

# Static Analysis of Functionally Graded Composite Beams

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**Abstract.** This paper presents a study of functionally graded (FG) composite beam. The FG material for the beam is considered to be composed of different layers of homogeneous material. The fiber volume fraction corresponding to each layer is calculated by considering its variation along the thickness direction ( $z$ ) according to a power law. Accordingly, the effective properties of the homogeneous layers are estimated and a beam composed of this FG material is modelled using the commercially available ANSYS software. The solid 186 layered structural solid element has been used for discretization of the model of the FG beam. The model developed is validated by comparing the results with those numerical results available in literature. Results are presented for simply supported and fixed boundary conditions for the FG beam. The stress distribution across the thickness of the FG composite beam has also been analyzed.

## 1. Introduction

Functionally Graded Materials (FGMs) are a group of inhomogeneous materials composed of two or more materials engineered to have continuously varying material properties along preferred directions [1]. The FG materials are microscopically heterogeneous and are made from mixture of two or more materials that are appropriate to achieve the desired objectives. The overall material properties of the FGMs are unique and different from the individual material that forms it. The mechanical properties that vary continuously in the preferred directions are Young's modulus, Poisson's ratio, Shear Modulus and density etc. Figure 1 shows a beam made of such FG material composed of two materials, ceramic and metal. The top and bottom surfaces of this FG beam are considered ceramic and metal rich, respectively, and the material properties vary across its thickness in a smooth and continuous manner. Such a beam can withstand high temperature gradient across its thickness while maintaining the structural strength and fracture toughness. The ceramic rich surface exposed to the high temperature provides thermal resistance due to its low thermal conductivity while the metallic constituent provides toughness of the plate.



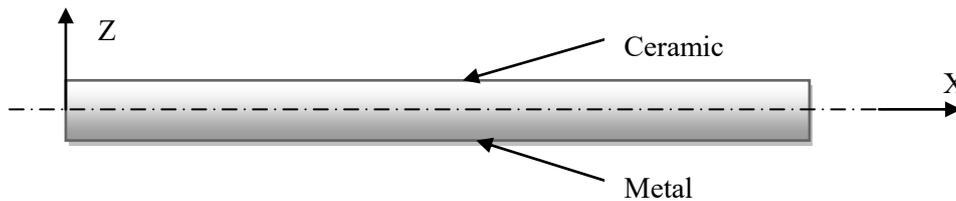


Fig. 1. Schematic representation of FG beam

With an aim to make suitable use of required properties of the materials, these FG materials are getting wide applicability in various branches of Engineering and Technology. Some typical applications of the FG materials include aircraft fuselages in the aerospace industry, rocking-motor casings in the military industry, packaging materials in the microelectronic industry, engine components in the automotive industry, human implants in the biomedical industry, heat exchanger tubes, space craft heat shields, fly wheels, Plasma facings for fusion reactors and so on [2].

In the recent years, researchers developed considerable interest in the analyses of FG beams. For example, kadoli et al. [3] studied the static behavior of FG metal–ceramic beams by using higher order shear deformation theory. They presented numerical results regarding the transverse deflection and axial and shear stresses for a FGM beam under uniform distributed load. Chen et al. [4] studied the elastic buckling and static bending analysis of shear deformable functionally graded porous beams based on the Timoshenko beam theory. The partial differential equation system governing the buckling and bending behavior of porous beams is derived based on the Hamilton's principle. The influence of variation of porosity distributions on the structural performance is highlighted to shed important insights into the porosity design to achieve improved buckling resistance and bending behavior. Li et al. [5] derived an analytical relationship between the bending solutions of FGM Timoshenko beams and the homogenous Euler–Bernoulli beams. Subsequently, they [6] developed a size-dependent functionally graded piezoelectric beam model using a variational formulation. This model contains three material length scale parameters and could capture the size effect. The static bending and free vibration problems of a simply supported beam were numerically solved. Next, Murína et al. [7] established a fourth-order differential equation for the beam deflection with longitudinal variation of the homogenized effective material properties. They achieved Homogenization of the varying material properties of the beams by extended mixture rules and laminate theory. The equilibrium and kinematic equations of the beam have been established.

