Mixed Convection Opposing Flow in a Vertical Porous Annulus-Two Temperature Model

Abdullah A AA Al-Rashed[3], Salman Ahmed N J[1], H M T Khaleed[2], T M Yunus Khan[2,4*], K S NazimAhamed[1]

[1]*Center for Energy Sciences, Dept. of Mechanical Engineering, University of Malaya, Kuala Lumpur, 50603 Malaysia*
[2]*Dept of Mechanical Engineering, Faculty of Engineering, Islamic University, Madinah Munawwarra, Kingdom of Saudi Arabia*
[3]*Dept. of Mechanical Engineering, University of Malaya, Kuala Lumpur, 50603 Malaysia*
[4]*Dept. of Mechanical Engineering, BVB College of Engineering & Technology, Hubli, India*
[1]*Dept. of Civil Engineering, Rural Engineering College, Hulkoti 582205, India*

*Corresponding author: E-mail:**yunus.tatagar@gmail.com*

Abstract. The opposing flow in a porous medium refers to a condition when the forcing velocity flows in opposite direction to thermal buoyancy obstructing the buoyant force. The present research refers to the effect of opposing flow in a vertical porous annulus embedded with fluid saturated porous medium. The thermal non-equilibrium approach with Darcy modal is considered. The boundary conditions are such that the inner radius is heated with constant temperature T_w the outer radius is maintained at constant temperature T_c. The coupled nonlinear partial differential equations such as momentum equation, energy equation for fluid and energy equation for solid are solved using the finite element method. The opposing flow variation of average Nusselt number with respect to radius ratio *Rr,* Aspect ratio*Ar and* Radiation parameter R_d for different values of Peclet number *Pe* are investigated. It is found that the flow behavior is quite different from that of aiding flow.

Key words: Mixed convection, Porous medium, Finite Element Method, Thermal non-equilibrium,

1. Introduction

The mixed convection theory has gained significant attention from the researchers due to its occurrences in wide variety of natural as well as artificial processes which accounts for many heat transfer industrial applications such as heat exchangers, thermal insulation of buildings, solar power generation. Thedetails of the different aspects of the convection and transport phenomenon are well documented in the books [1-5]. The significance of the porous annular cylindrical geometry is well known due to its vast applications in industry as well as in the research field. The convective heat transfer analyses in the porous medium along with its effect on radiation, viscous dissipation, masstransferin different geometries by eminent researchers are discussed in details [6-25]. Thermal non-equilibrium approach which comparatively yields more accurate result than the thermal equilibrium approach has been reported in many investigations so far [26-37]. Two-dimensional steady mixed convection in a vertical porous layer was investigated numerically by Saeid and Pop [38] using TNE model.Duwairi et al. [39] studied the effects of oscillating plate temperature on transient mixed convection heat transfer from a porous vertical surface embedded in a saturated porous medium with internal heat generation or absorption, using the Galerkin's method using FEM. Studies addressing the various aspects of the mixed convection analyses using TNE modelling have been reported by Bera et al. [40] and Manish et al. [41]. Kumari and. Pop [42] studied mixed convection boundary layer flow past a horizontal circular cylinder embedded in a bidisperse porous medium. In our previous works, the TNE model was applied for mixed convection in vertical cylinder fixed with saturated porous medium [43]. In the present study, the Effect of aspect ratio, radiation parameter and radius ratio on mixed convection flow in a vertical porous annulus is considered.

