling between solid and fluid phases. It can be observed that \overline{Nu}_f , \overline{Nu}_s and \overline{Nu}_t increase with increase in Peclet number Pe [39] and radius ratio Rr of the vertical annulus. The radius ratio Rr is the ratio of porous thickness to inner radius. The discrepancy in solid and fluid temperature increases at higher radius ratio since there is large difference between the fluid and solid heat transfer rates at higher Rr .

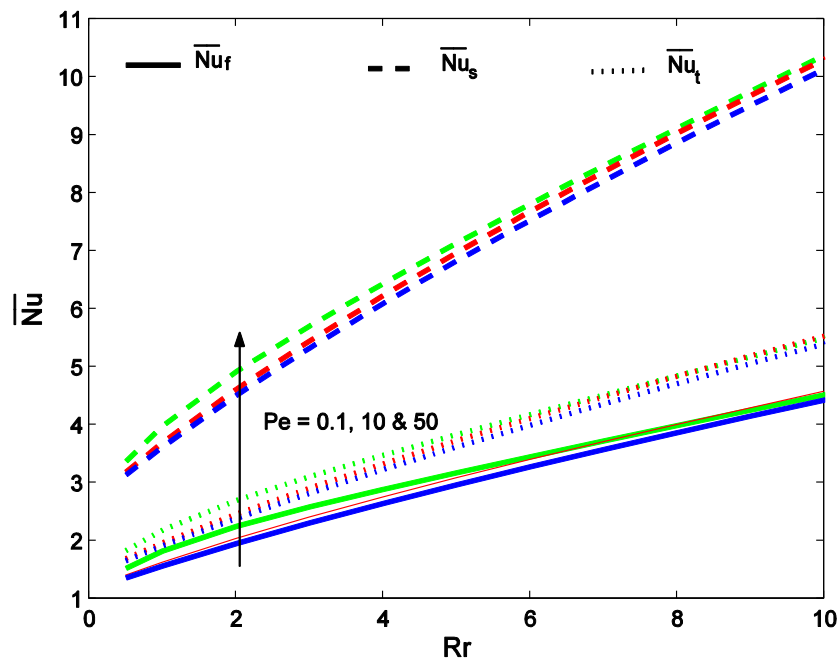


Figure 2: Nusselt number variations with respect to Rr and Pe

Figure 3 shows the variation of \overline{Nu}_s and \overline{Nu}_t with respect to Ar of vertical annulus for aiding flow, for $Ra = 100$, $Rr = 2$, $\gamma = 50$, $H = 50$ and $Rd = 1$. The aspect ratio Ar is the ratio of porous thickness to height of annulus. The graphs are plotted for only two values of Ar , i.e. 1 and 5 beyond which the variations in \overline{Nu}_s and \overline{Nu}_t are so negligible that the lines overlap each other owing to difficulty in distinguish from each other. There is slight increase in the Nusselt number of solid phase when aspect ratio Ar is changed from 1 to 5 but there is no significant change in the fluid and total Nusselt number when Ar is varied from 1 to 5. Figure 4 illustrates the effect of Rd on \overline{Nu}_s and \overline{Nu}_t for aiding flow, for $Ra = 100$, $Rr = 2$, $\gamma = 1$, $Ar = 5$ and $H = 1$. It is observed that \overline{Nu}_s and \overline{Nu}_t increase almost linearly with increase in Rd , and there is not much variation for \overline{Nu}_f . The increase in \overline{Nu}_s is much higher compared to the increase in \overline{Nu}_t for different values of Pe .