Papers are also available concerning the analysis of functionally graded plates and shells. Ferreira et al. [8] studied static deformations characteristics of FG square plates of various aspect ratios using meshless collocation method and a third-order shear deformation theory. Effects of aspect ratio of the plate and the volume fractions of the constituents on the centroidal deflection are obtained. It is observed that when Poisson's ratio of the two constituents are nearly equal, then the two homogenization techniques give results that are close to each other. However, for widely varying Poisson's ratio, the two homogenization schemes give quite different results. Fernando et al. [9] studied the static analysis of three-dimensional, anisotropic, elastic plates composed of FG materials using a discrete layer theory in combination with the Ritz method. The method is validated by solving the problem of a single simply supported FGM plate. Zenkour [10] studied the static behavior of a rectangular FG plate under simply supported condition and subjected to uniform transverse load. Abrate et al. [11] investigated static, buckling and free vibration deflections of FG plates by using classical plate theory, FSDT model and HSDT model. The prediction of the ratios between the natural frequencies of FG plates and those of similar plates made out of homogeneous isotropic material are presented. Zenkour and Sobhy [12] studied the thermal buckling of functionally graded material (FGM) sandwich plates using the sinusoidal shear deformation plate theory. The Material properties and thermal expansion coefficient of the sandwich plate faces are assumed to be graded in the thickness direction according to a simple power-law distribution in terms of the volume fractions of the constituents. Bernardo et al. [13] studied the static and free vibrations analyses of FGM plates by

using a package of different methods and models based on the first-order shear deformation theory. They also studied the performance and adequacy of the developed models is carried out through a set of illustrative cases focused on the study of static and free vibrations behaviour of plate structures. Recently, Kumar et al. [14] proposed a new lamination scheme through the design of a graded orthotropic fiber-reinforced composite ply for achieving continuous variations of material properties along the thickness direction of laminated composite plates. The numerical results reveal the changes of laminate-rigidity and maximum values of stresses while achieving the continuous variations of material properties and stresses across the thickness of laminate by mean of aforesaid conversion/lamination scheme.

For solving a variety of problems, finite element softwares like ANSYS, ABACUS etc. are being widely used these days. The problems include static and dynamic structural analysis (both linear and non-linear), heat transfer and fluid flow problems and also acoustic and electromagnetic analysis. Concerning the analysis of different structures by using these packages, research work has already been carried out. Rao et al. [15] investigated the static and dynamic analysis of functionally graded (FG) shell structures using ANSYS under different loading conditions. The responses obtained for FG shells are compared with the homogeneous shells of pure ceramic ( $\text{Al}_2\text{O}_3$ ) and pure metal (steel) shells. Sadowski et al. [16] presented several numerical models of structural parts of airplanes made of different composites. In this analysis, different structures were modeled by using ABACUS software by applying different boundary conditions. The finite element (FE) analysis and the mechanical response of the structural elements were compared with previously formulated simplified analytical models. Huang et al. [17] made the functionally graded  $\text{Al}_2\text{O}_3$ - $\text{ZrO}_2$  composite by using powder metallurgy method. Finite element softwares have also been used to simulate stress distribution and fracture of these composites under impact [18]. Research works are also available on the mathematical modeling and analysis of FG structures [19, 20]. From the review of literature presented here, it seems that analysis of functionally graded composite beams using ANSYS software as well as experimental determination of their bending characteristics are not available, hence the objectives of this paper.

## 2. Development of finite element model

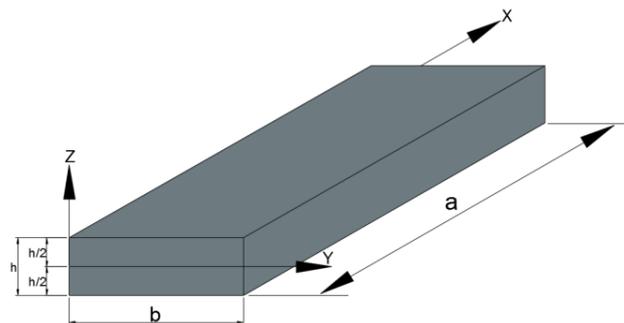


Fig. 2 Functionally Graded Composite Beam

Figure 2 illustrates a Functionally Graded composite beam. The length and the thickness of the beam are denoted as  $a$  and  $h$  respectively, while the width is taken as  $b$ . The mid surface of the beam is considered as the reference plane (Fig. 2) and for estimating the graded elastic properties of the FG material, the concept of homogeneous layers within the FG material proposed by Reiter et al. [19, 20] is followed. Accordingly, the FG material is considered to be composed of different layers of homogeneous material. The fiber volume fraction varies along the thickness direction ( $z$ ) according to a power law and the fiber volume fraction corresponding to each layer is calculated.