2. Physical Model

Consider a porous medium fixed between inner and radii of an annulus as shown in Figure1. The inner wall of the annulus is heated to the constant temperature T_w whereas the outer wall is maintained at the constant temperature T_∞, such that $T_w > T_\infty$. The medium is subjected to mixed convection with opposing flow i.e. the external fluid is made to flow from top of cylinder towards the bottom section

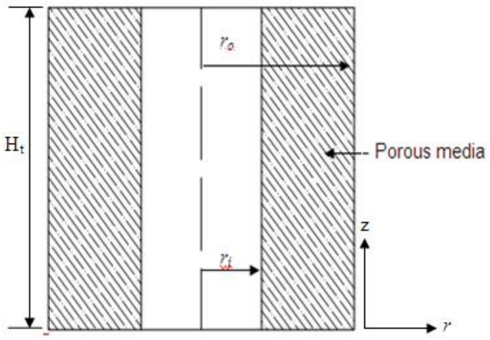

Figure 1.Vertical Annular Cylinder

Following assumptions are applicable [7, 15]

- The fluid flow obeys Darcy law
- The fluid and solid matrix of porous medium are in thermal equilibrium
- There is no phase change
- The fluid properties are constant except the variation of density with temperature.

The relevant governing equations for mixed convention in annulus are given as:

Momentum equation

$$\frac{\partial^2 \overline{\psi}}{\partial \overline{z}^2} + \frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\frac{\partial \overline{\psi}}{\partial \overline{r}}\right) = \frac{\overline{r}Ra}{Pe}\frac{\partial \overline{T_f}}{\partial \overline{r}} \quad (1)$$

Energy equation for fluid

$$Pe\left(\frac{\partial \overline{\psi}}{\partial \overline{r}}\frac{\partial \overline{T_f}}{\partial \overline{z}} - \frac{\partial \overline{\psi}}{\partial \overline{z}}\frac{\partial \overline{T_f}}{\partial \overline{r}}\right) = \left(\frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\left(1+\frac{4R_d}{3}\right)\overline{r}\frac{\partial \overline{T_f}}{\partial \overline{r}}\right) + \frac{\partial^2 \overline{T_f}}{\partial \overline{z}^2}\right) + H\left(\overline{T}_s - \overline{T}_f\right) \quad (2)$$

Energy equation for solid

$$\left(\frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}\left(\left(1+\frac{4R_d}{3}\right)\overline{r}\frac{\partial \overline{T_s}}{\partial \overline{r}}\right) + \frac{\partial^2 \overline{T_s}}{\partial \overline{z}^2}\right) = H\gamma\left(\overline{T}_s - \overline{T}_f\right) \quad (3)$$

The various non-dimensional parameters used are:

$$\bar{r} = \frac{r}{L_{ref}} \;,\; \bar{z} = \frac{z}{L_{ref}} \;,\; \bar{T} = \frac{(T - T_o)}{(T_w - T_\infty)} \;,\; where \;\; T = \frac{(T_w - T_\infty)}{2},$$

Stream Function $\bar{\psi} = \frac{\psi}{\alpha \phi L_{ref}}$,

Inter-phase heat transfer coefficient $H = \frac{h L_{ref}}{\phi k_f}$

Radiation parameter $R_d = \frac{4\sigma n^2 T^3_\infty}{\beta_R K_s}$

Thermal conductivity ratio $\gamma = \frac{\phi k_f}{(1-\phi) k_s}$, (4)

Peclet number $Pe = \frac{V_o L_{ref}}{\phi \alpha}$,

Rayleigh number $Ra = \frac{g\beta \, \Delta T K L_{ref}}{\nu \alpha}$

The boundary conditions are

At $\bar{r} = \bar{r}_i$, $\bar{\psi} = 0$, $\bar{T}_f = \bar{T}_s = \frac{1}{2}$,

At $\bar{r} = \bar{r}_o$, $\bar{\psi} = 0$, $\bar{T}_f = \bar{T}_s = -\frac{1}{2}$ (5)

At $\bar{z} = Ar$ $\bar{\psi} = -1$

The Nusselt number is calculated according to:
For fluid,

$$\overline{Nu}_f = \frac{1}{\bar{z}} \int_0^{\bar{z}} \frac{\partial T_f}{\partial \bar{r}} \bigg|_{\bar{r}=\bar{ri},\bar{ro}} \overline{dz} \tag{6}$$