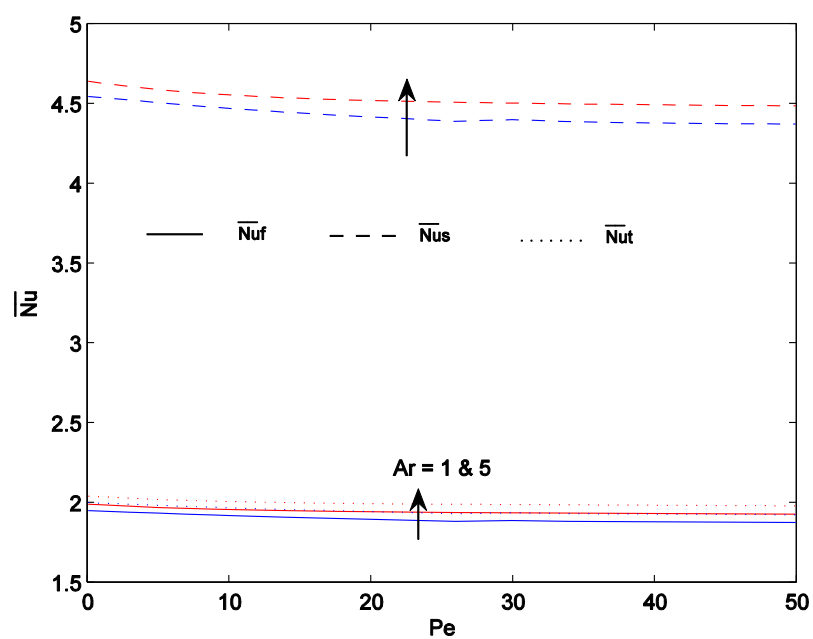


Figure 3: Nusselt number variations with respect to Pe and Ar

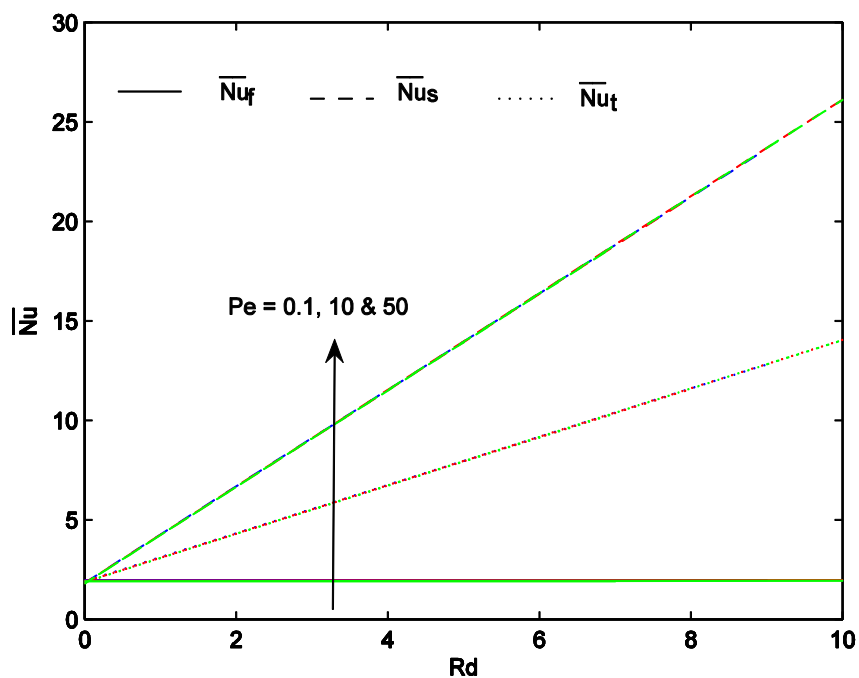


Figure 4: Nusselt number variations with respect to Rd and Pe

4. Conclusion

The current work highlights the aiding flow in an annular porous medium. The governing equations are solved by using finite element method. It is found that the fluid, solid and total Nusselt number increases with increase in Peclet number and radius ratio of the vertical annulus for aiding flow. It is noted that there is slight increase in the Nusselt number of solid phase when aspect ratio is increased. It is further noted that the solid Nusselt number is affected to greater extent when radiation parameter is increased.

5. References

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