Based on the location of a layer within the FG model, the effective material properties of each layer are determined using the Mori-Tanaka method. Following this, for the present model, each layer within the lamina is assumed to be homogeneous and the fiber volume fraction varies for the different layers. The effective properties of the homogeneous layers are estimated considering the

layer as a discrete representative volume element (RVE). The beam shown here (Fig. 2) is considered to be composed of  $(2N+1)$  number of layers where the middle layer is denoted as  $(N+1)$ th layer. Each layer of the lamina is considered to be composed of fiber reinforced in epoxy matrix and it is assumed that the fiber and matrix phase are perfectly bonded. Also, the material properties for both the phases are assumed to be linearly elastic.

The thickness of the fiber is denoted by  $a_f$  while that of the fiber-matrix pack for each layer is denoted by  $a_c$  (Fig. 3a). The volume fractions of fiber and matrix phases within the fiber-matrix pack are denoted by,  $v_f$  and  $v_m$ , respectively and the variation of fiber volume fraction among the various layers is considered according to a simple power law. The maximum value of this volume fraction is obtained in the  $k^{th}$  ( $k=N+1$ ) layer of the lamina while it decreases to a minimum value at both the surfaces in a symmetrical manner. Though the power law provides a continuous variation of fiber volume fraction along the thickness direction, for a particular layer, this is calculated based on the number of layers in the lamina. As all the layers of the lamina are of equal thickness, the variation of fiber volume fraction among the layers is obtained by the variation of fiber thickness which in turn is achieved by varying the volume of the fiber among the layers (Fig. 3b).

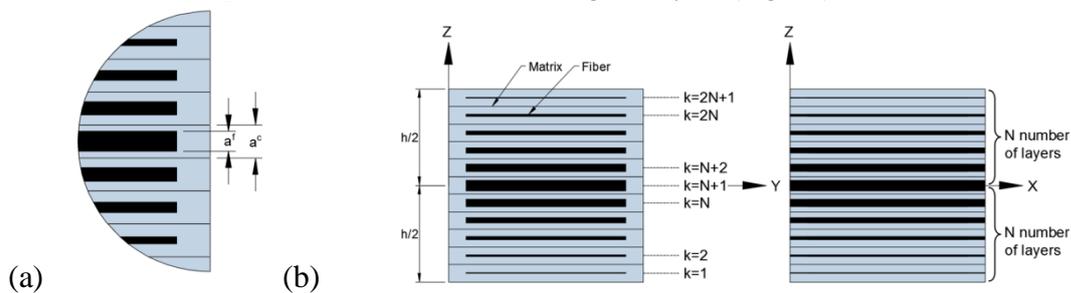


Fig. 3(a) Fiber Matrix Pack (b) Layer-Wise Variation of volume fraction for the present FG material

The FG beam considered here is modeled in the commercially available ANSYS software (ANSYS 15.0). The ANSYS Design modeler was used to generate the FG composite beam model. The beam dimension for the model was considered in accordance with the ASTM standard. Figure 4 explains the 3D beam model in the ANSYS platform for the present analysis.

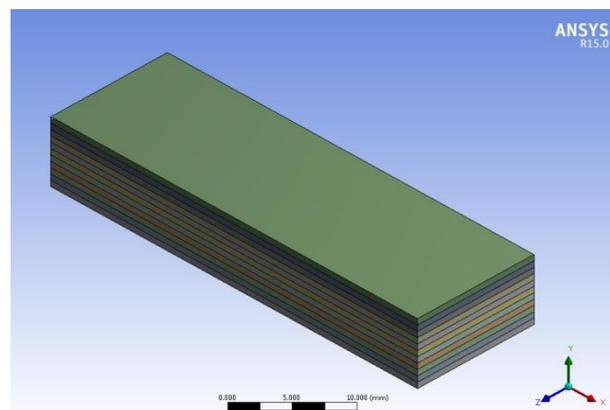


Fig. 4. Representation of the 3D Beam Model

The material properties of the FG beam vary along the thickness direction and accordingly, eleven layers are considered for the model to obtain the material properties. Though the layer structure does not capture the gradual change in material properties but a sufficient number of layers can