For solid,

$$\overline{Nu}_s = -\frac{1}{\bar{z}} \int_0^{\bar{z}} (1 + \frac{4}{3} Rd) \frac{\partial T_s}{\partial \bar{r}} \bigg|_{\bar{r}=\bar{ri},\bar{ro}} \overline{dz} \tag{7}$$

The total Nusselt number is given by the relation.

$$\overline{Nu}_t = (\frac{-1}{\gamma+1}) \frac{1}{\bar{z}} \times \int_0^{\bar{z}} \left\{ \gamma \frac{\partial \bar{T}_f}{\partial \bar{r}} \bigg|_{\bar{r}=\bar{ri},\bar{ro}} \overline{dz} + (1 + \frac{4}{3} R_d)(\frac{\partial T_s}{\partial \bar{r}})_{\bar{r}=\bar{ri},\bar{ro}} \right\} \overline{dz} \tag{8}$$

3. Results and Discussion

The coupled partial differential equations are solved by using finite element method with the help of triangular elements. A computer code is generated to solve the finite element equations.An iterative process is adopted to arrive at the final solution of $\bar{\psi}, \bar{T}_f \; and \; \bar{T}_s$. Figure 2 depicts the Nusselt number variation of opposing flow for different values of *Rr* and *Pe*, plotted with *Ra* = 100, $\gamma = 5$, *H* = 10, *Ar*= 5and *Rd* = 1.It is obvious that $\overline{Nu_f}, \overline{Nu_s}$ and $\overline{Nu_t}$ increase with increase in *Rr* for different values of *Pe*. The Nusselt number increases with *Rr* as in case of thermal equilibrium model [12]. It is further noted that the solid Nusselt number is higher than that of fluid Nusselt number

Figure 3 shows the variation of $\overline{Nu_s}$ and with respect to *Ar* of vertical annulus for aiding flow, for *Ra* = 100, *Rr* = 2, $\gamma = 50$, *H* = 50 and *Rd* = 1. The aspect ratio *Ar* is the ratio of porous thickness to height of annulus. The graphs are plotted for only two values of *Ar*, i.e.1 and 5 beyond which the variations in $\overline{Nu}_s$ and are so negligible that the lines overlap each other owing to difficulty in distinguish from each other. There is slight increase in the Nusselt number of solid phase when aspect ratio *Ar* is changed from 1 to 5 but there is no significant change in the fluid and total Nusselt number when *Ar* is varied from 1 to 5.Figure 4 illustrates the effect of *Rd* on $\overline{Nu}_s$ and for aiding flow, for *Ra* = 100, *Rr* = 2, $\gamma = 1$, *Ar*= 5and *H* = 1. It is observed that and increase almost linearly with increase in *Rd*, and there is not much variation for . The increase in is much higher compared to the increase in for different values of *Pe*.