reasonably approximate the material gradation. Orthotropic materials properties were considered for these layers according to those computed in the AUTODESK composite software. It may be noted that similar properties were also predicted by Reiter et al. [19] based on the micromechanical analysis. The solid 186 layered structural solid element was used for discretization of the FG beam model as shown in Fig. 5. This Solid 186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior and is defined by 20 nodes each having three degrees of freedom. Hexahedral mesh has been employed for the model and the quality of the mesh was checked by the orthogonal quality of the elements and their skewness.

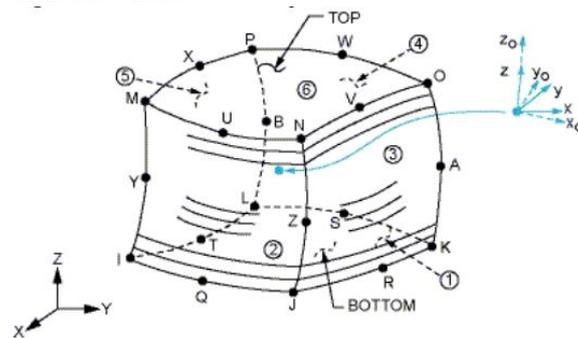


Fig. 5. Representation of Solid 186 Element

### 3. Results and Discussion

In this section, results are presented using the model developed in the previous section. The beam dimensions considered here for presentation of results are according to the ASTM standard and are taken as  $a=100$  mm,  $h=6.93$  mm and  $b=12.7$ mm and unless otherwise mentioned, the loading conditions were assumed to be static in nature. First the beam modeled here in the ANSYS environment is validated using the results available in literature. For this, static structural analysis is adopted and maximum deflections for the beam for various uniformly distributed loading conditions are obtained. Then the load deflection curve is plotted for this FG beam and compared with that presented by earlier researchers [14]. This comparison is explained in Fig. 6 which indicates that the results obtained in both the cases are almost indistinguishable thus validating the present method of modeling of FG beam in the ANSYS environment.

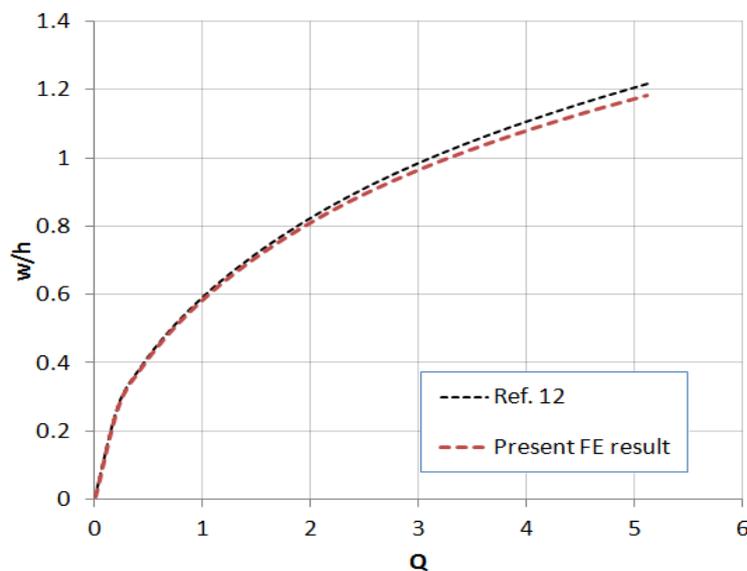


Fig. 6 Comparison of nondimensional transverse deflection at midspan of Present FG beam with Ref. [14]

Figure 7 illustrates the non dimensional load-deflection curve for the classical laminate beam ( $0^0/90^0/0^0$ ) and the present FG laminated composite beam. In both the cases the beam is subjected to uniformly distributed transverse load and it can be observed from Fig. 7 that the FG beam shows lesser rigidity compared to that of the classical beam. Figure 8 shows the load deflection curve for a fixed FG beam and it can be observed that maximum deflection is reduced as compared to those obtained for simply supported beam thus indicating the rigidity of the beam.

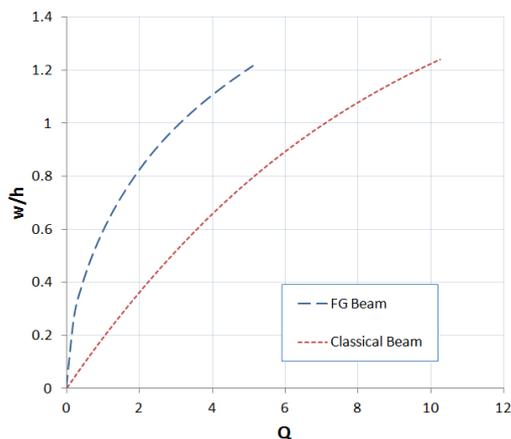


Fig. 7 Nondimensional Load-deflection curve for the FG beam and Classical laminated beam

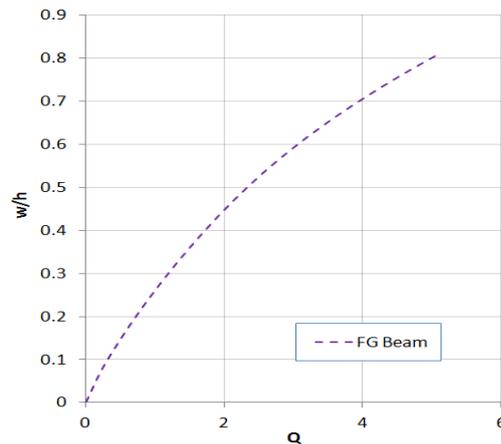


Fig. 8 Nondimensional Load-deflection curve for the Fixed-fixed FG beam

Another aspect of the present study is to investigate the stress distribution across the thickness of the FG beam. The applied load for deformation of the FG composite beam and the classical laminated beam were considered to calculate the non dimensional stresses across the thickness of the FG beam and are explained in Fig. 9. It may be observed from Fig. 9 that the stress distribution of the FG laminated beam is continuously varying while the deviation of the distribution of stress occurs at the inter laminar surfaces of the classical laminated beam is significant. But this sudden deviation in the stress distribution is reduced in case of FG laminated beam.

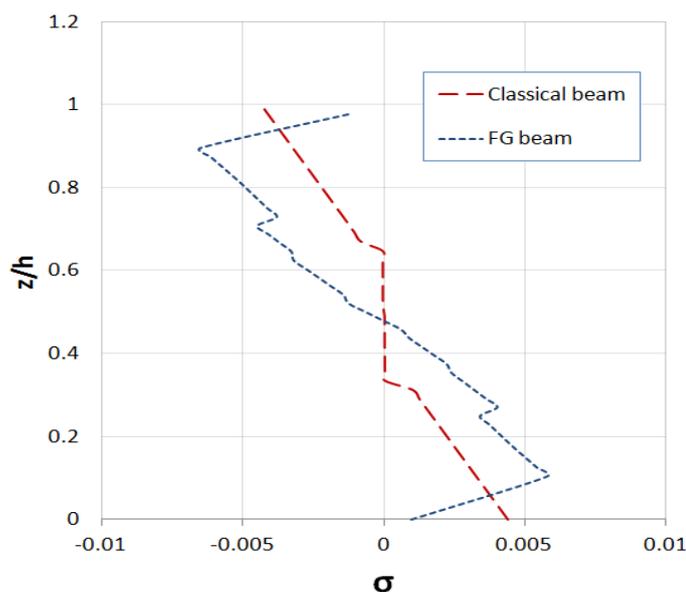


Fig. 9 Stress distribution (nondimensional) across the thickness of the FG composite beam and conventional laminated beam.

#### 4. Conclusion

In this paper, study has been carried out on the modeling of functionally graded (FG) composite beam. The FG material is assumed as of different layers of homogeneous material. The fiber volume fraction corresponding to each layer is calculated by considering its variation along the thickness direction ( $z$ ) according to a power law. Accordingly, the effective properties of the homogeneous layers are estimated and a beam composed of this FG material is modeled in the ANSYS environment. The present model is validated by comparing the results with those numerical results available in literature. Results are presented for simply supported and fixed boundary conditions for the FG beam. The distribution of stress across the thickness indicates a smooth variation of the same compared to those obtained in case of the conventional laminated beams.

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