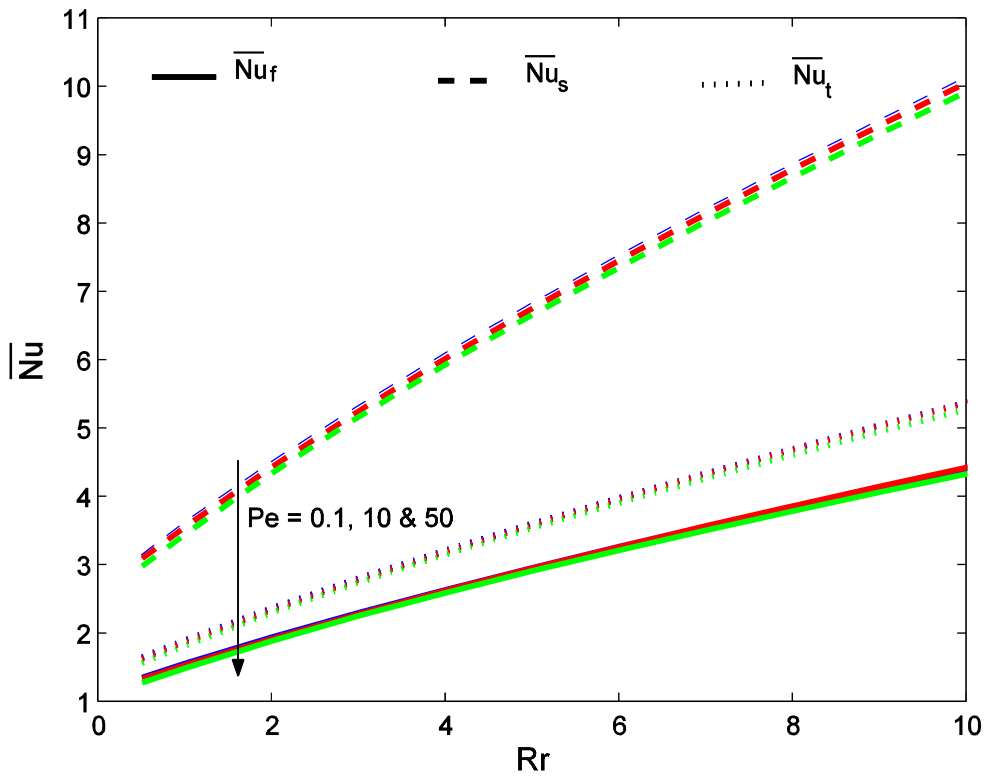

Figure 2: Average Nusselt number for opposing flow with respect to *Rr*and*Pe*

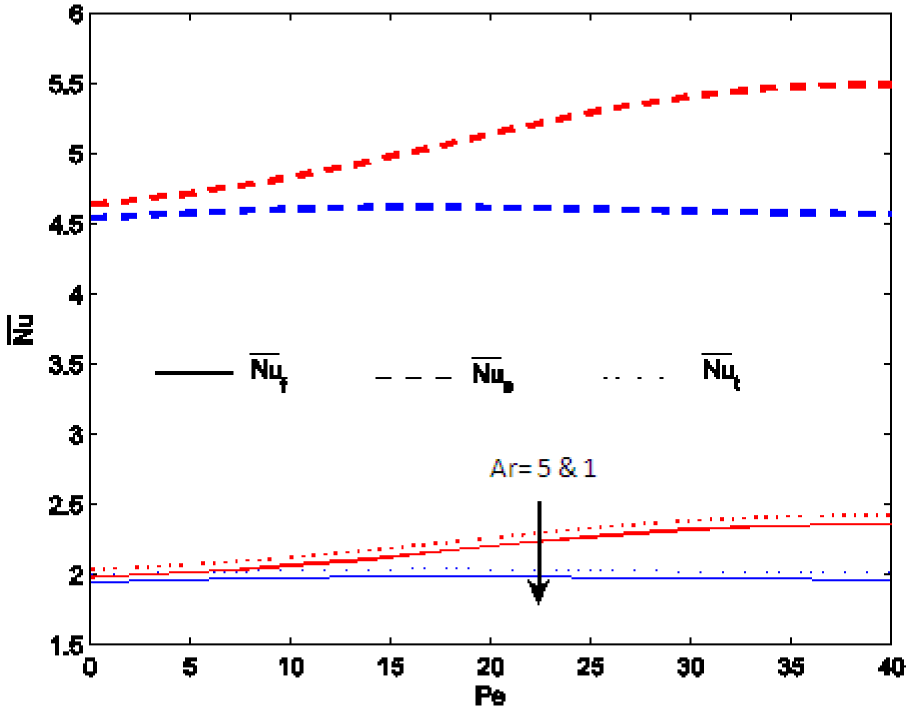

Figure 3: Average Nusselt number variation for opposing flow with respect to *Pe*and*Ar*

4. Conclusion

An opposing flow mixed convection in an annular vertical porous cylinder is investigated with respect to radius ratio, aspect ratio etc. It is found from current work that the Nusselt number increases with increase in radius ratio of annulus. It is further noted that the effect of aspect ratio on Nusselt number is more pronounced at higher Peclet number.

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